Network-Aware Distributed Algorithms

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Acknowledgements

- Guanfeng Liang
- Lewis Tseng
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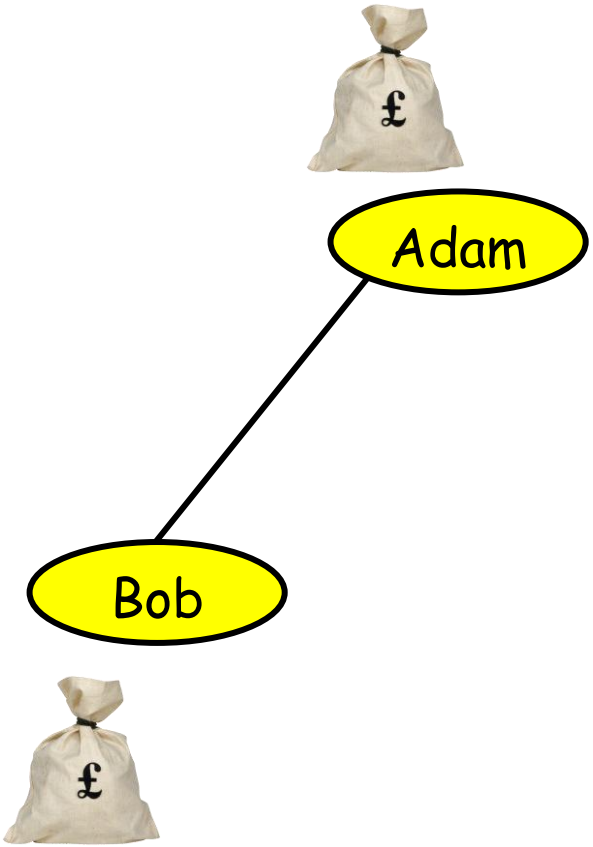
Work supported in part by

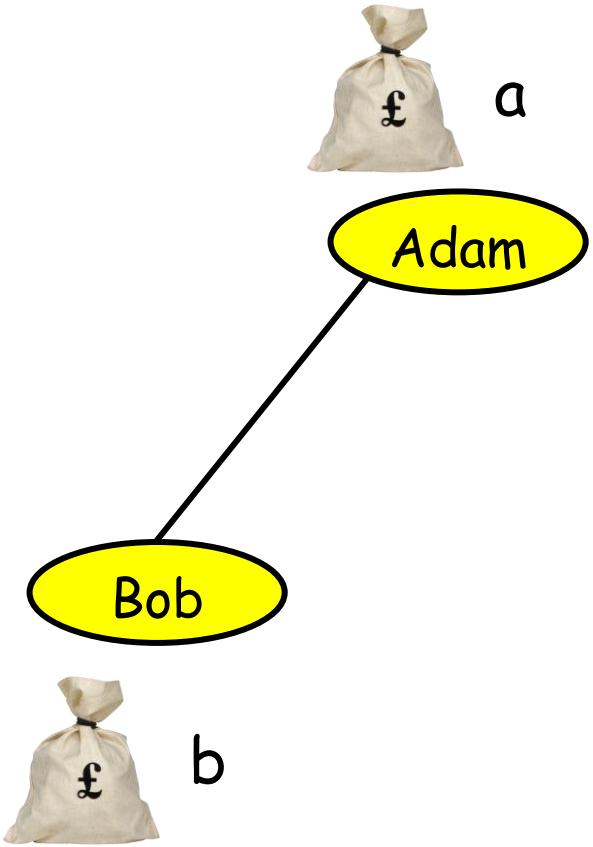
Puzzler

Puzzler



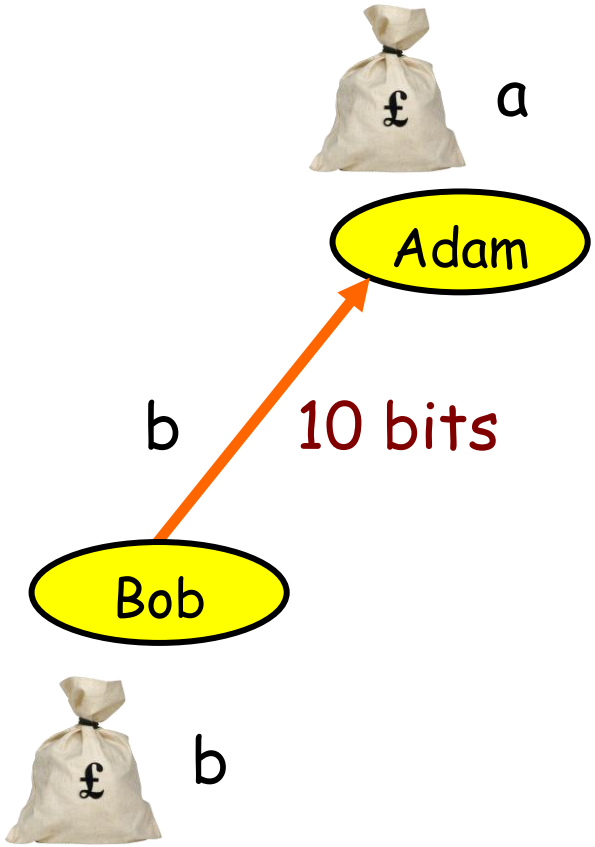
Puzzler

10-bit inputs



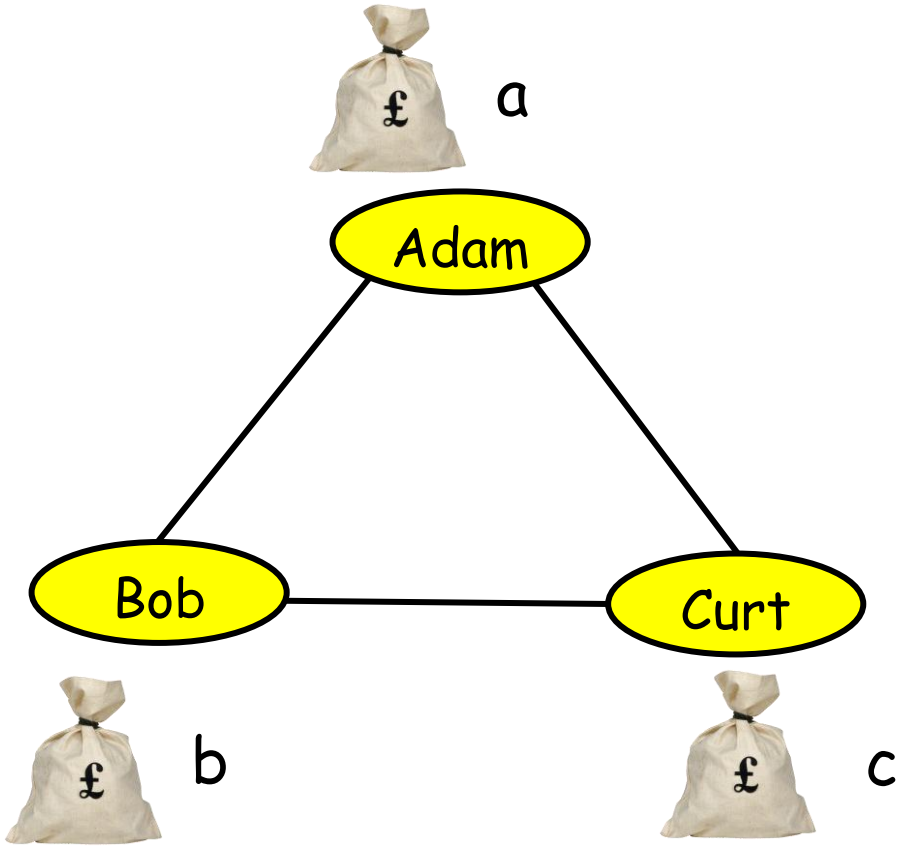
Puzzler

10-bit inputs



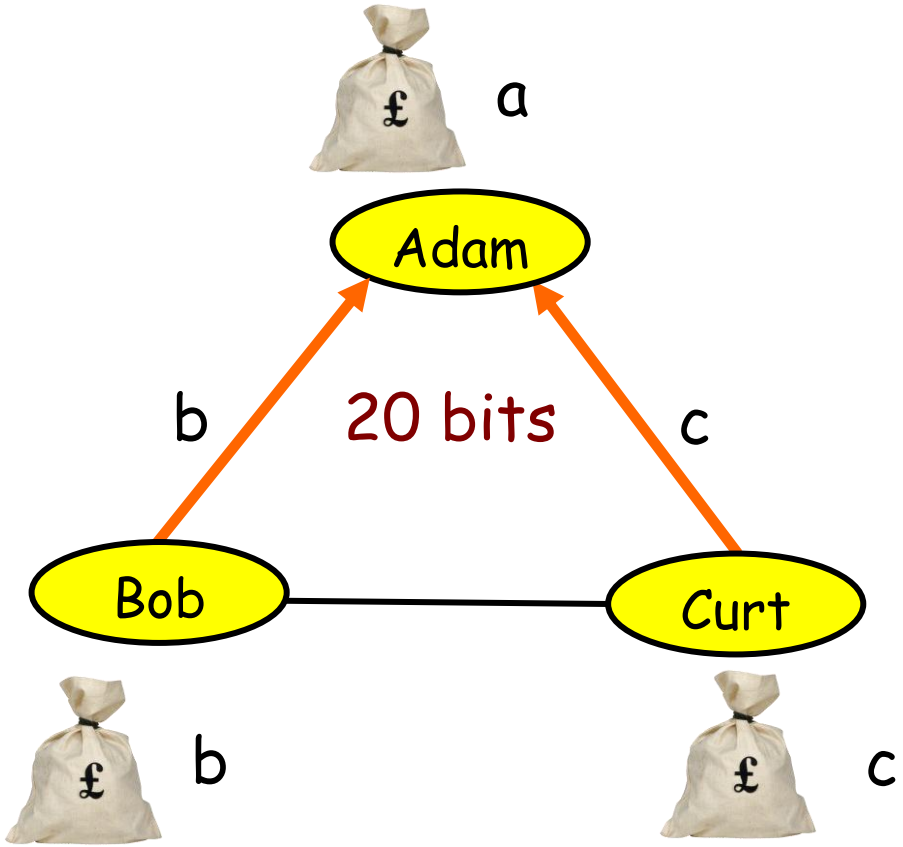
Puzzler

10-bit inputs



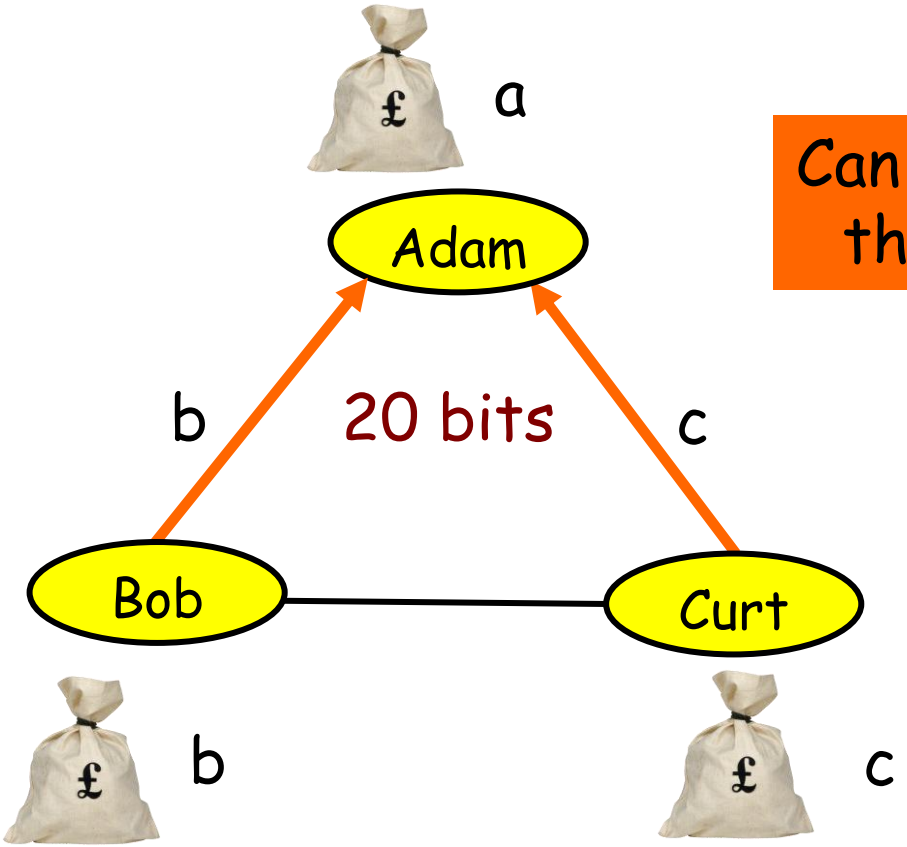
Puzzler

10-bit inputs



Puzzler

10-bit inputs

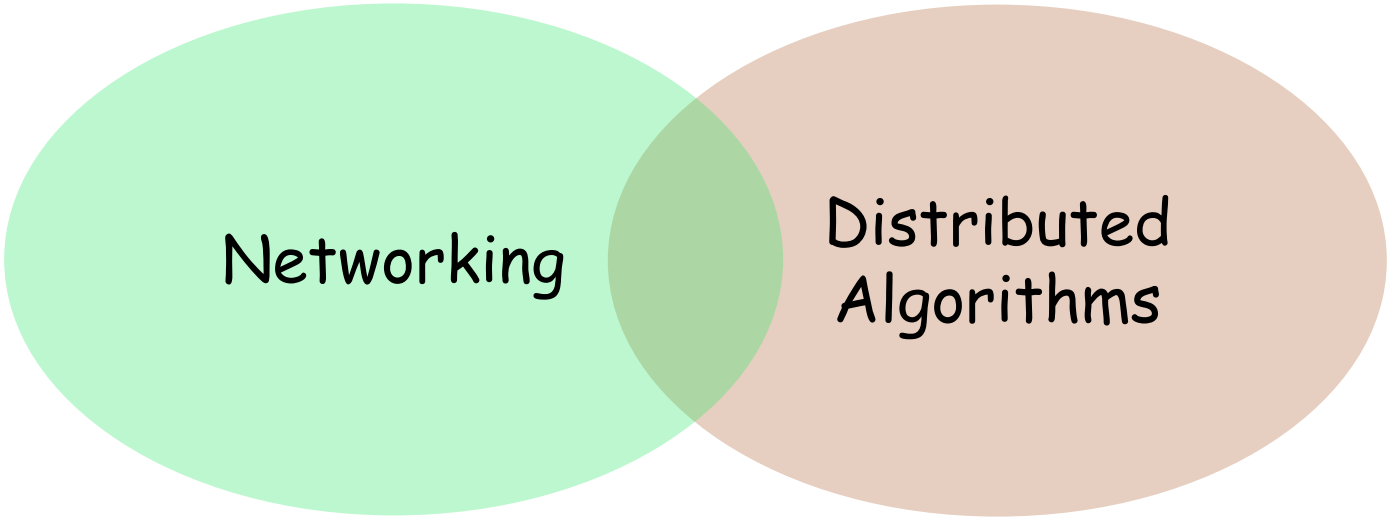


Can we do better than 20 bits ?

Now back to the

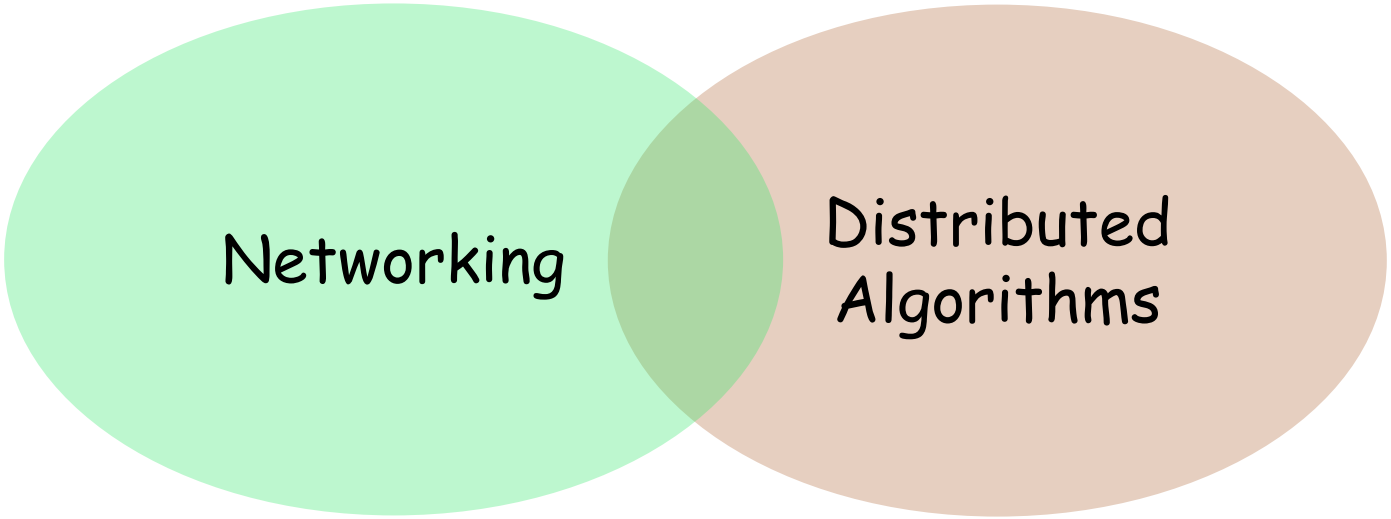
originally scheduled program …

Network-Aware Distributed Algorithms



Distributed Algorithms & Networking

- Problems with overlapping scope
- But cultures differ



Networking

"Accurate" network models

Constants matter

Distributed Algorithms

Simple network models

Emphasis on order complexity

Networking

"Accurate" network models

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Information transfer (typically "raw" info)

Distributed Algorithms

Simple network models

Emphasis on order complexity

Networking

Distributed Algorithms

"Accurate" network models

Constants matter

Information transfer (typically "raw" info)

Simple network models

Emphasis on order complexity

Computation affects communication

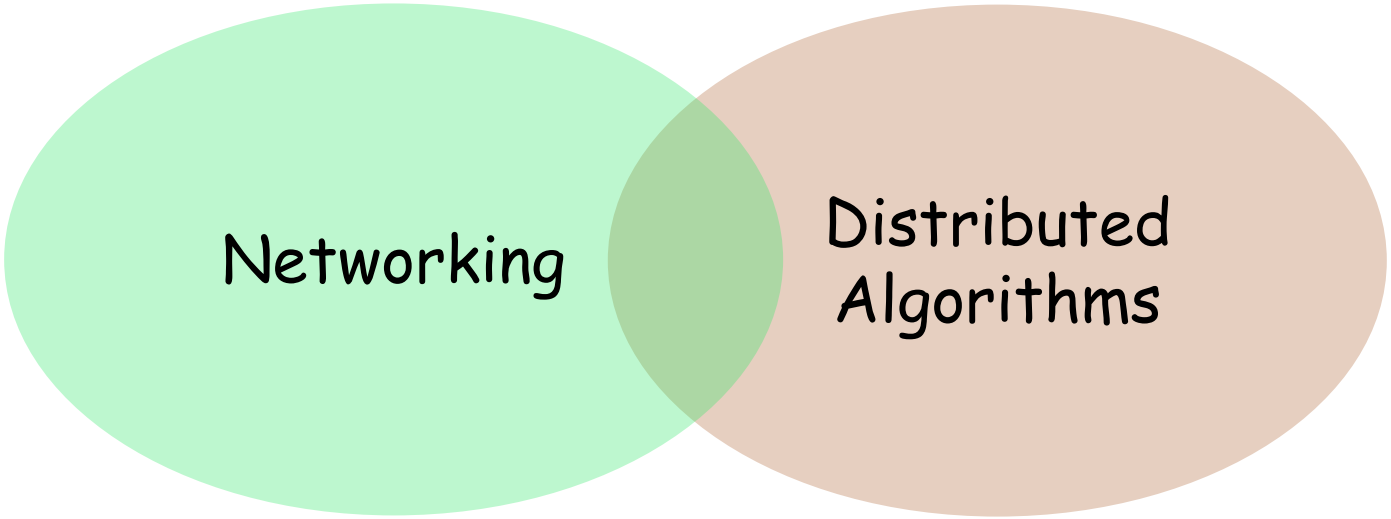
Popular Network Models

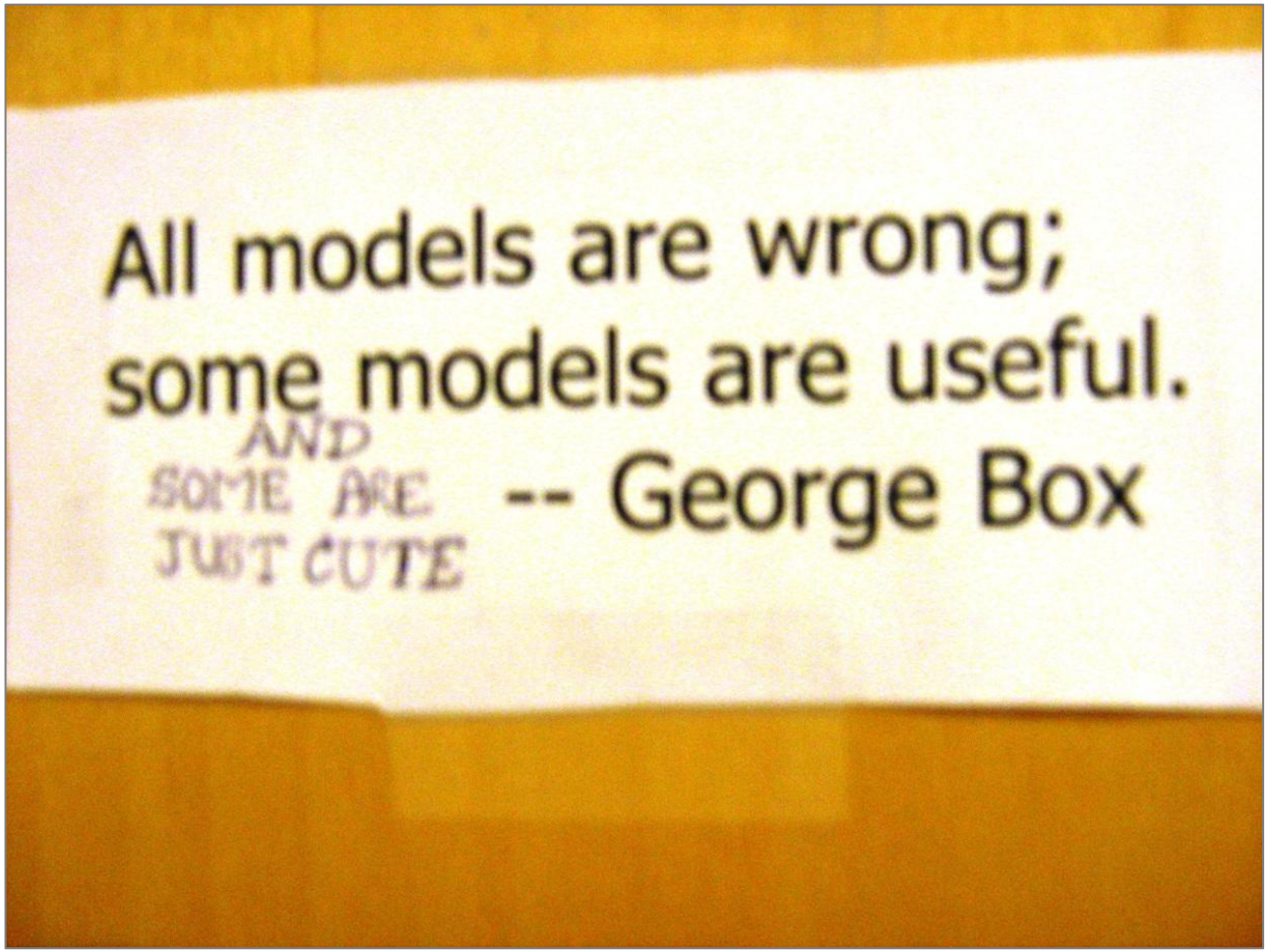
- Point-to-point graphs
- Broadcast channel
- Unit disk graph (wireless broadcast)
- SINR threshold model (wireless interference)

Unsurprising Insight

- "Accurate" network models

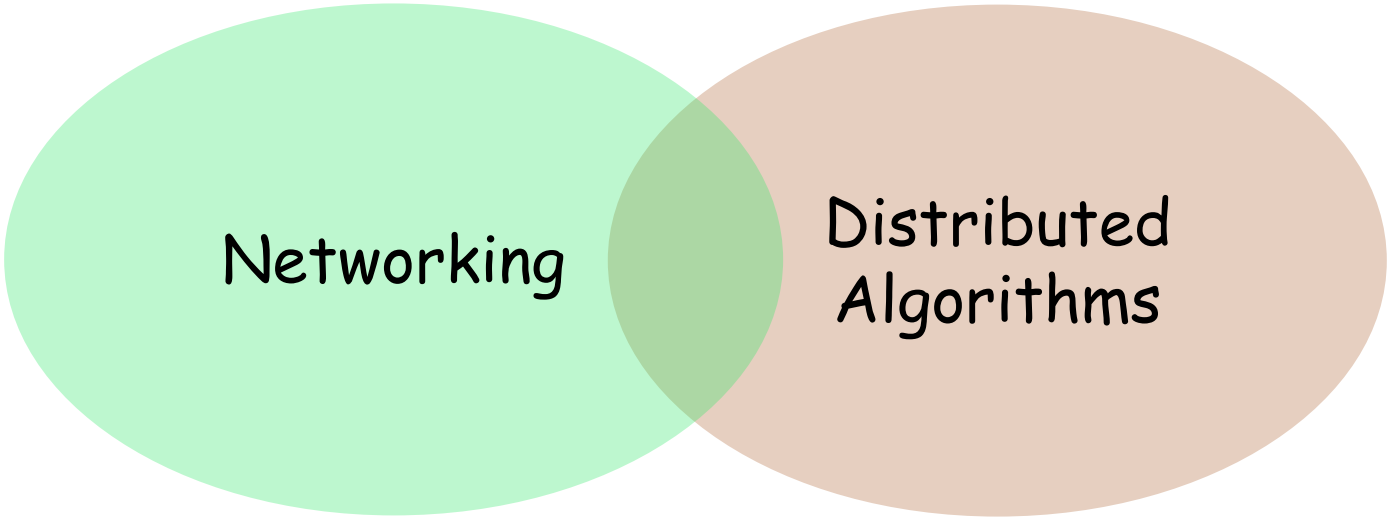
  can lead to more interesting problems




All models are wrong;
some models are useful.
AND
SOME ARE
JUST CUTE
-- George Box

This talk

- Example ... consensus



Consensus

- Multiple parties / agents / nodes
  - Initial input at one or more nodes
- All nodes agree in the end
- Some notion of validity for agreed value

Consensus … Dictionary Definition

- Majority of opinion
- General agreement

Consensus

Consensus

Validity: Decide on 

Consensus

Validity: Decide on ??

Majority rule

Consensus

Validity: Decide on 

Majority rule

Consensus

Validity: Decide on ??

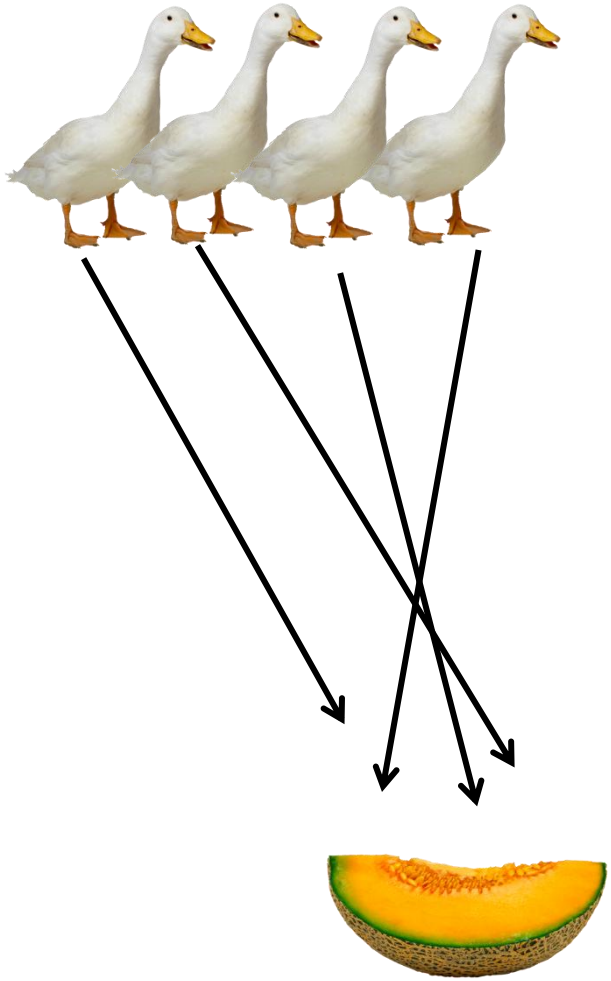
Average consensus

Consensus

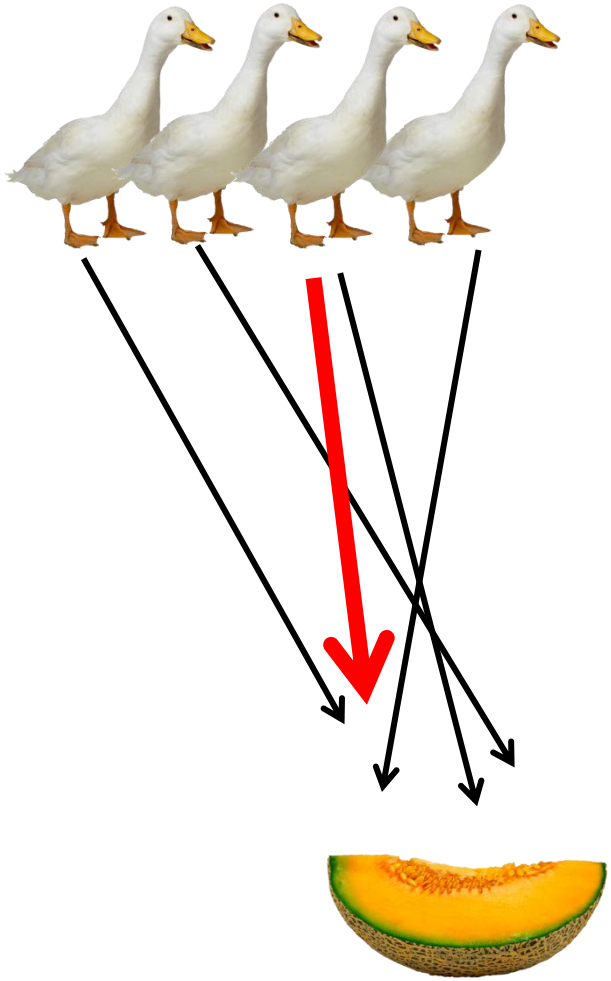
Validity: Decide on 

Average consensus

Flock of Birds (or Robots)



Flock of Birds (or Robots)



Average consensus

Many Faces of Consensus

- All nodes have non-null input / only a subset do
- No failures / failures allowed (node/link)
- Synchronous/asynchronous
- Deterministically correct / probabilistically correct
- Exact agreement / approximate agreement
- Global communication / local communication

Consensus in Practice

- Fault-tolerant file systems
- Fault-tolerant servers
- Distributed control
- Social networks

This talk

- Byzantine broadcast
- Average consensus

This talk

- Byzantine broadcast
- Average consensus

Byzantine Broadcast

Client-Server Model

Client

command A

Server

state 0

Client-Server Model

Client

command A

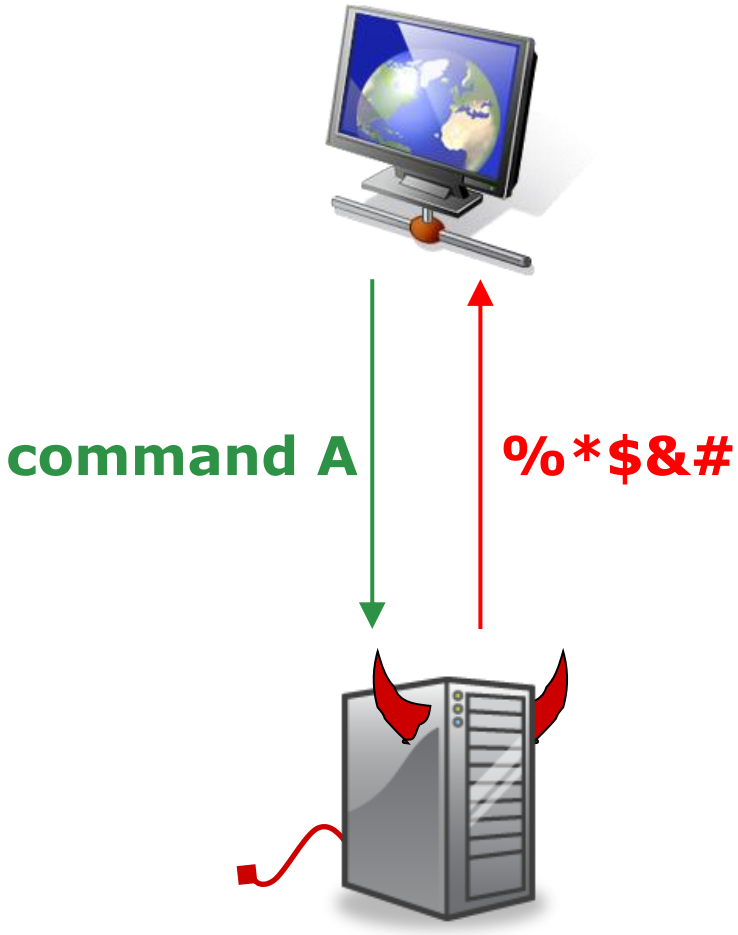
response (A)

Server

state A

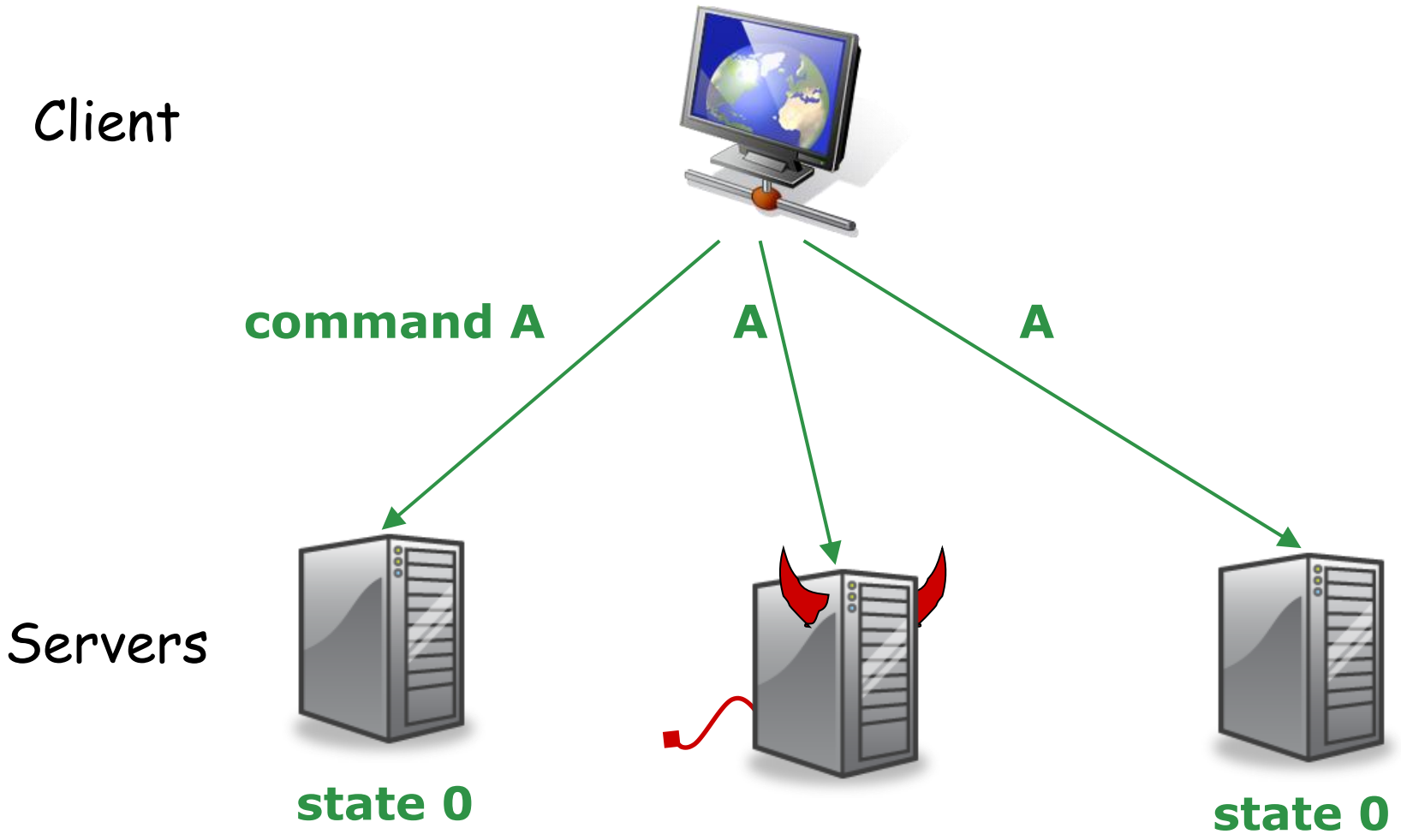
Client-Server Model

Client

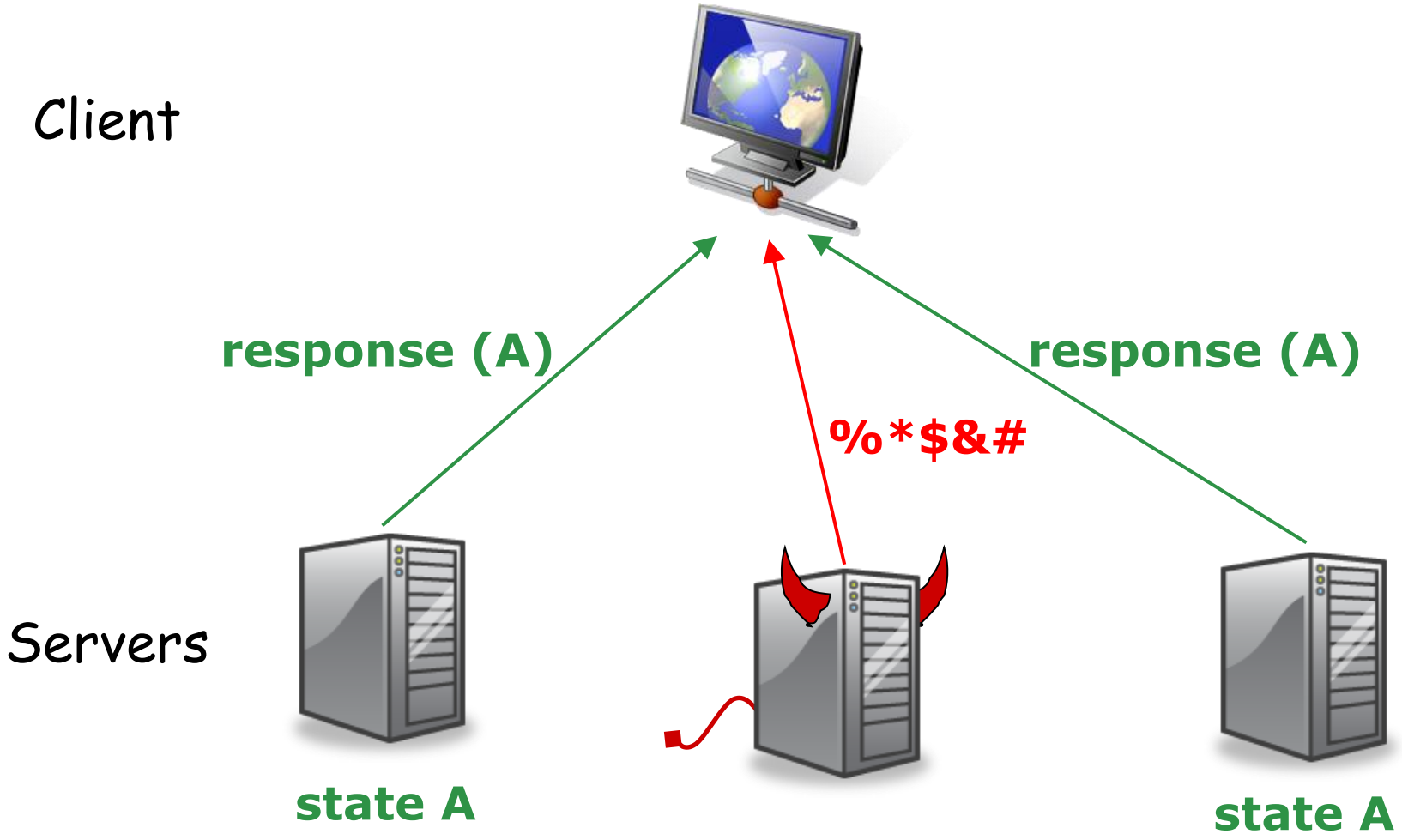


Server

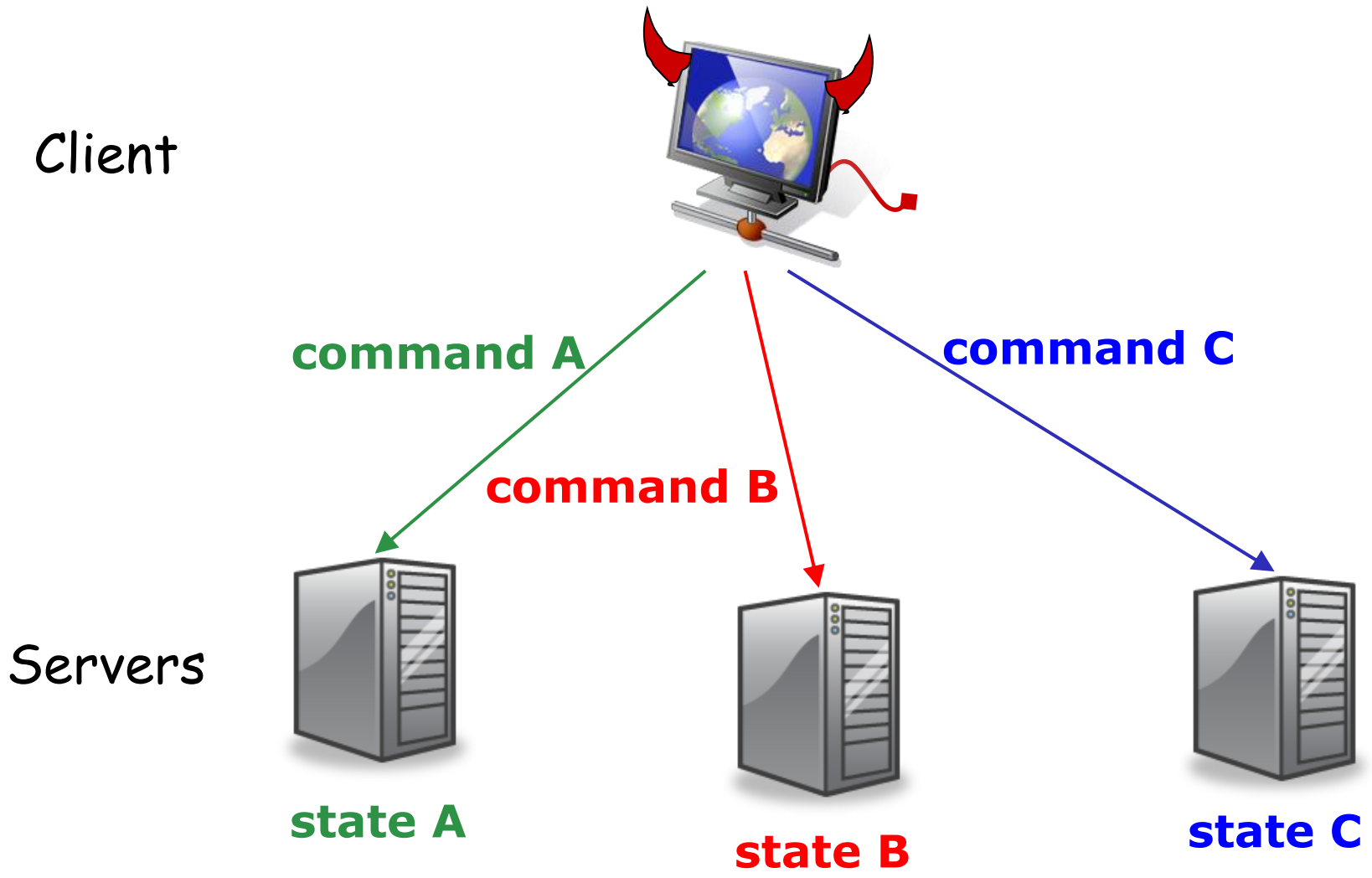
Replicated Servers



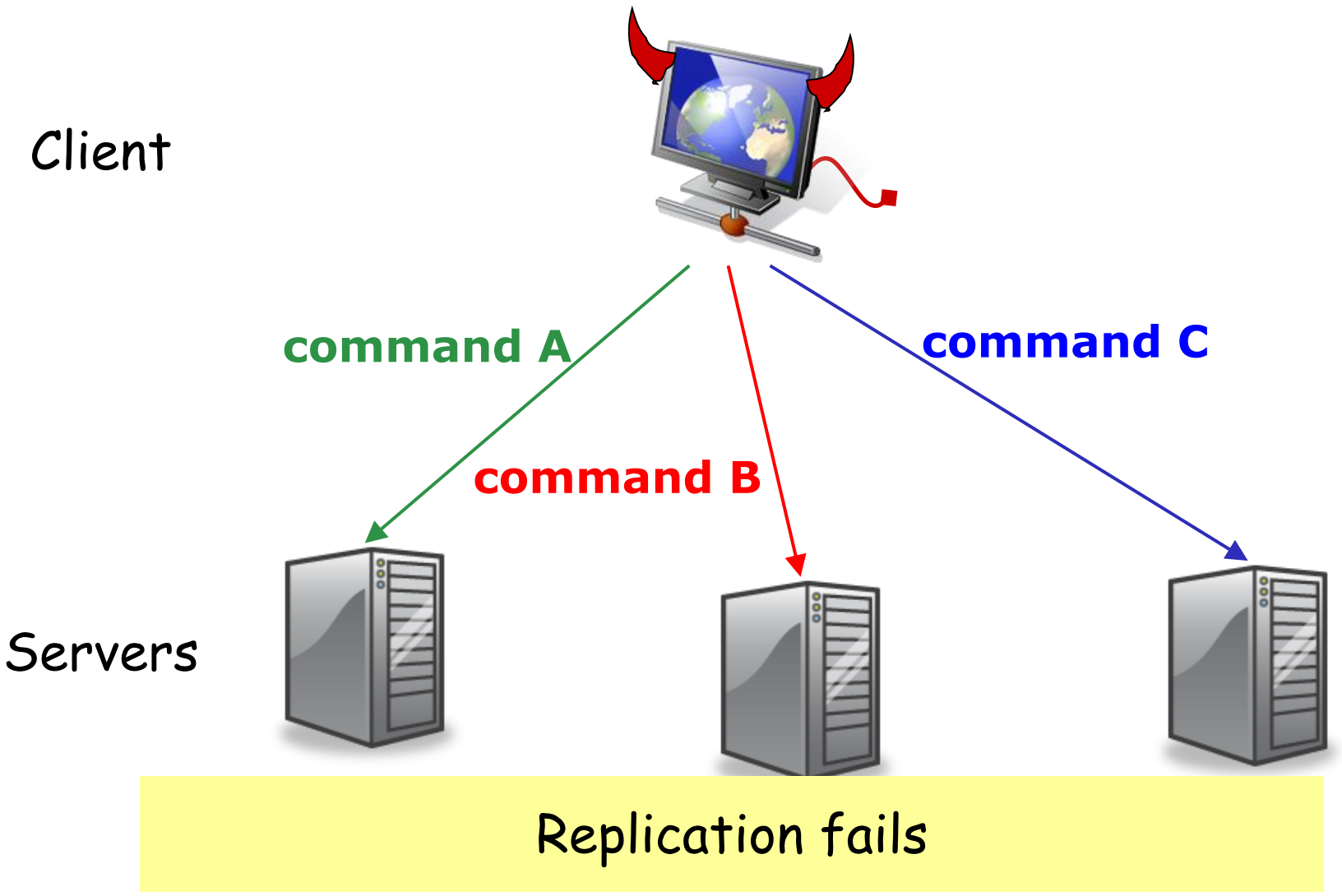
Replicated Servers



Replicated Servers



Replicated Servers



Replicated Servers

Client

command A

command B

command C

Servers

Must agree on same request

Byzantine Broadcast

- Source node S broadcasts to others
- n – 1 other nodes

Byzantine Broadcast

Source S an input (command)

- Fault-free nodes agree on identical value
- S fault-free ➔ agree on its input
- Up to f Byzantine node failures

Byzantine Fault Model

- Nodes may fail
- Arbitrarily bad behavior
  - Packet tampering
  - Packet dropping

… anything goes

Many Faces of Consensus

- All nodes have non-null input / only a subset do
- No failures / failures allowed (node/link)
- Synchronous/asynchronous
- Deterministically correct / probabilistically correct
- Exact agreement / approximate agreement
- Global communication / local communication

Byzantine Broadcast

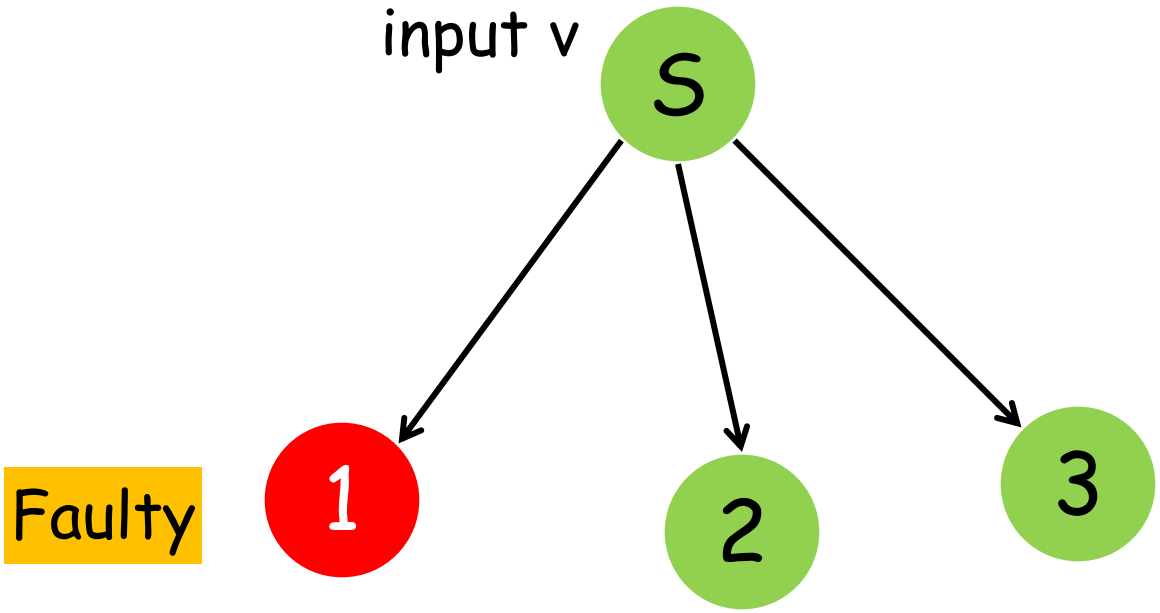
Example algorithm [Lamport,Shostak,Pease 1982]

- 4 nodes
- At most 1 faulty node

n = 4
f = 1

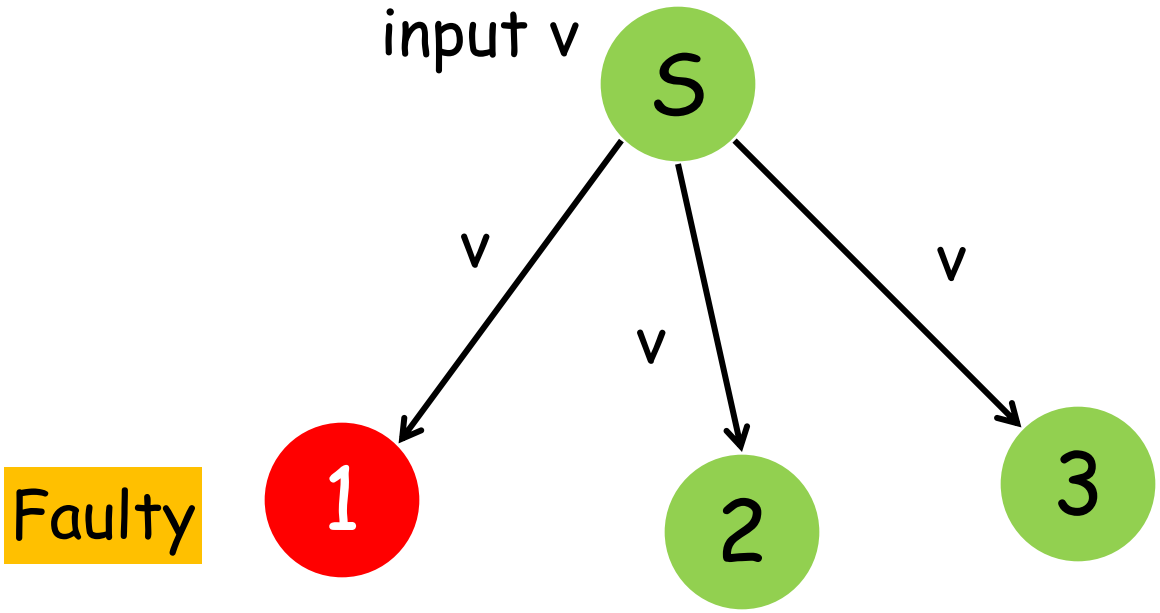
Byzantine Broadcast

n = 4

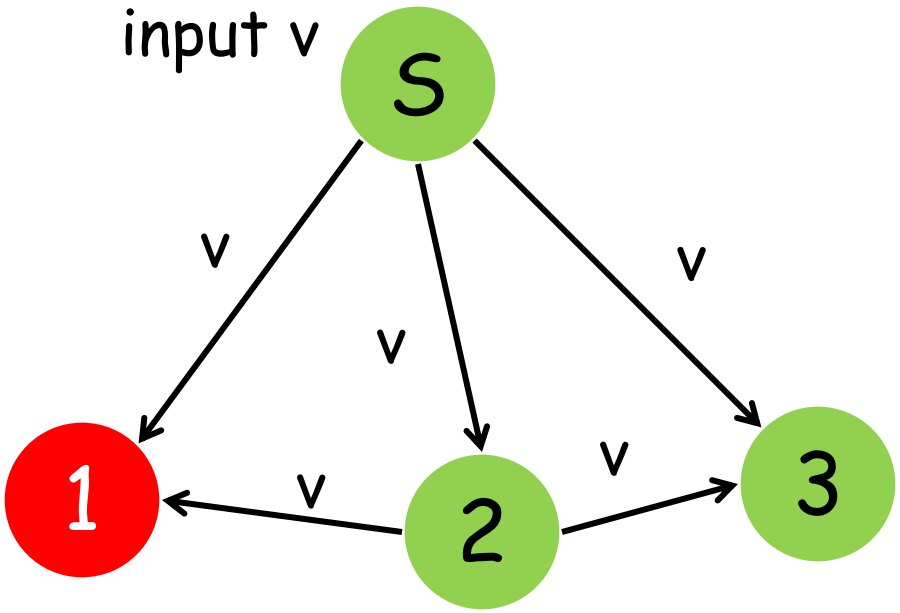


Byzantine Broadcast

n = 4



Broadcast



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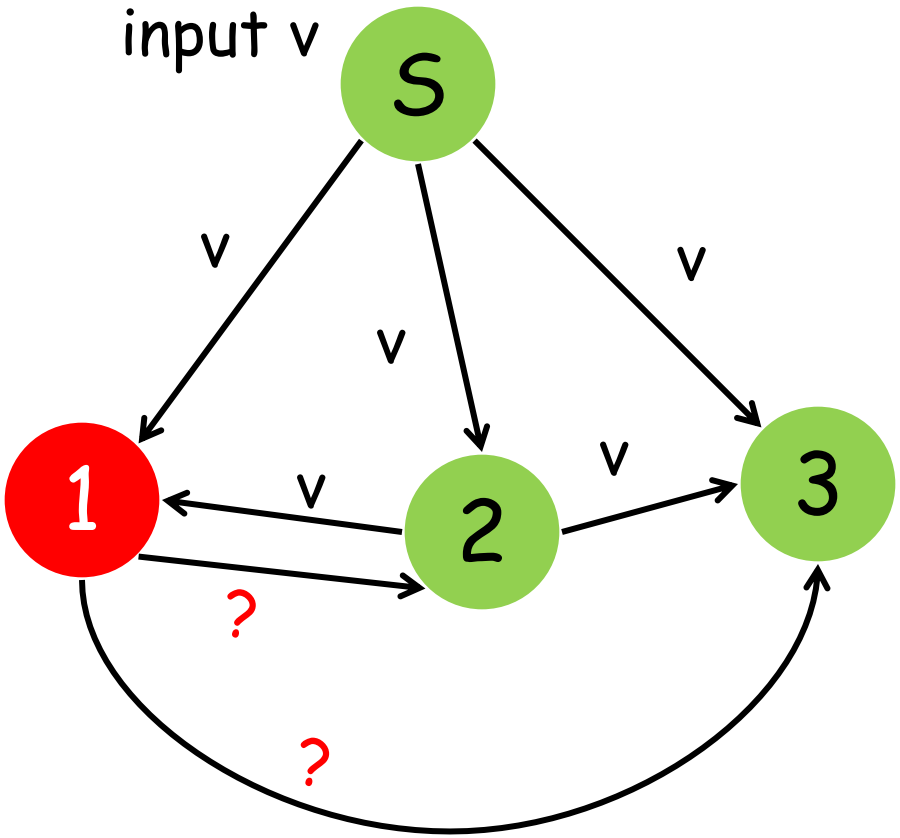

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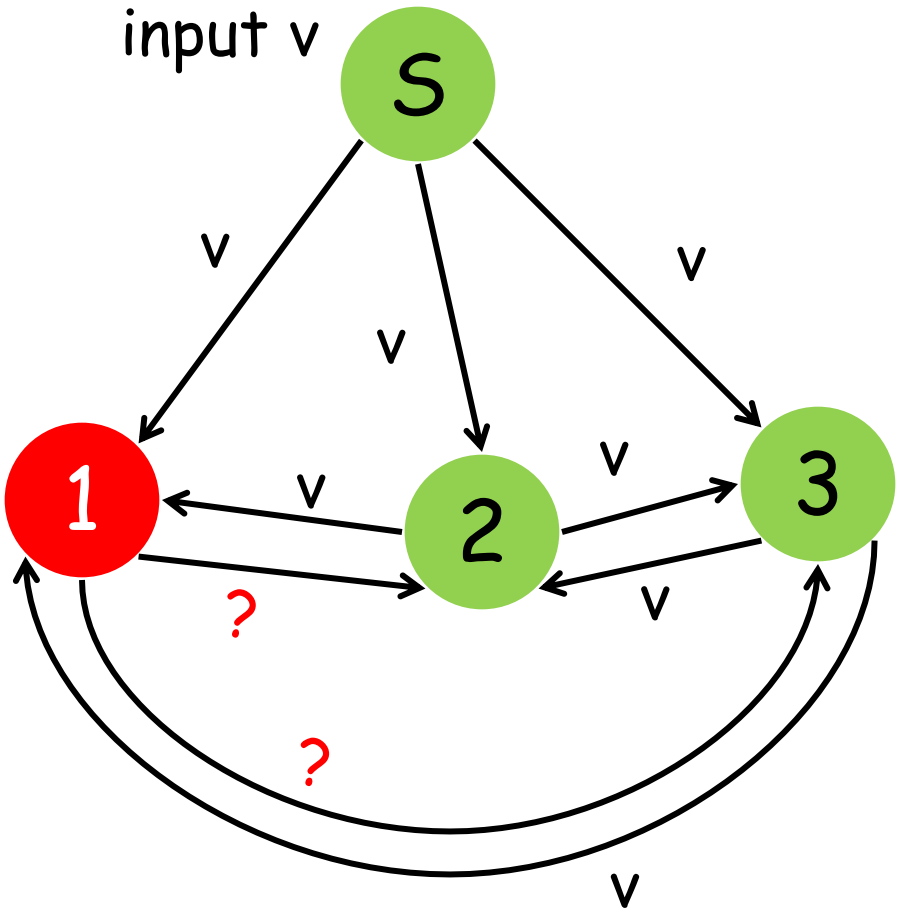
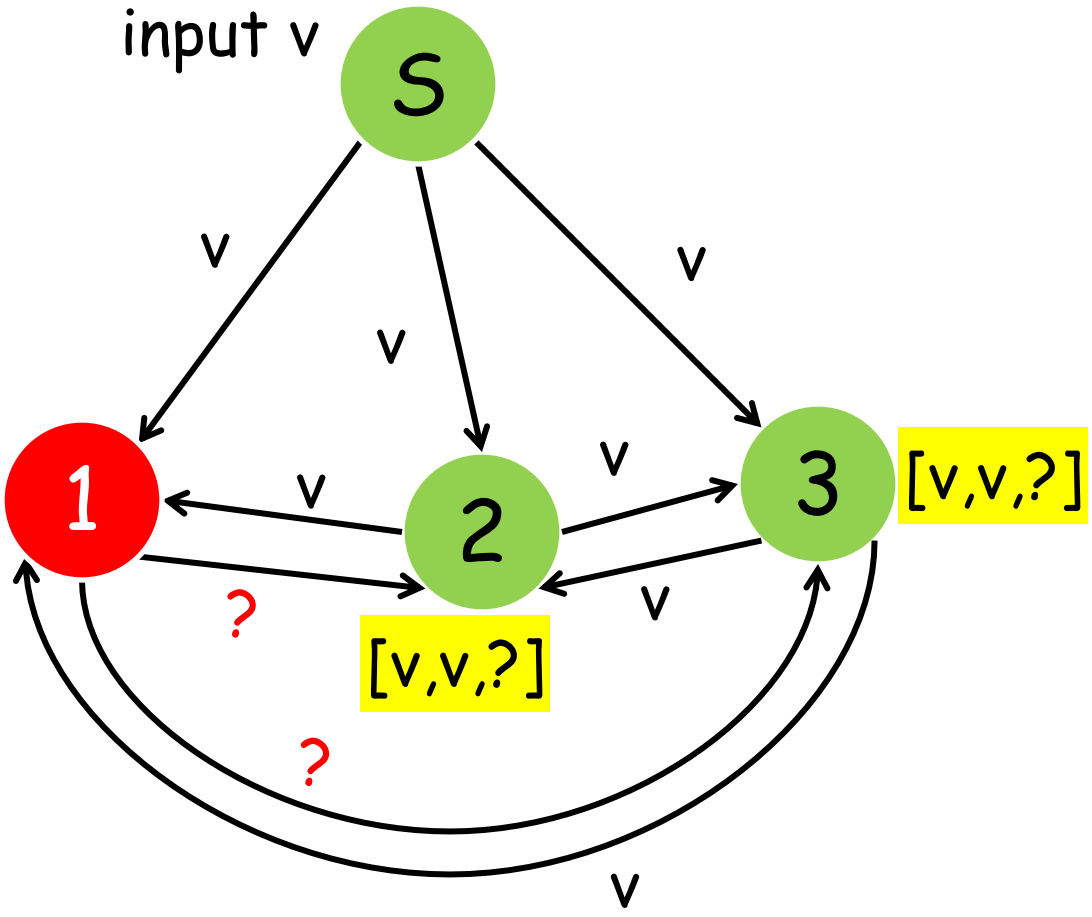
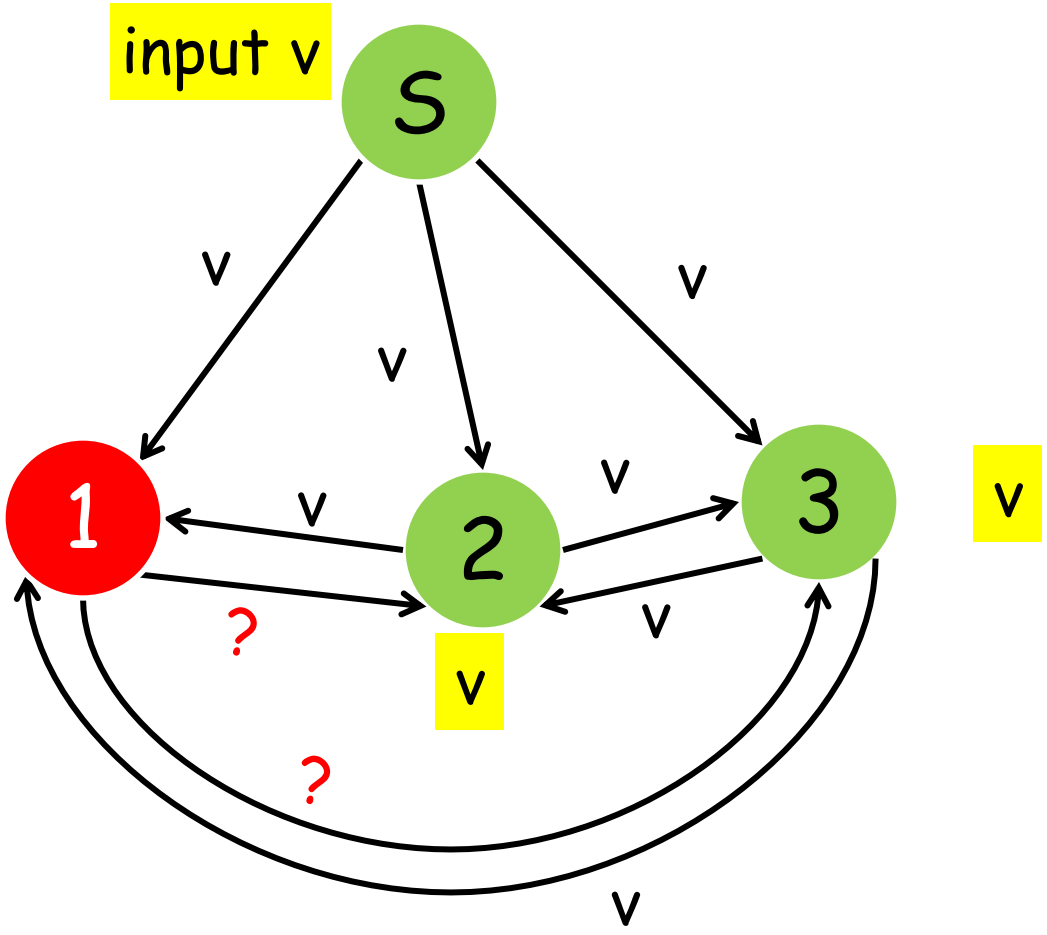


Broadcast



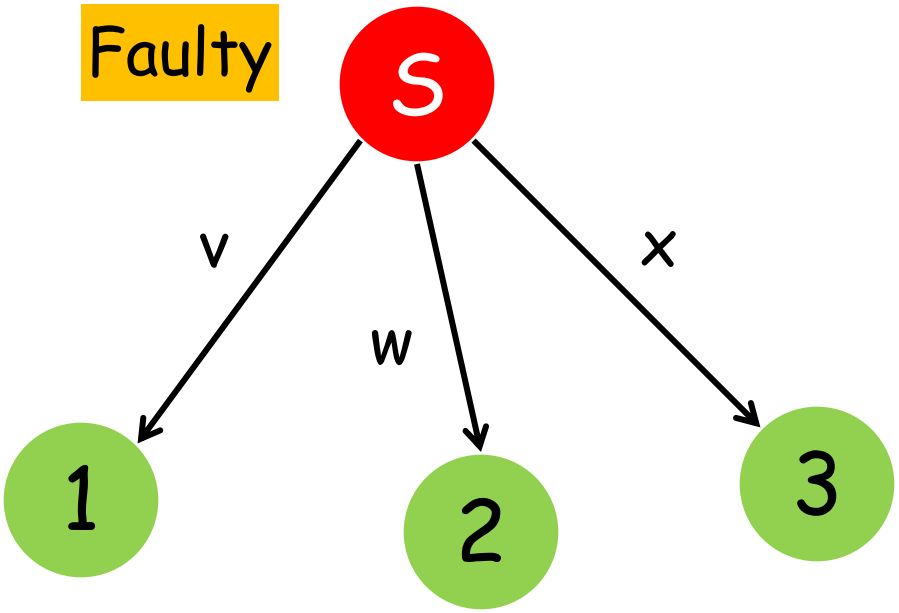
Broadcast



Majority vote

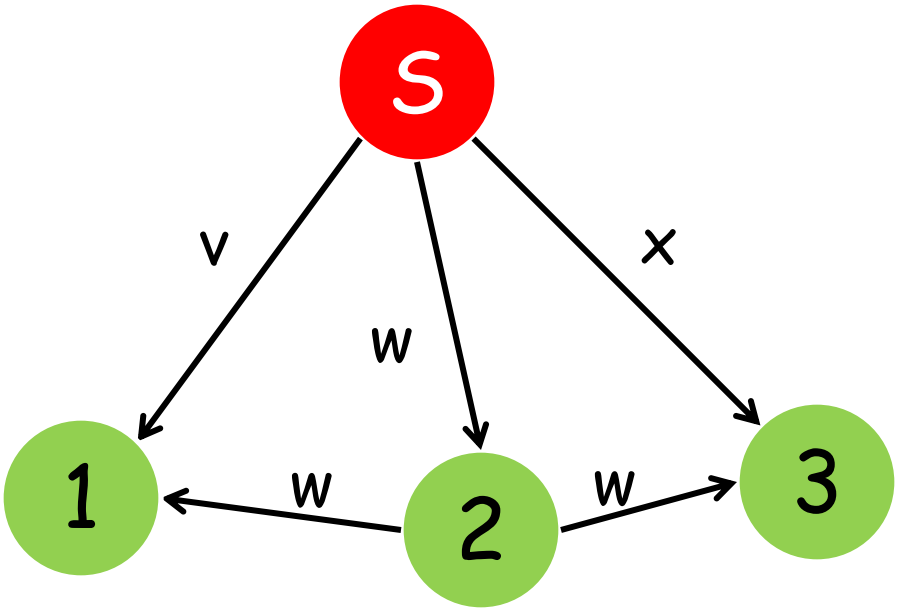
➔ Correct

Broadcast

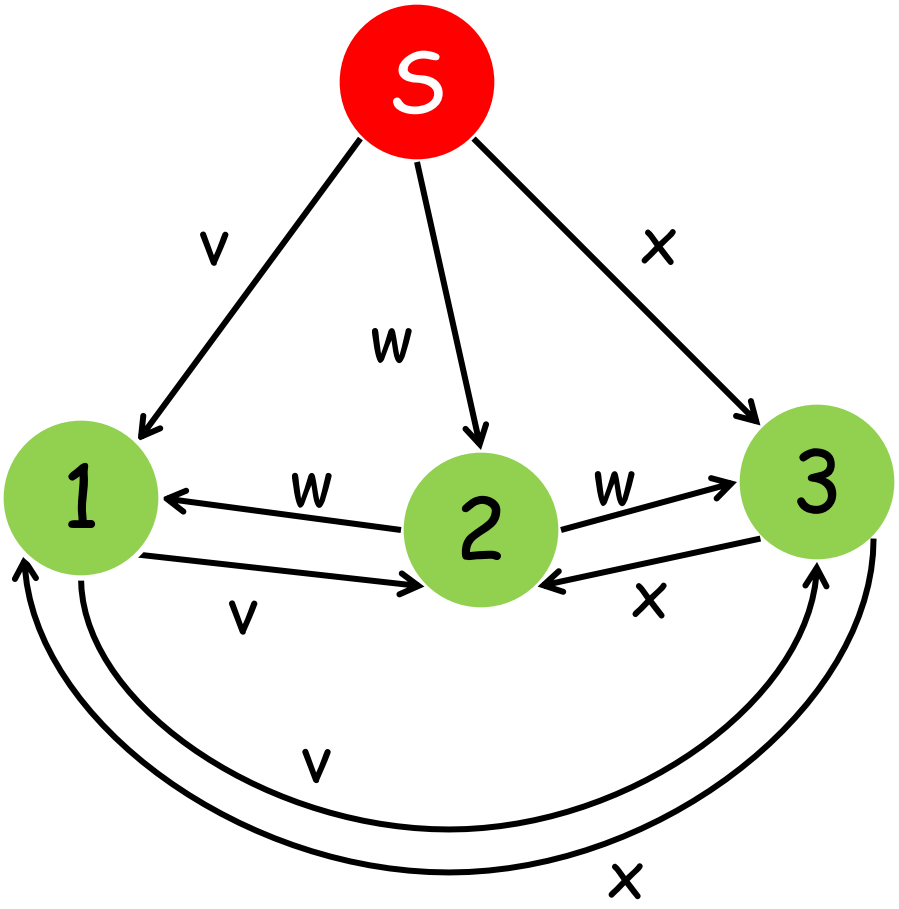


Bad source may attempt to diverge state at good nodes

Broadcast



Broadcast



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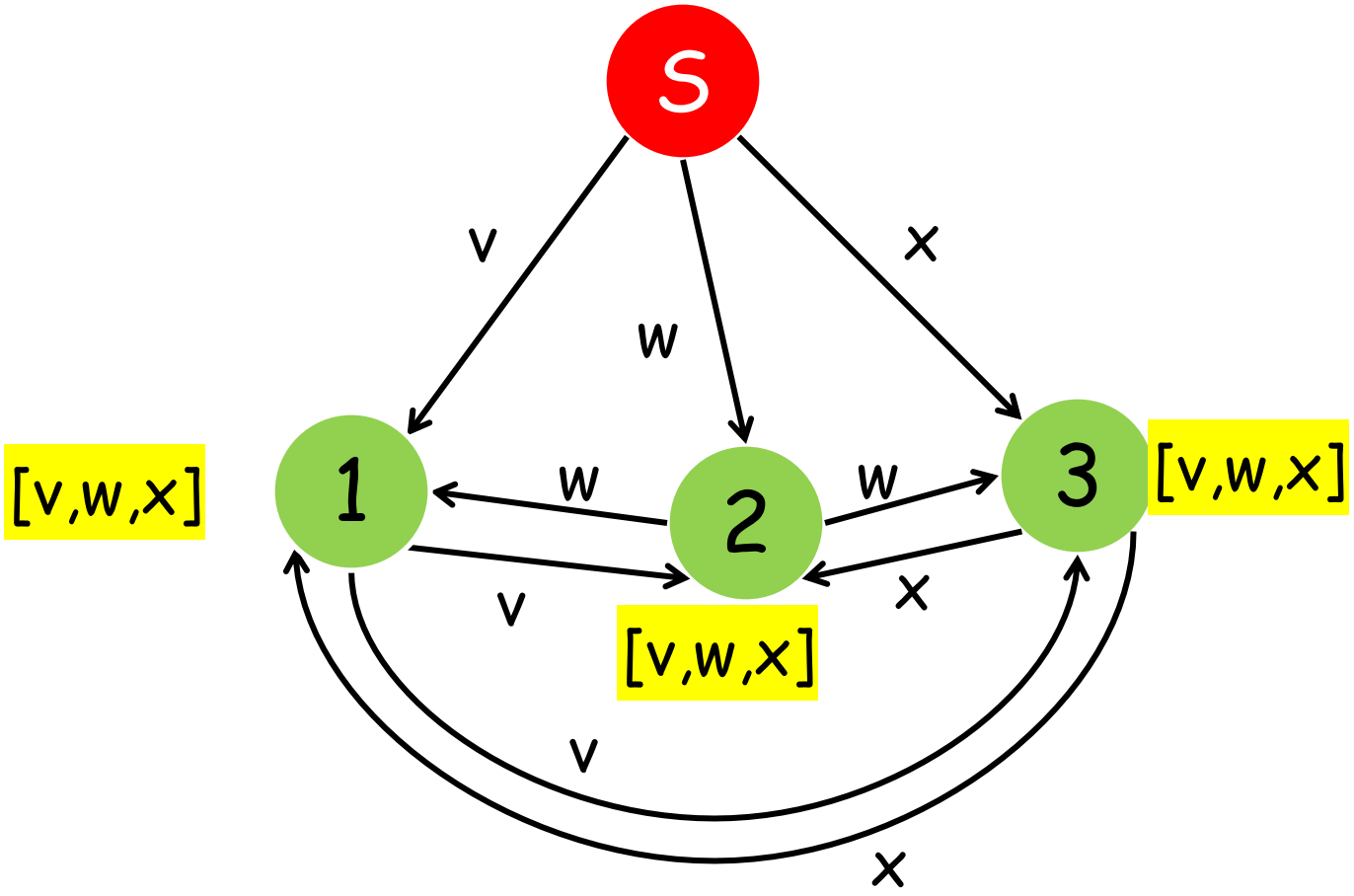

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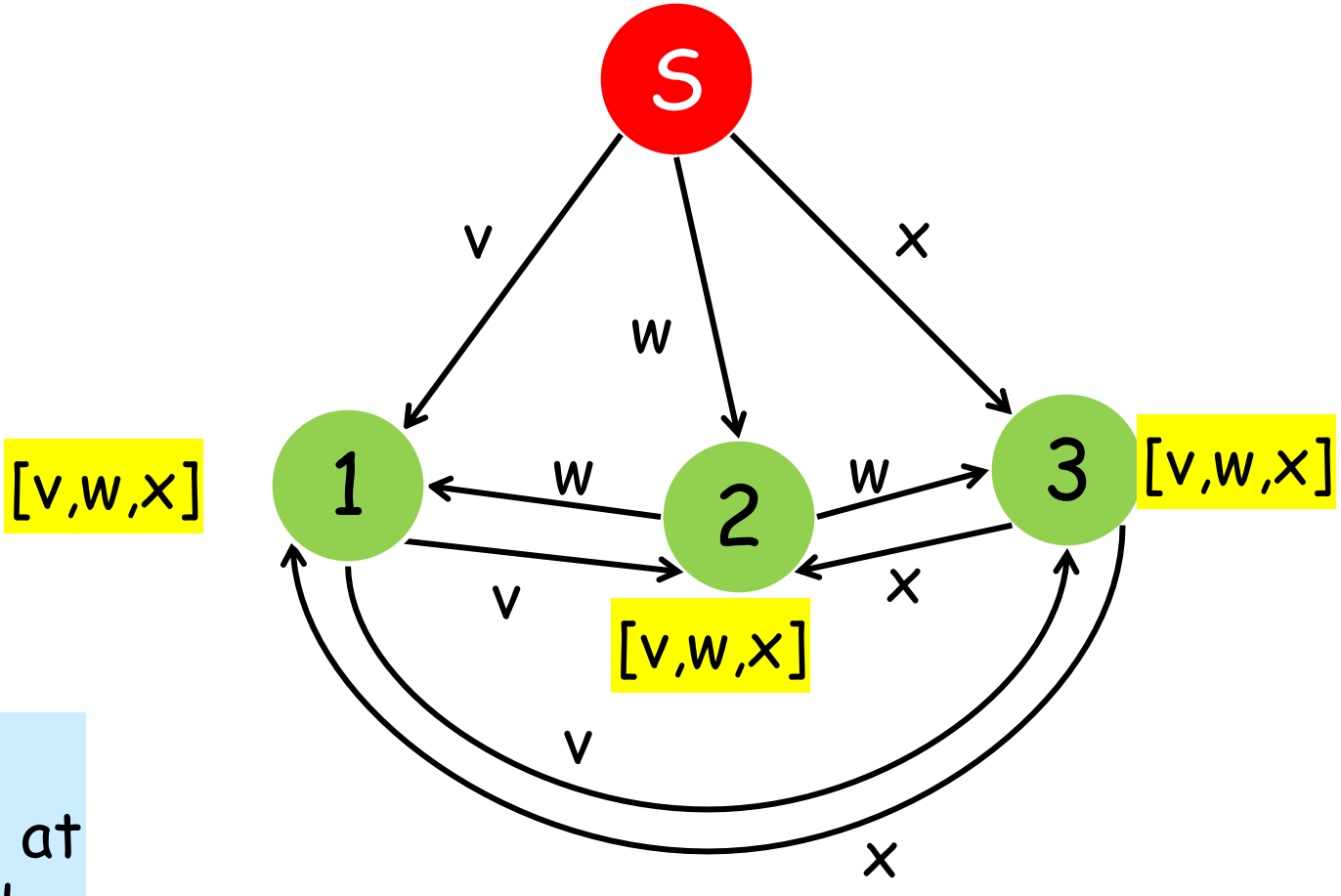
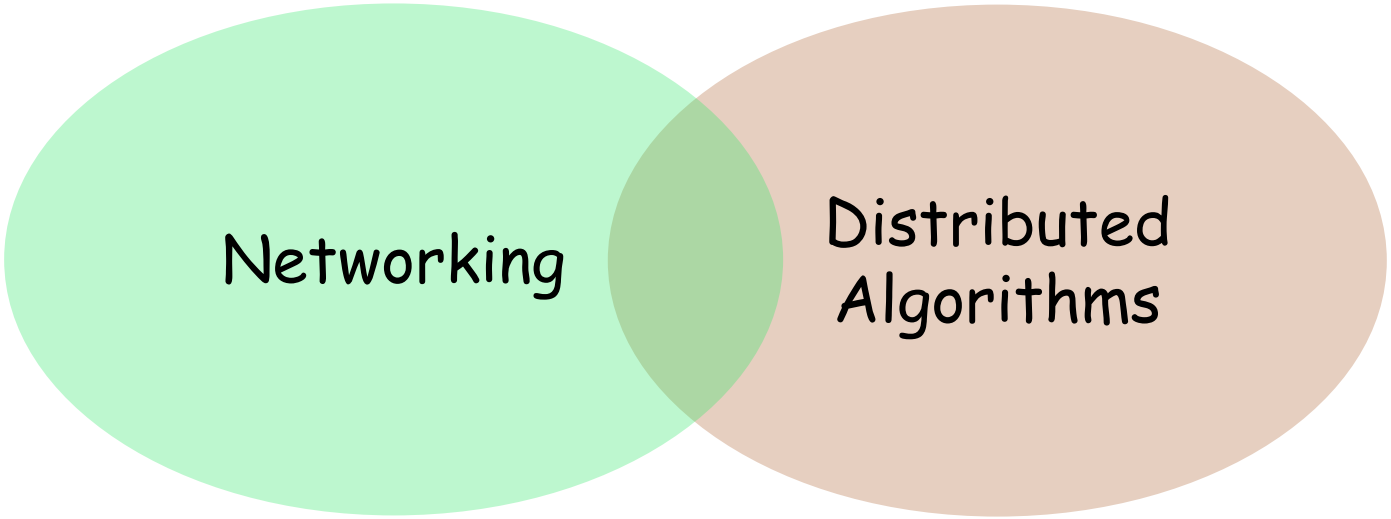


Vote identical at good nodes

Known Bounds

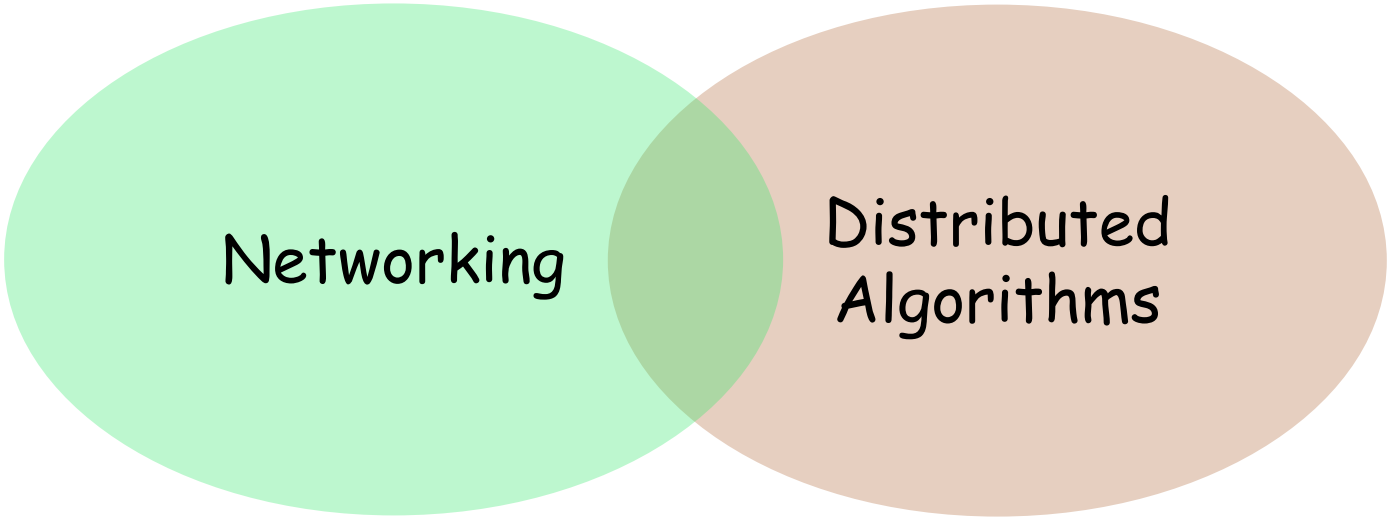
- $n \geq 3f + 1$ nodes to tolerate f failures
- Connectivity $\geq 2f + 1$
- $\Omega(n^2)$ messages in worst case
- $f+1$ rounds of communication

Impact of Network



Impact of Network

- How to quantify the impact ?



Metric 1:
Communication Cost per Bit

$$\frac{\text{Total communication cost (in bits)}}{\text{Number of bits of Byzantine broadcast}}$$

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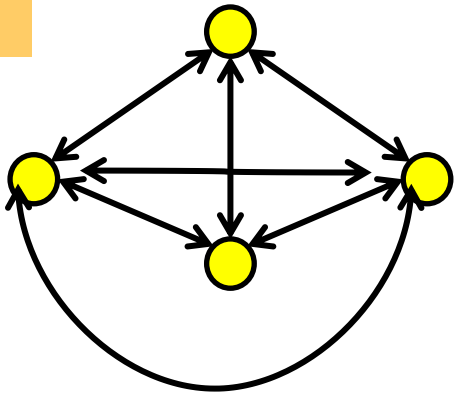
$$\frac{\text{Total communication cost (in bits)}}{\text{Number of bits of Byzantine broadcast}}$$

Ignores network characteristics



Metric 2: Throughput

- Borrow notion of throughput from networking

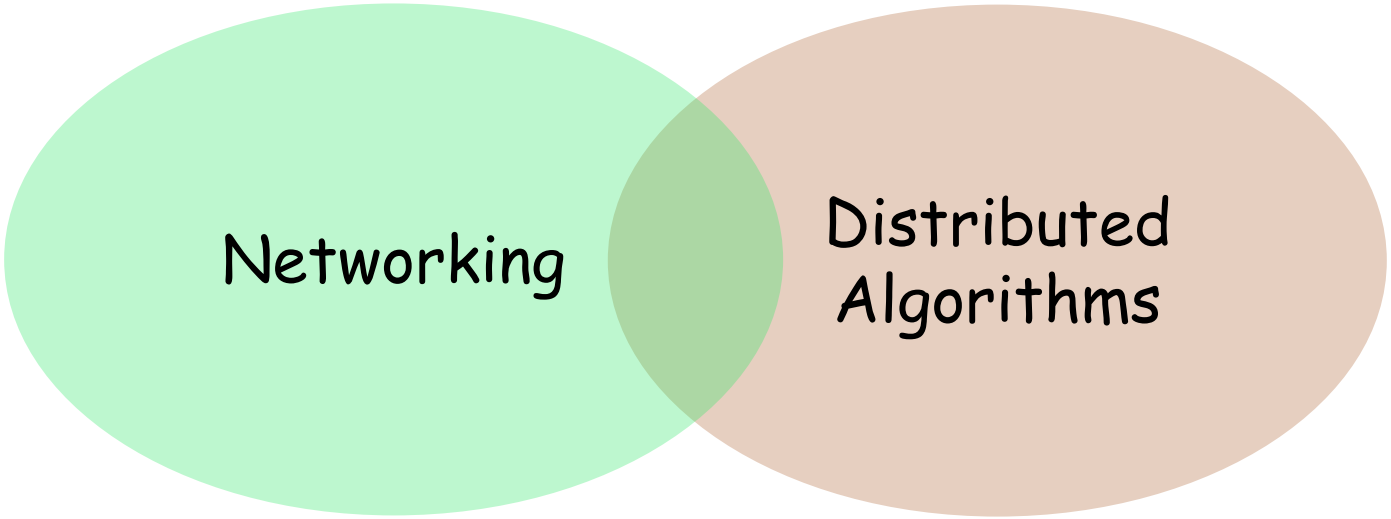
- *b(t)* = number of bits agreed upon in *[0,t]*

$$Throughput = \lim_{t \to \infty} \frac{b(t)}{t}$$

Impact of Network

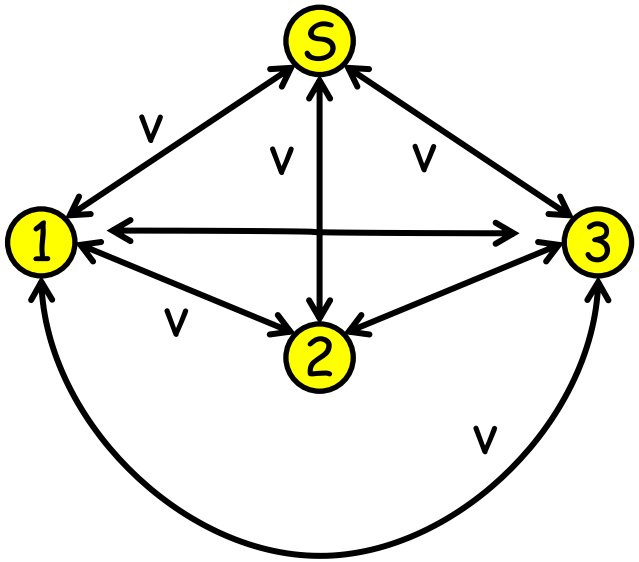
- How does the network affect

  Byzantine broadcast/consensus ?



Consider earlier algorithm …

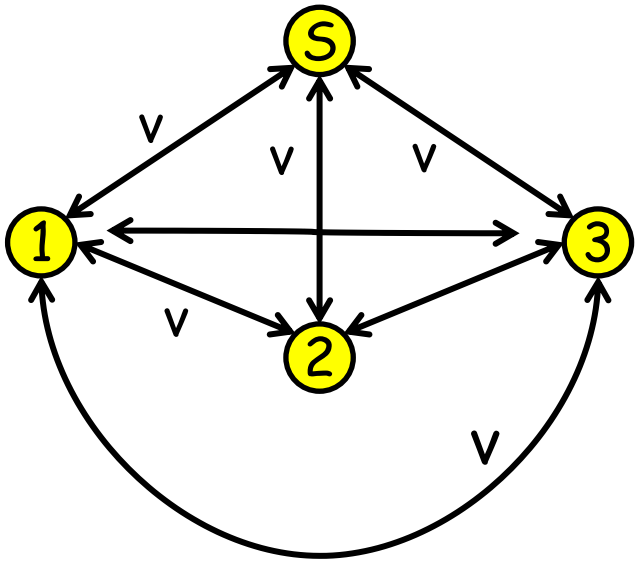
- All data sent on each link once
  - ➔ broadcast throughput 10



Each directed link rate = 10

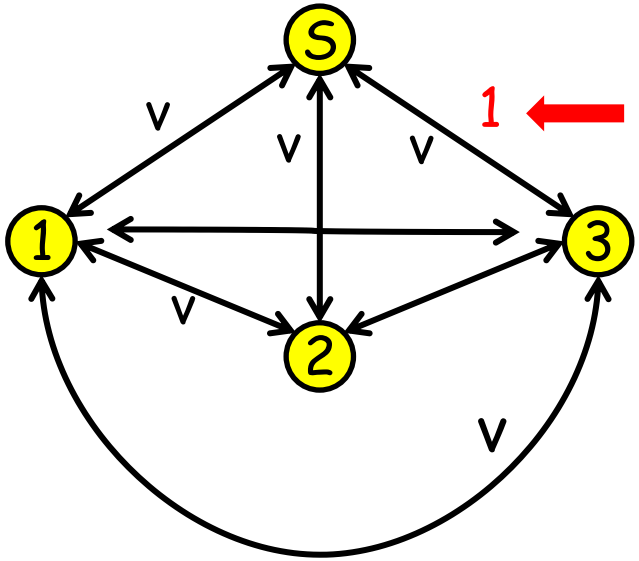
Example

- broadcast throughput 10



Each directed link rate = 10

- broadcast throughput 1

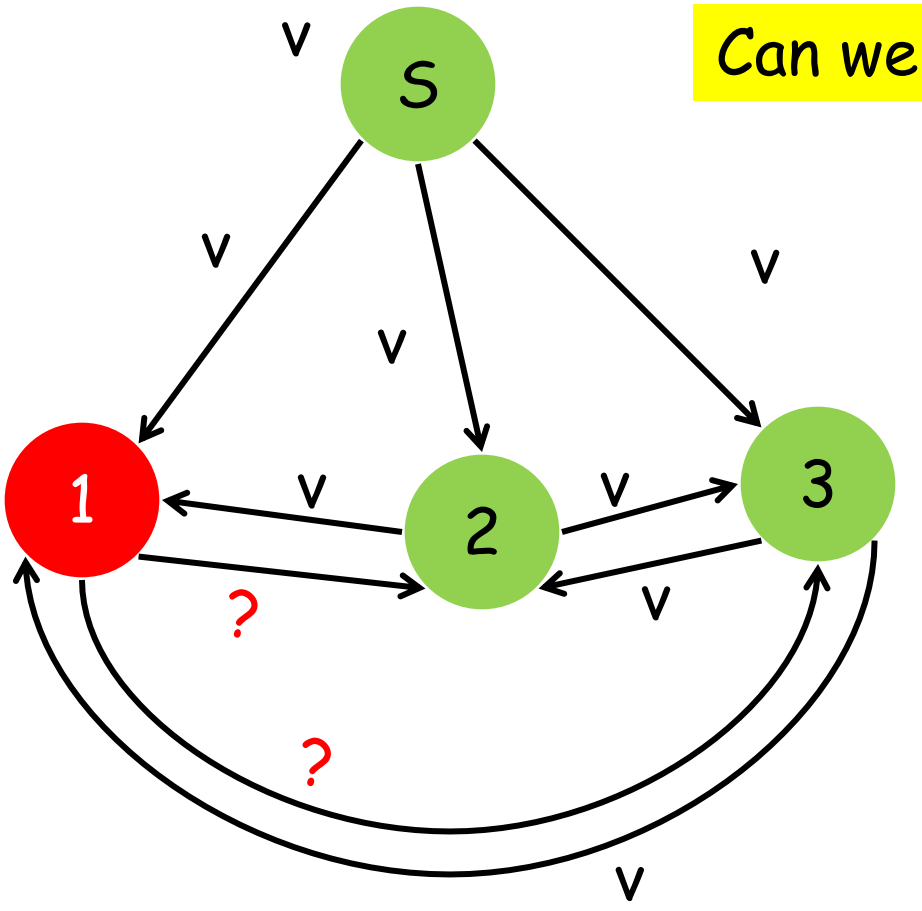


All other links = 10

Point-to-Point Networks

- How to best exploit available link capacity ?
  - Symmetric case
  - Asymmetric case

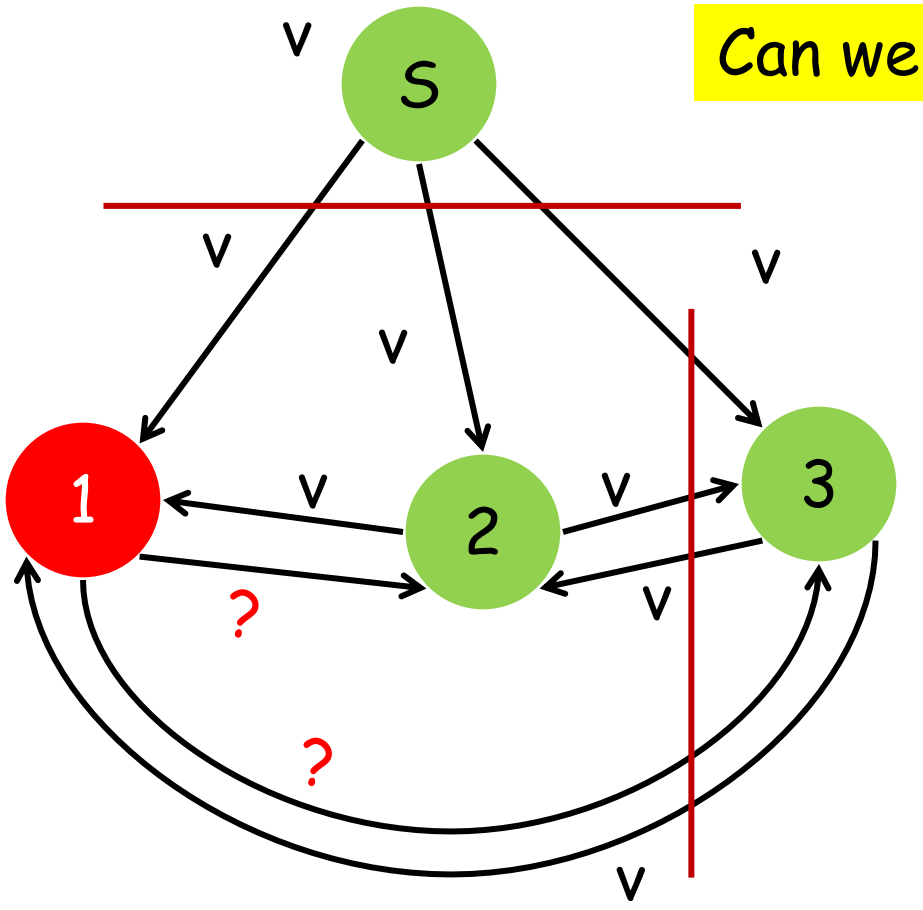
Symmetric Case



Can we do better ?

Symmetric Case

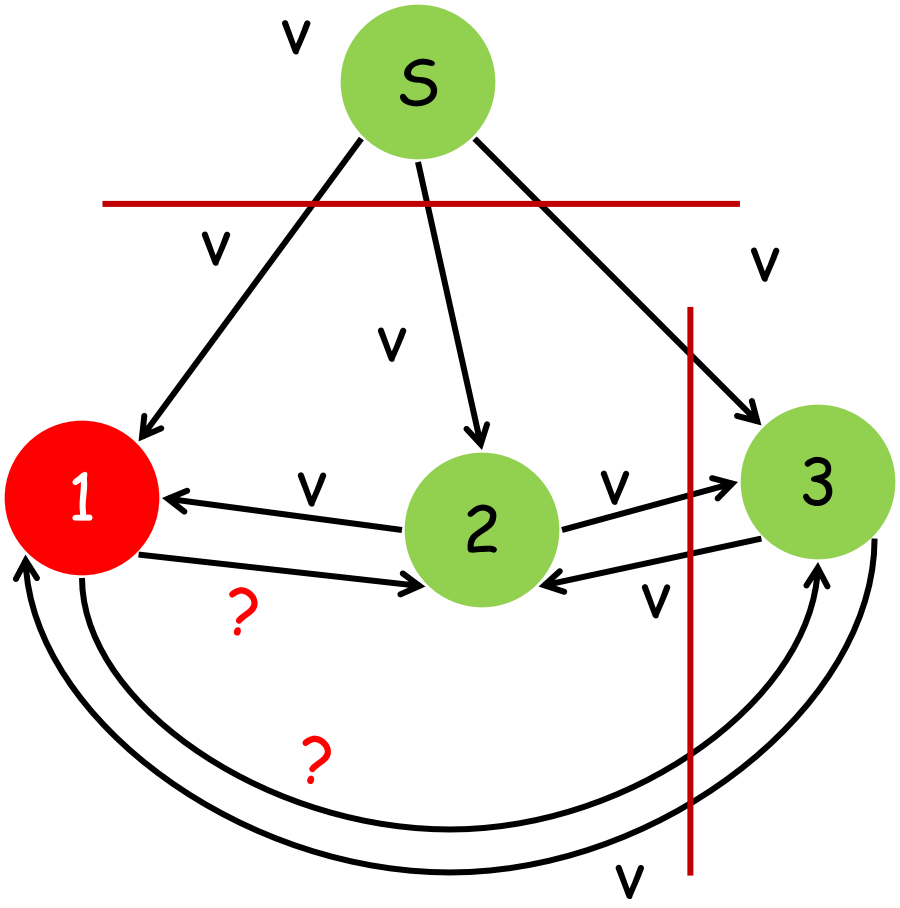
- "Replication" code



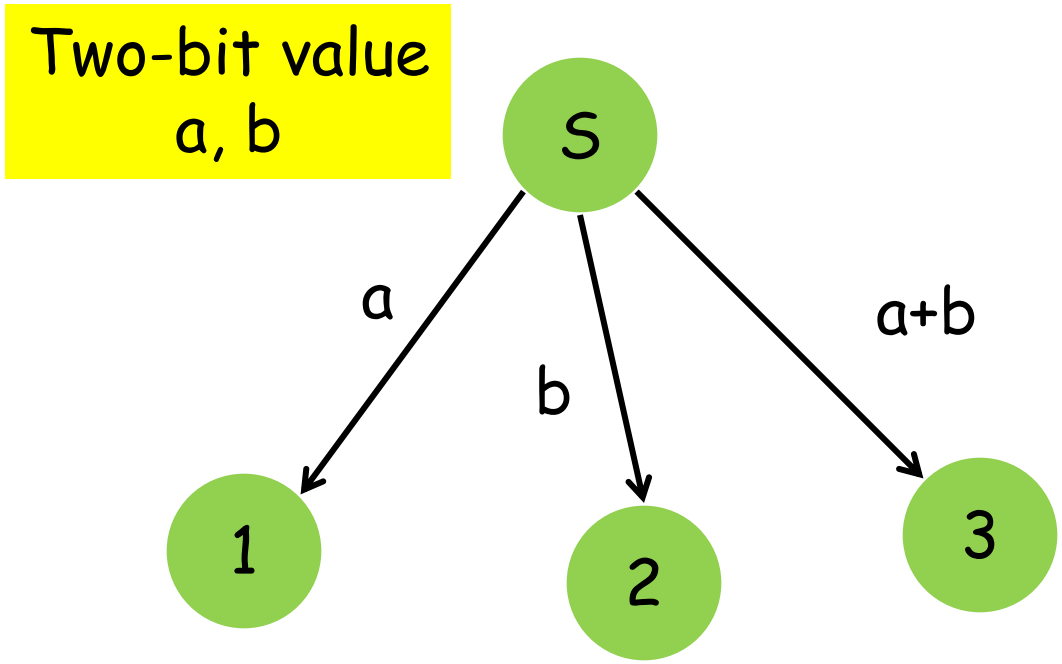
Can we do better ?

Can we do better ?

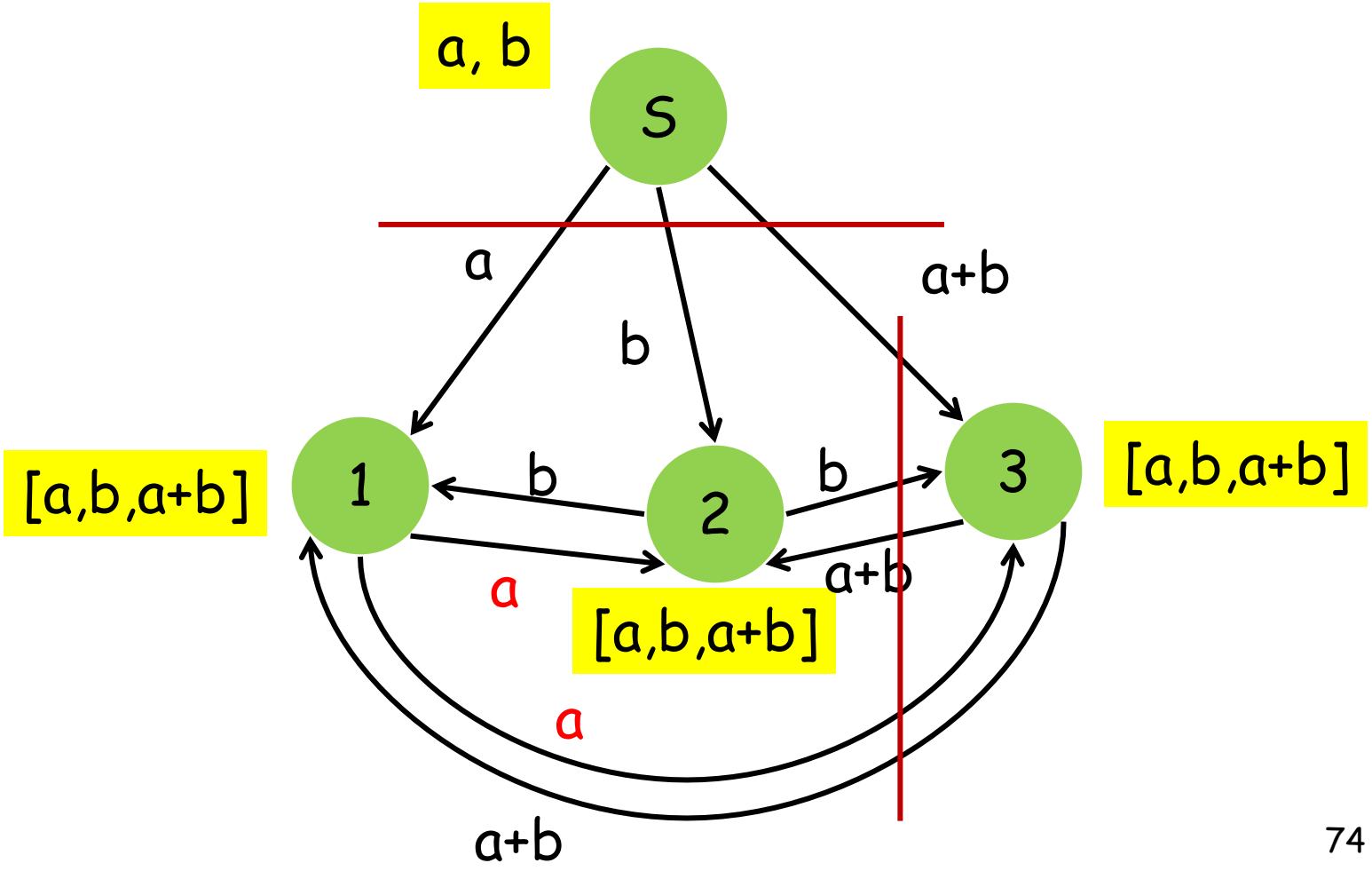
- More efficient code … standard tool in Communication



Make Common Case Fast



Make Common Case Fast



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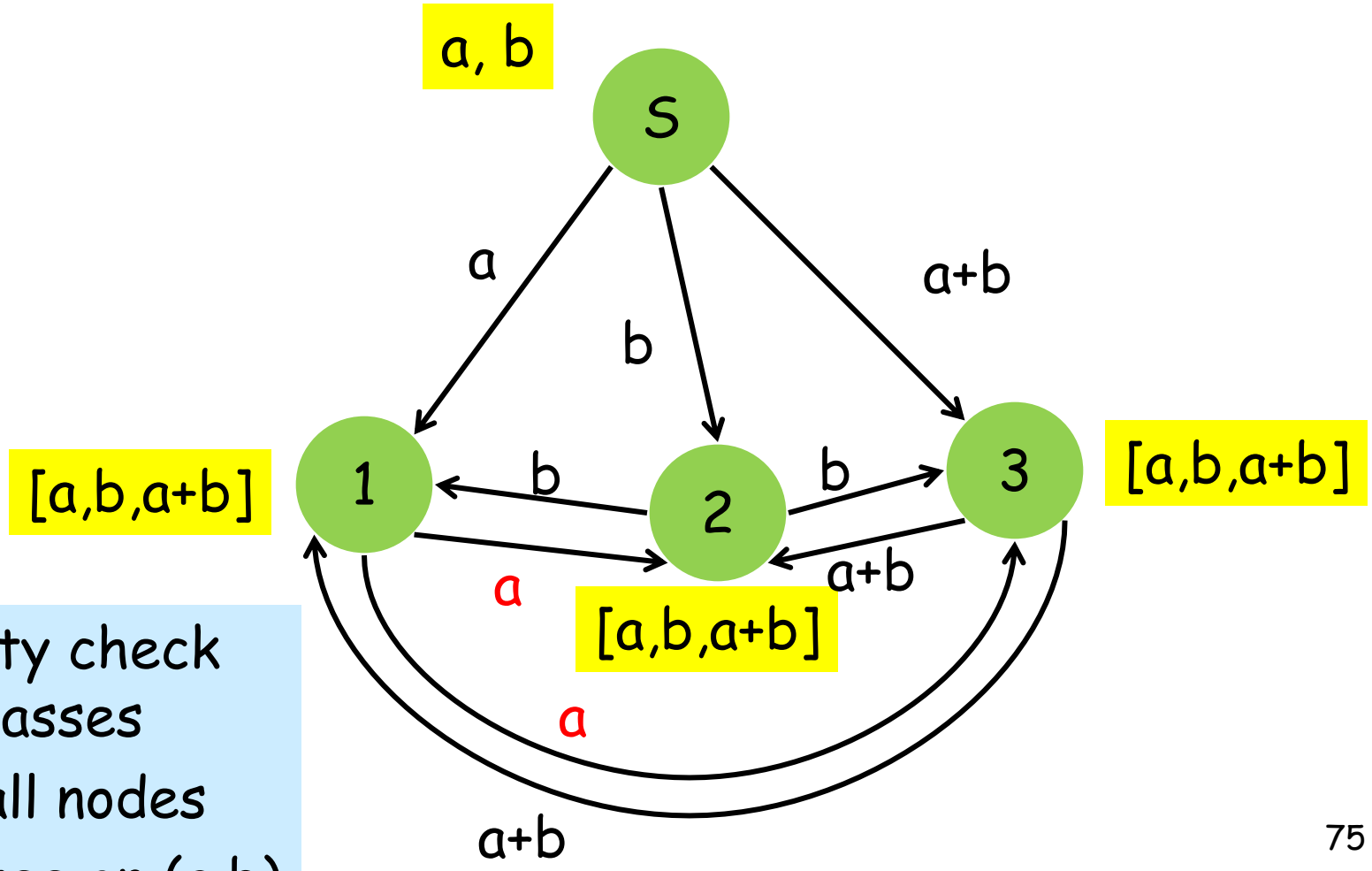
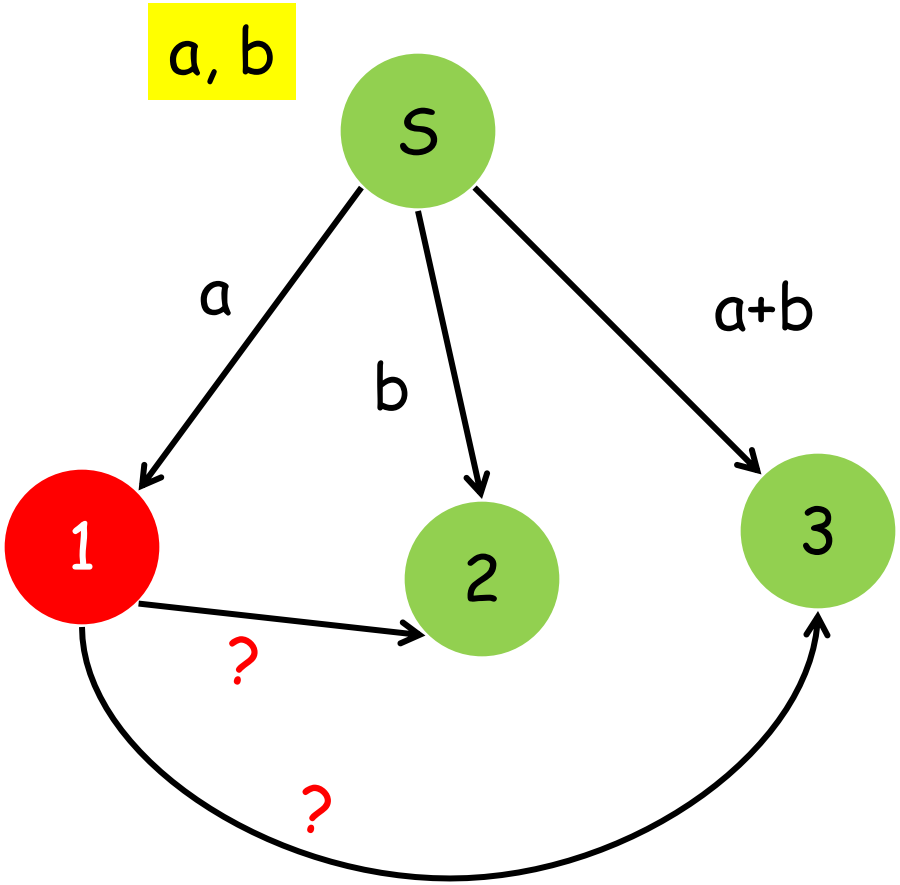


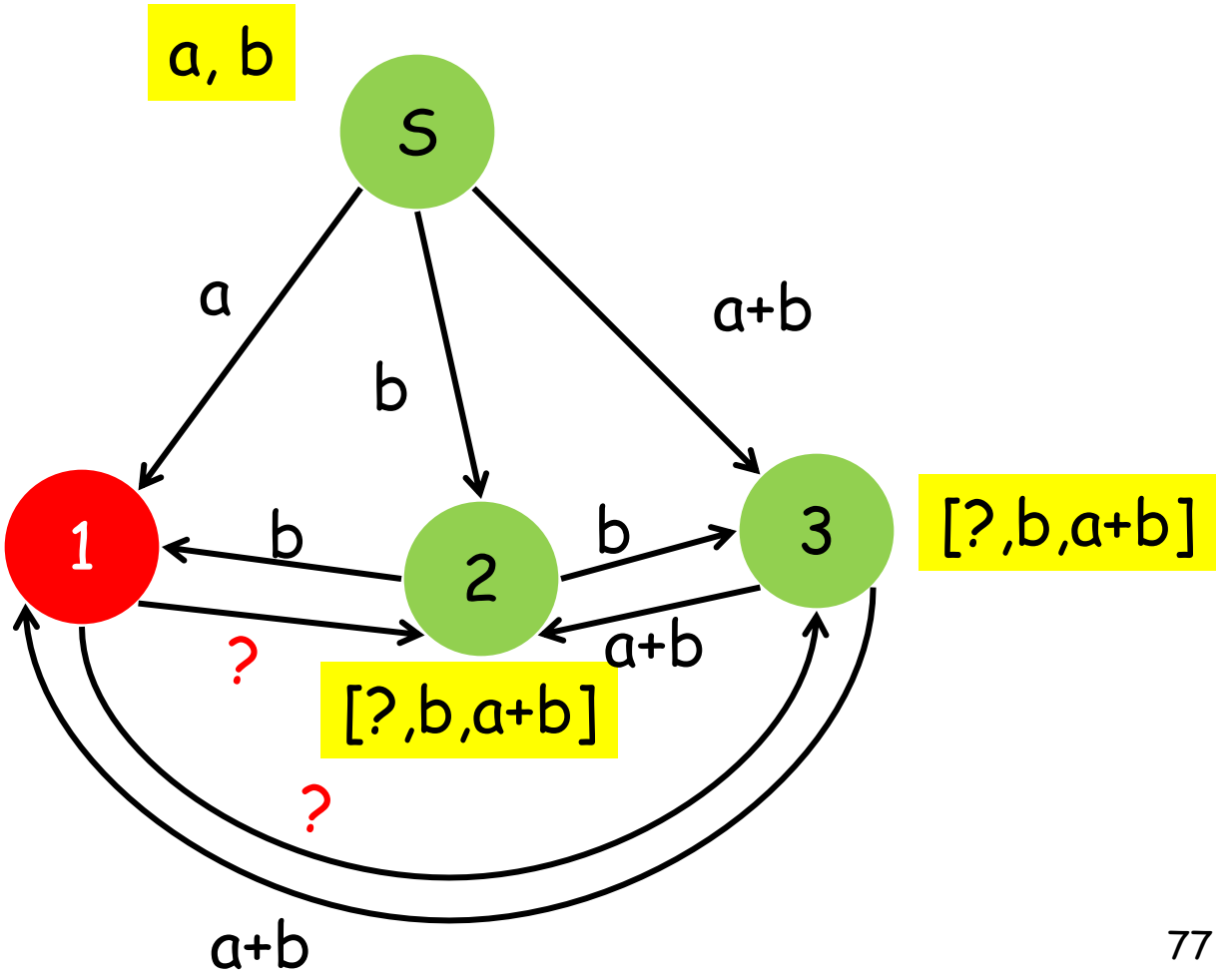
Parity check passes at all nodes

➔ Agree on (a,b)

Make Common Case Fast

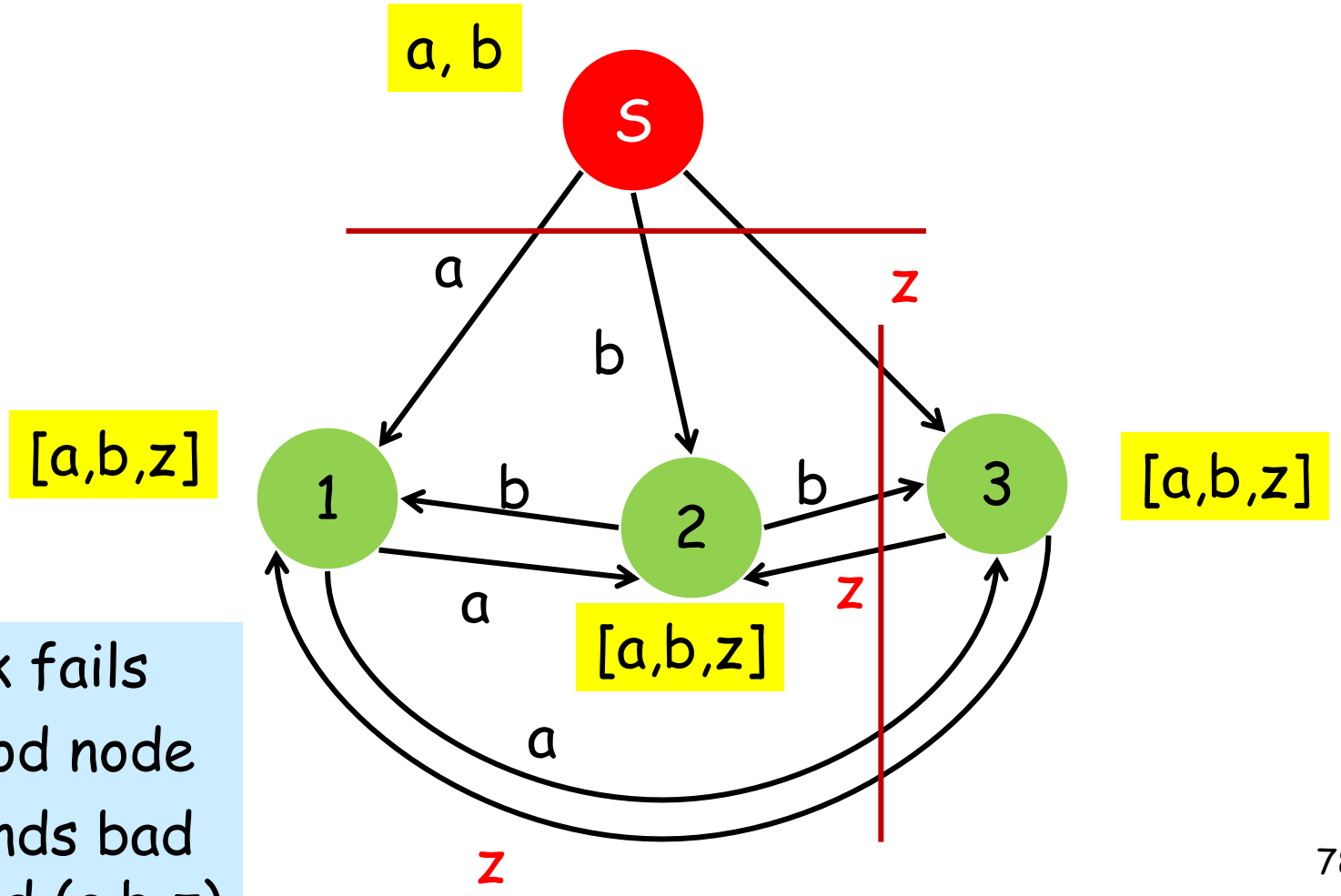


Make Common Case Fast



Parity check fails at a node if A misbehaves

Make Common Case Fast



Check fails at a good node if S sends bad codeword (a,b,z)

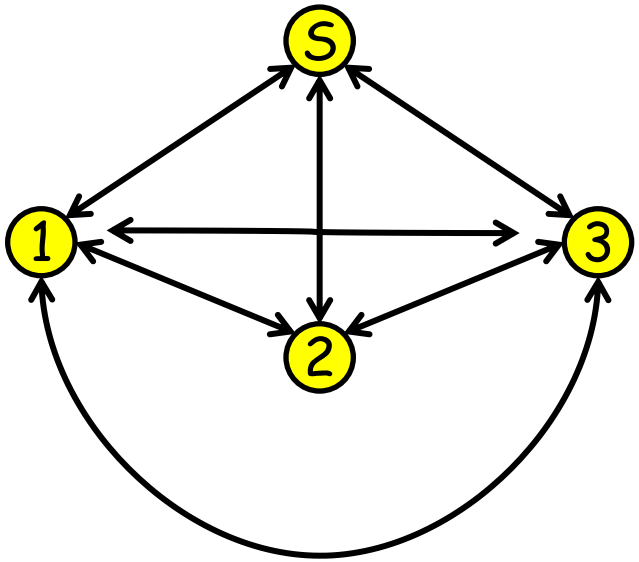
After Failure Detection

- More work required after failure detection
- But not too many times

Symmetric Case

- Per link capacity R

  ➔ Byzantine broadcast rate (n-1-f)R Optimal



n = 4
f = 1
➔ 2R

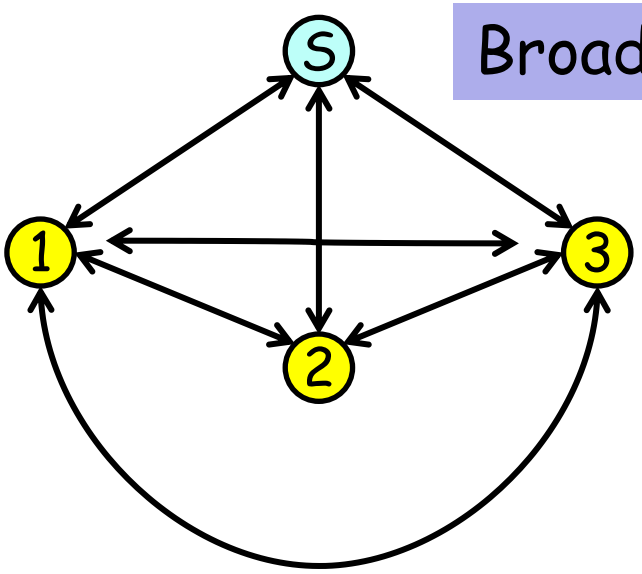
Arbitrary Networks

Optimal Byzantine Broadcast algorithm unknown

➔ Throughput within constant factor

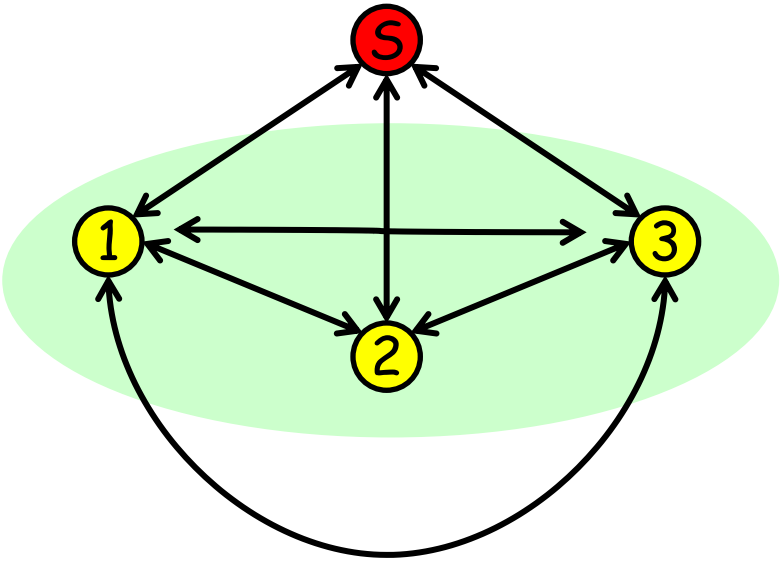
Algorithm Sketch

- Broadcast data without fault tolerance

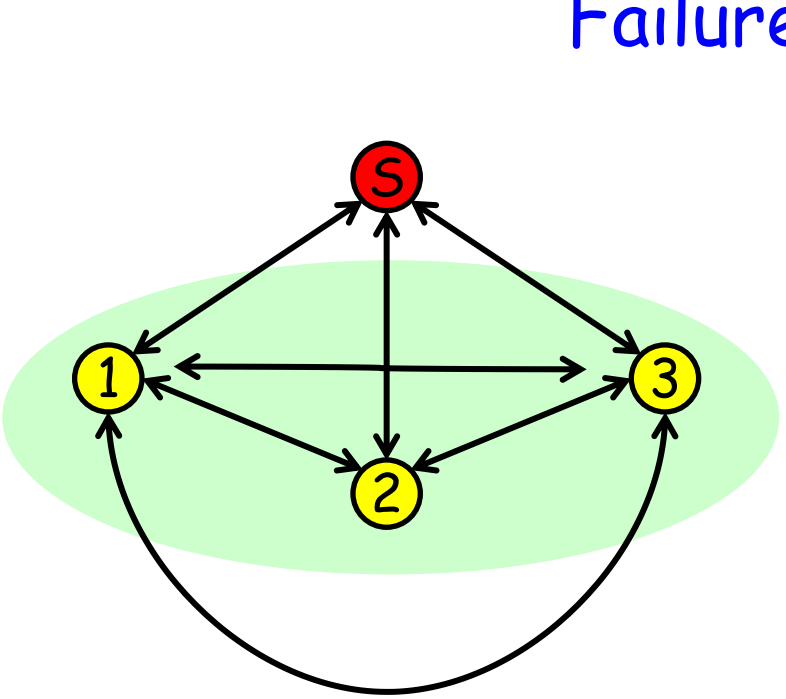


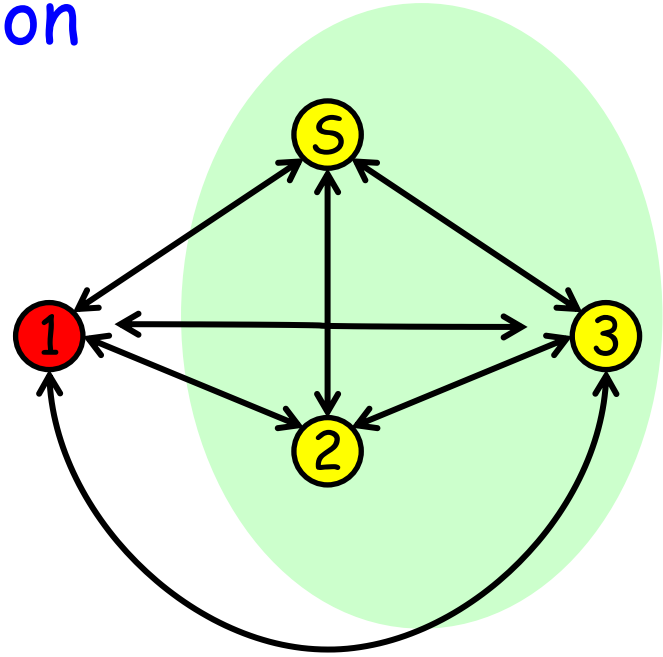
Broadcast from S

Failure Detection

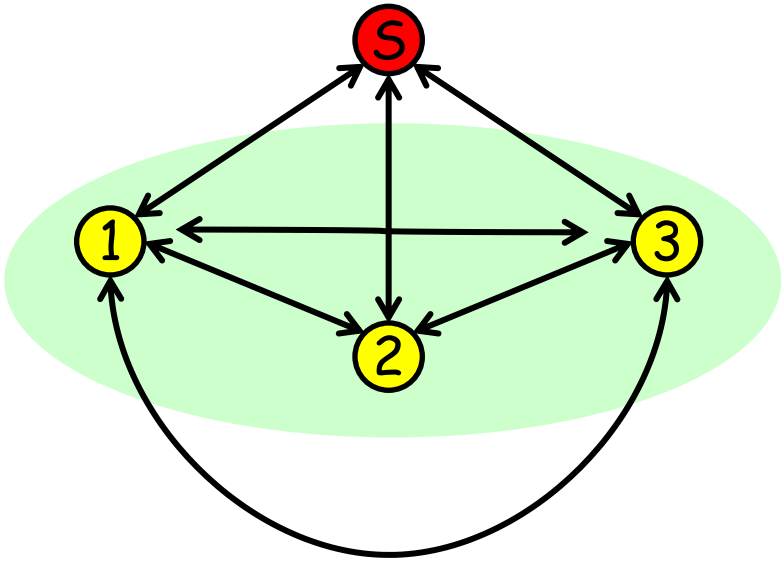


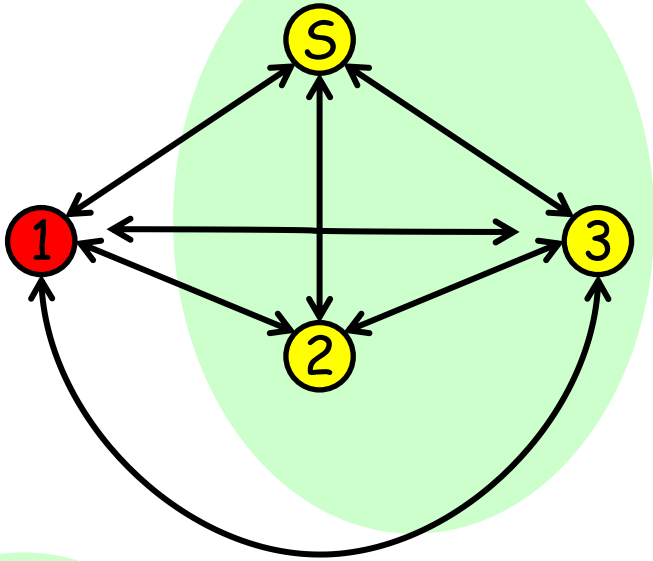
Failure Detection

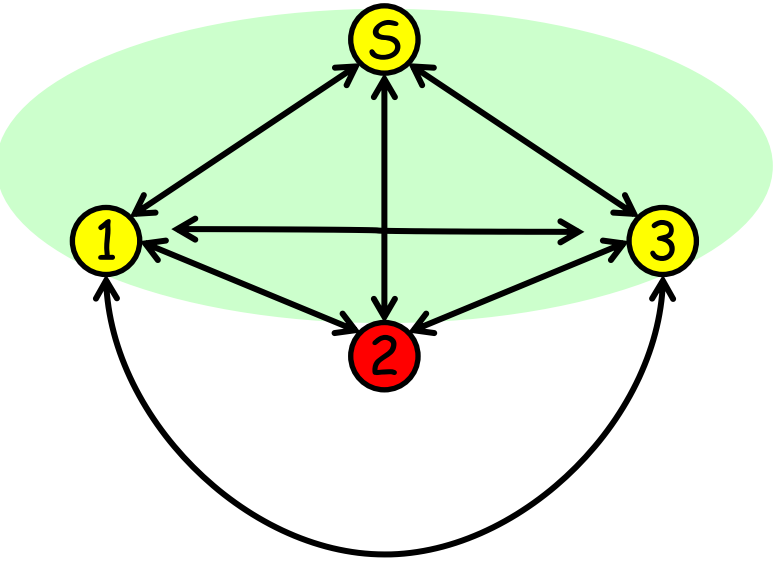


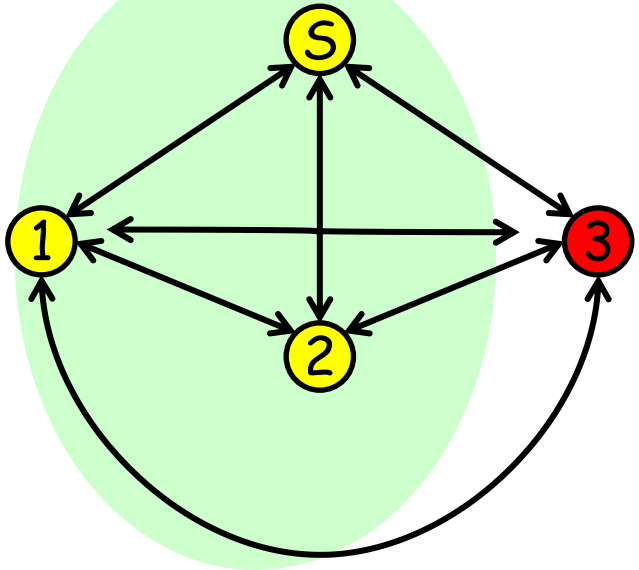


Failure Detection

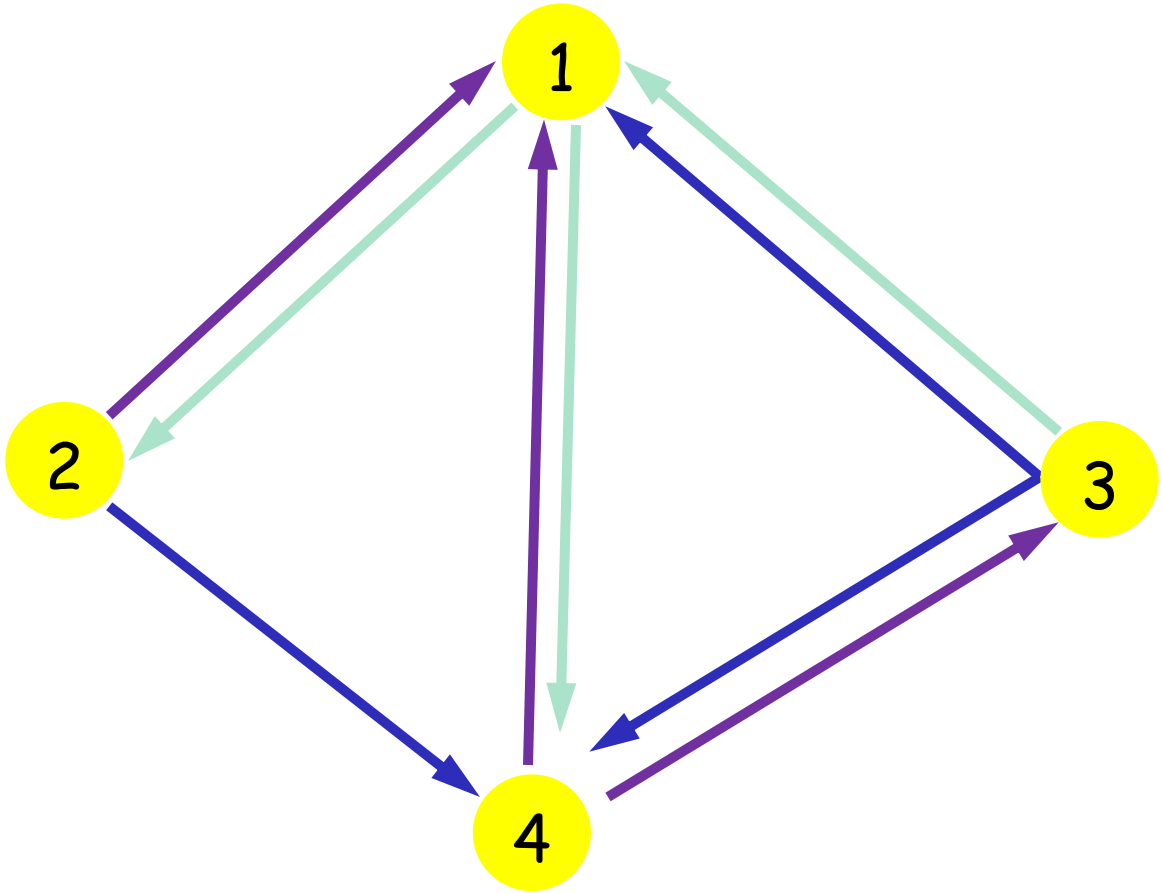






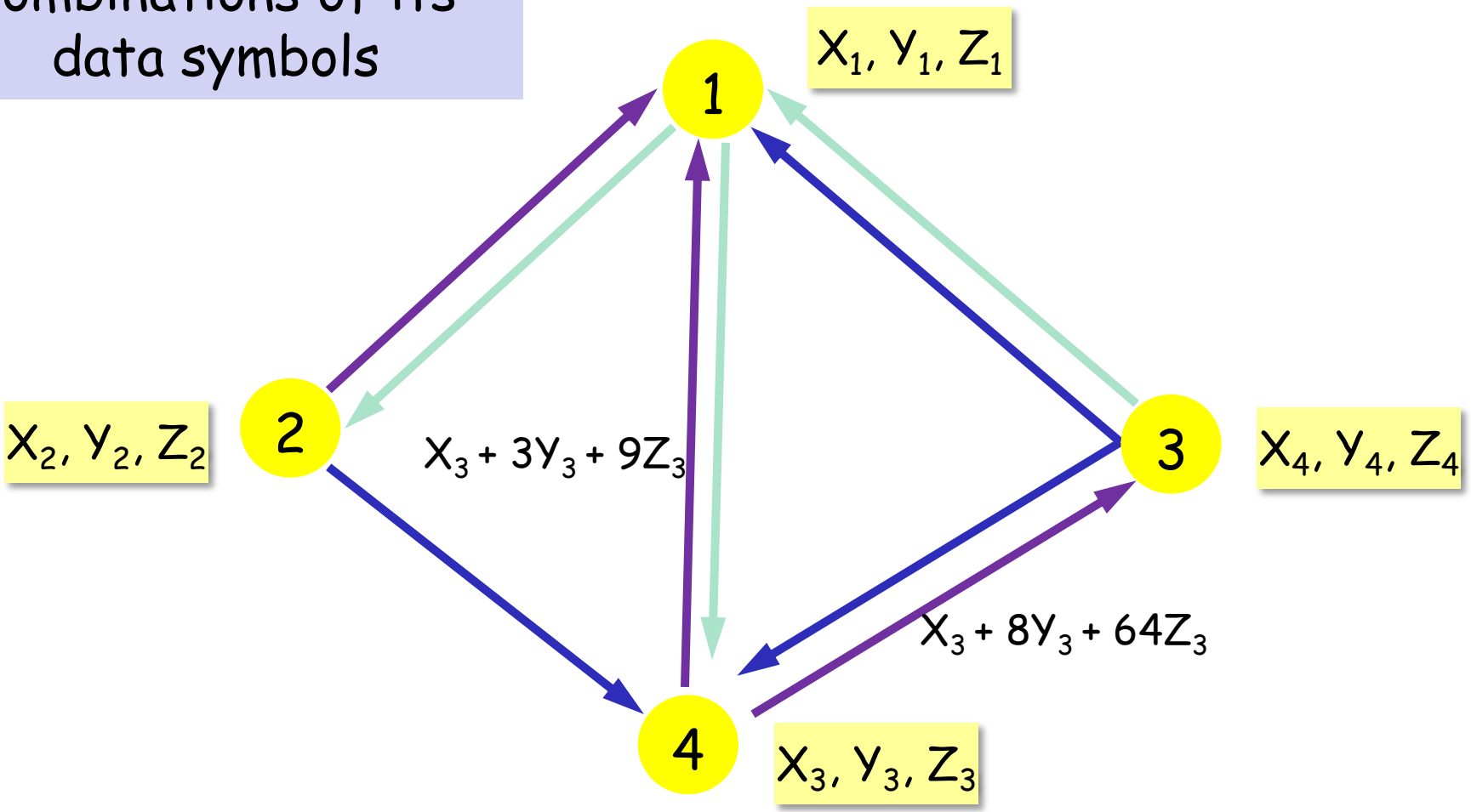


Local Coding



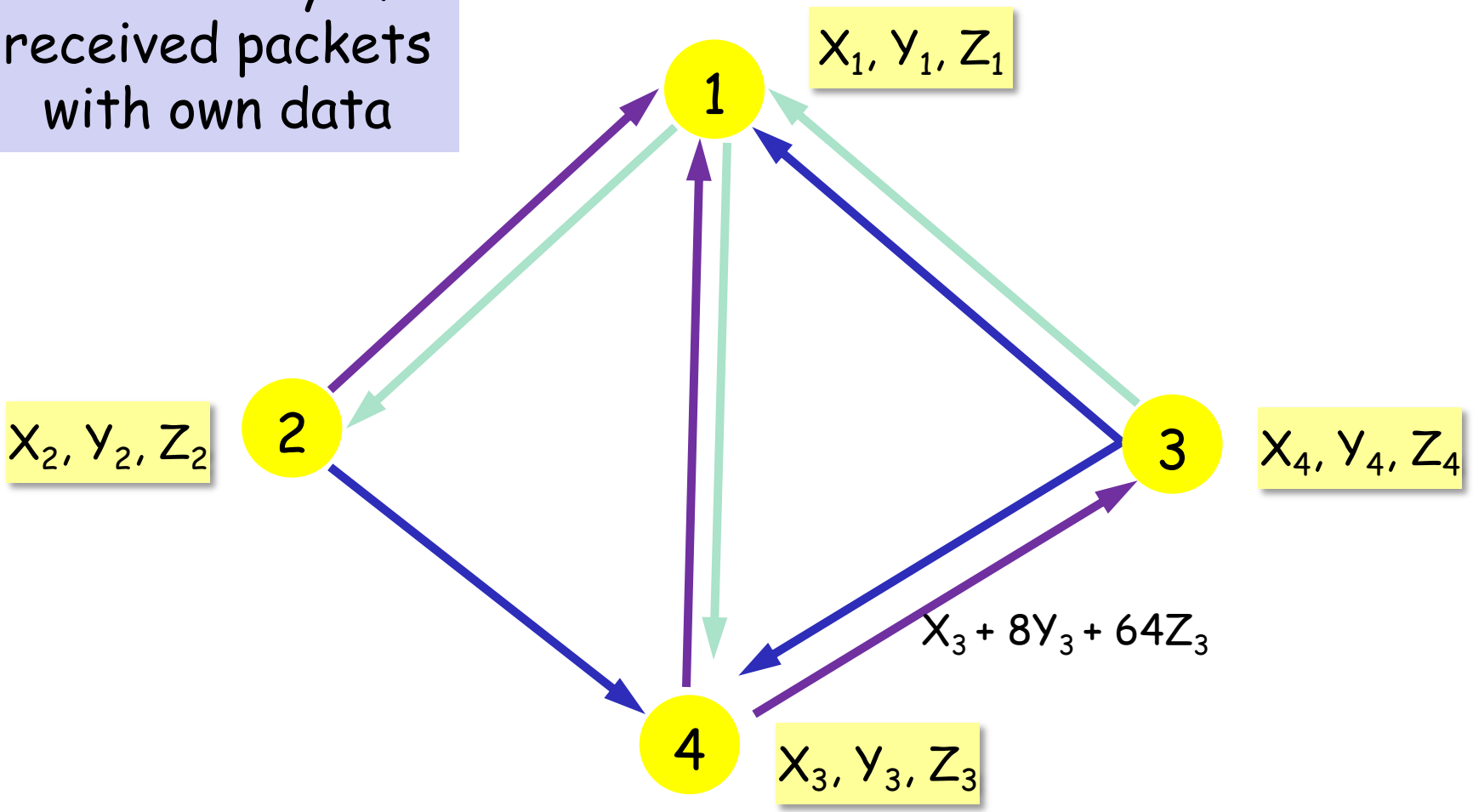
Each directed link can carry 1 symbol

Each node sends linear combinations of its data symbols



Each node checks consistency of received packets with own data

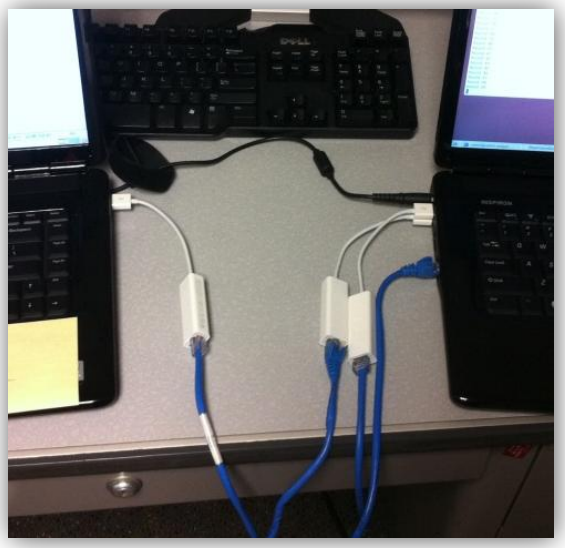
$X_1 + 3Y_1 + 9Z_1 = X_3 + 3Y_3 + 9Z_3$?

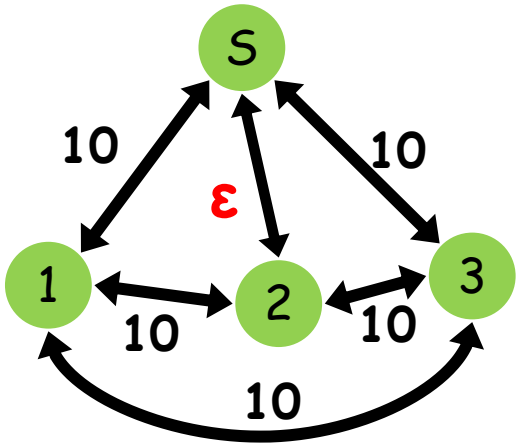


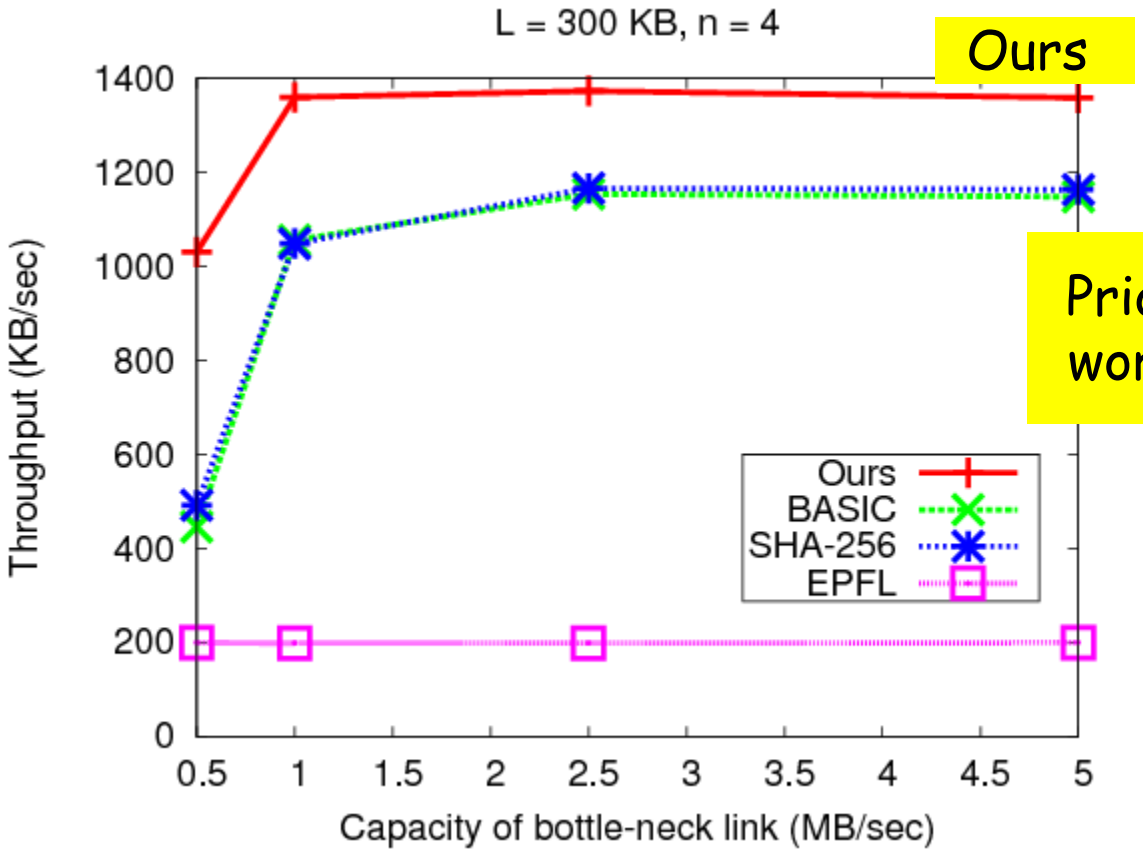
Failure Detection

- Equality function
- Faulty nodes should not be able to make unequal values appear equal
- Utilize link capacities

Experimental Evaluation



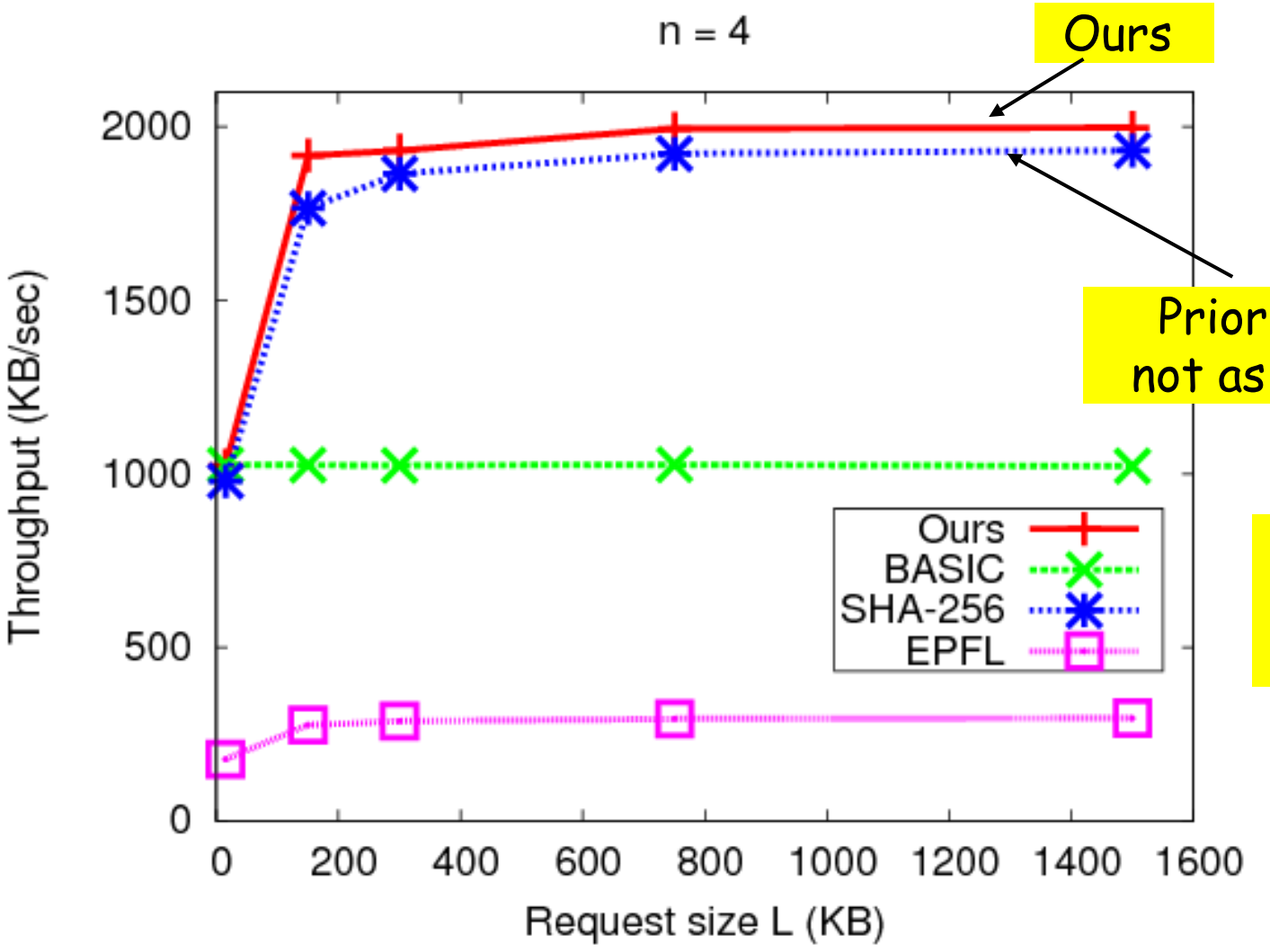




Ours

Prior work

Ethernet: Failure-Free Case



Wrap-Up

This Talk

- Byzantine broadcast

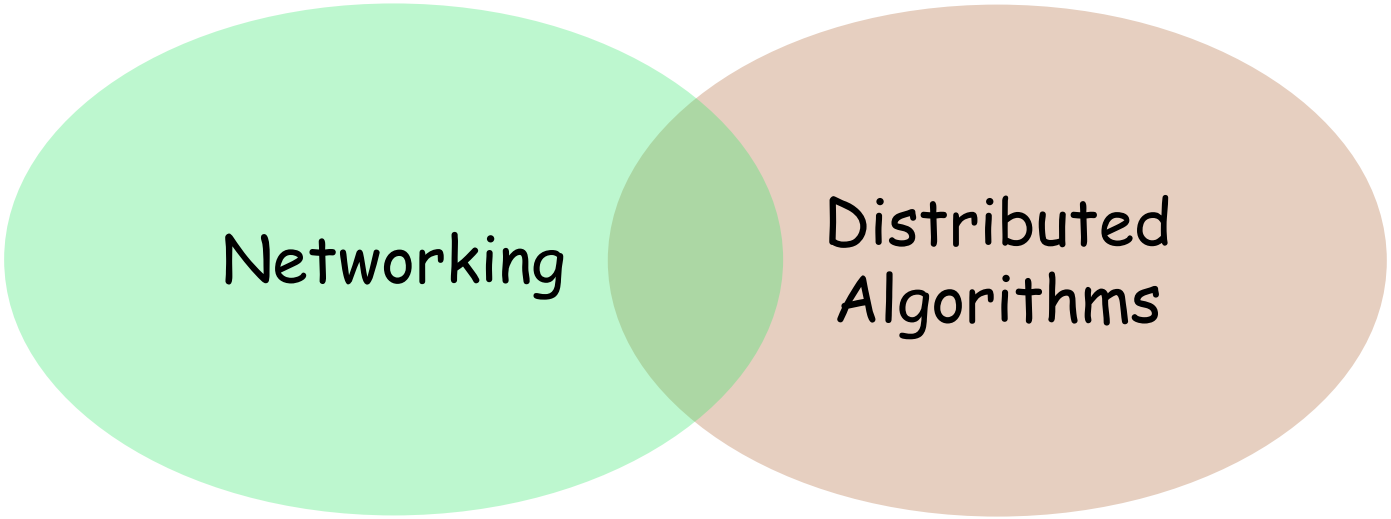
- To illustrate

  impact of network

  on algorithm design & performance

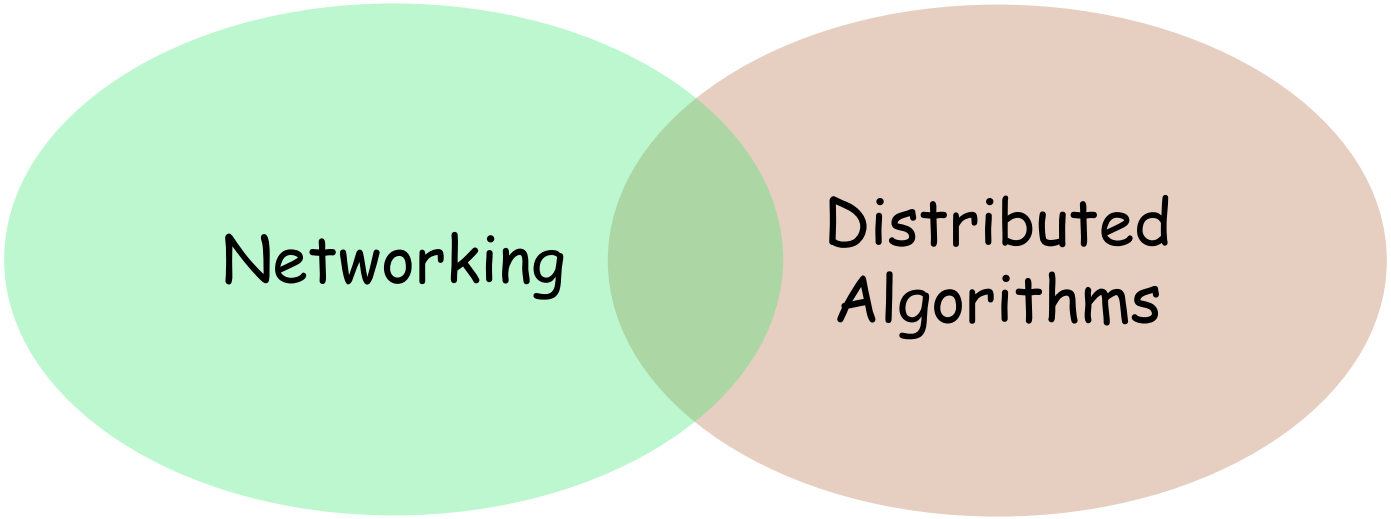
Rich Problem Space

- More realism in network model can change solutions quite significantly



Rich Problem Space

- Networks … wired, wireless
- Computations … many of interest
- Metrics … how to capture impact of networks?

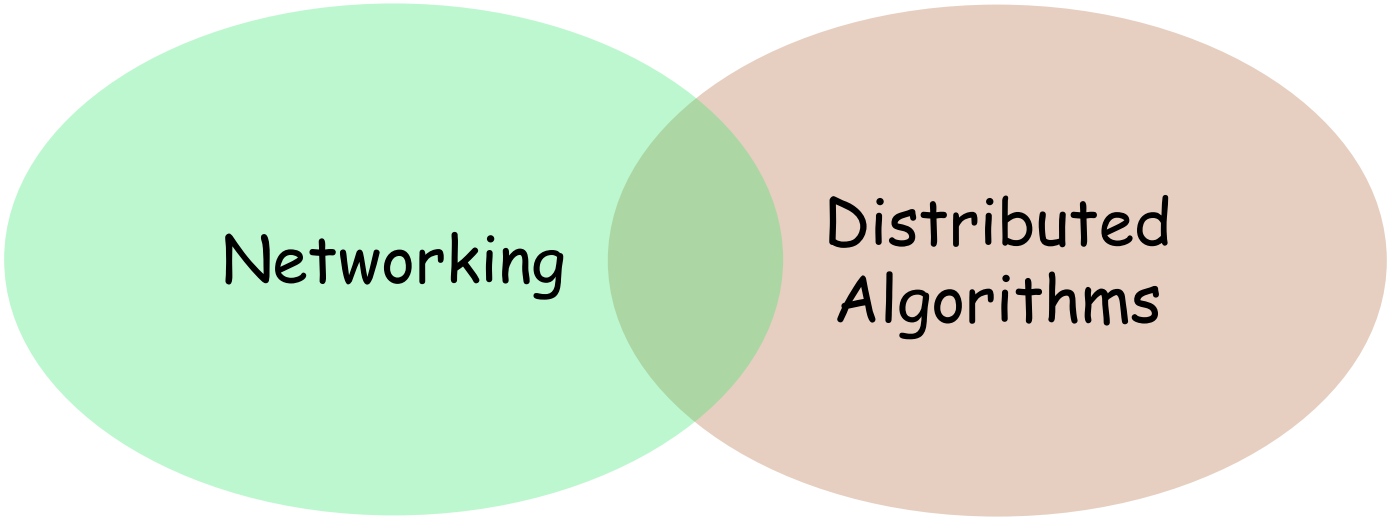


Rich Problem Space

Need new ways to

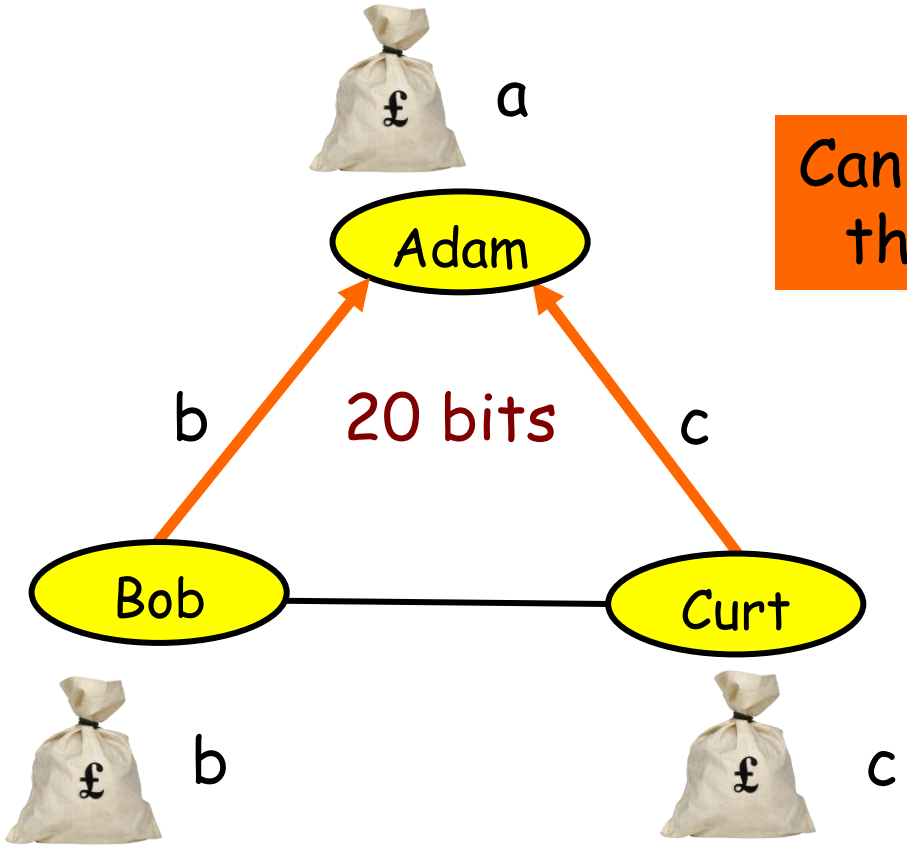
formulate & solve

old problems



Thanks!

Puzzler



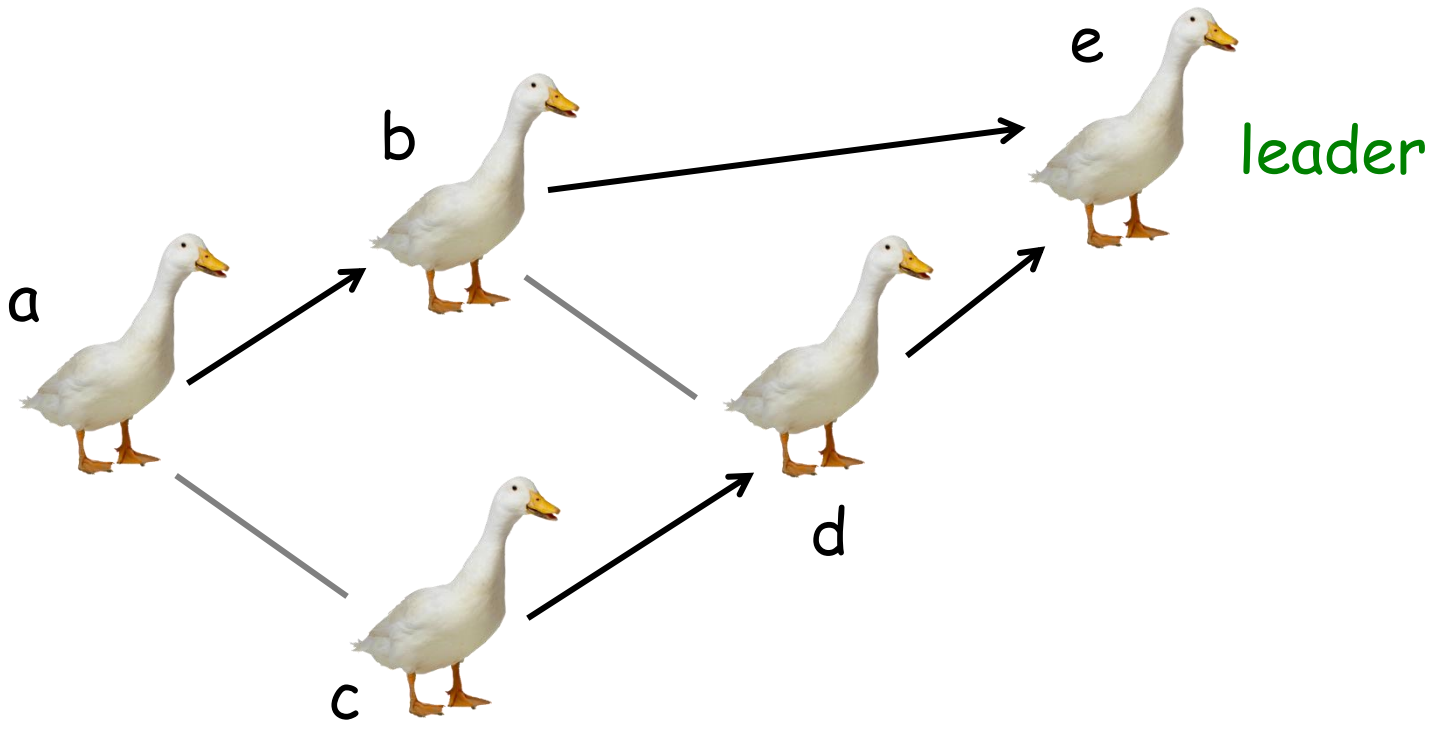
Can we do better than 20 bits ?

Thanks!

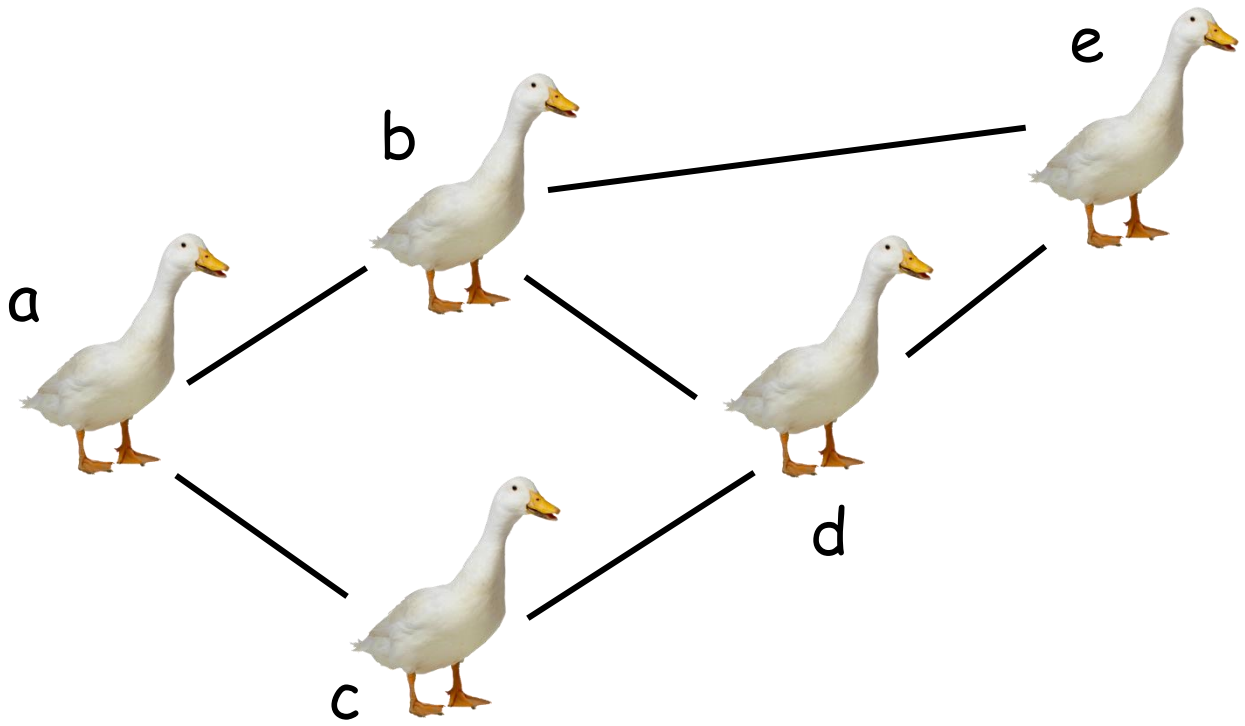
Thanks!

Average Consensus

- Centralized solution

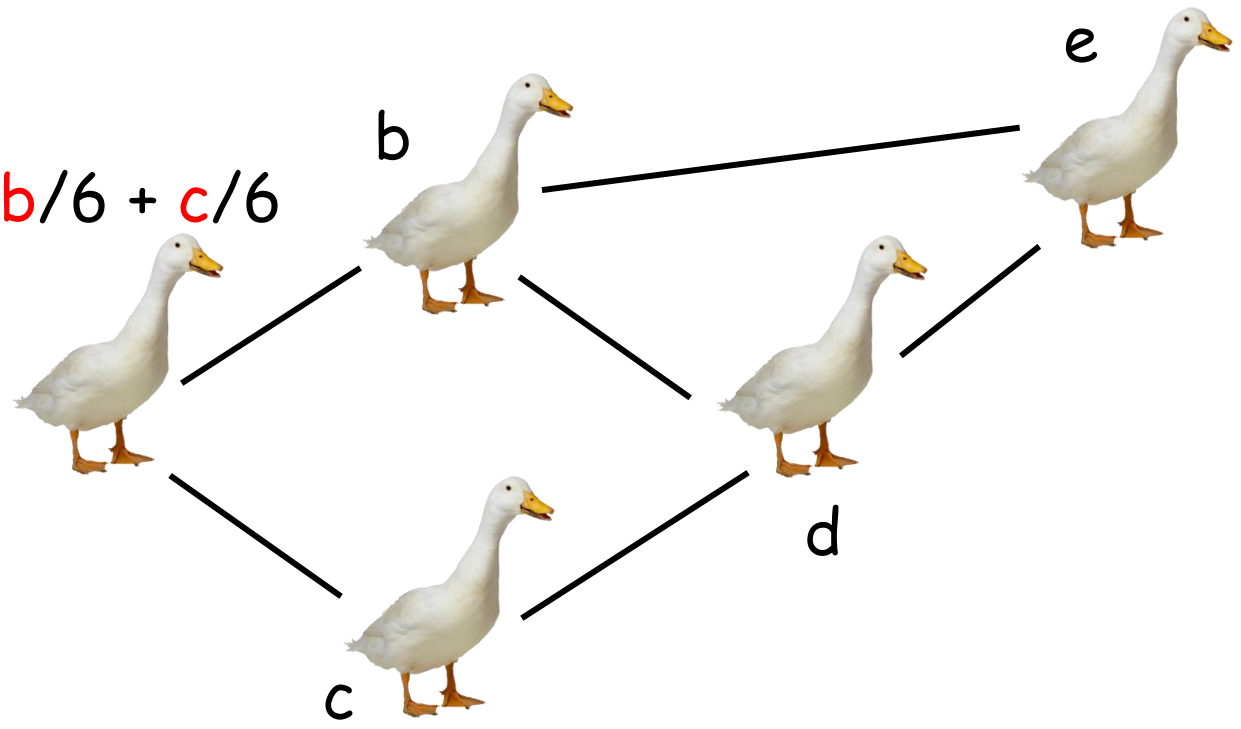


Iterative Average Consensus

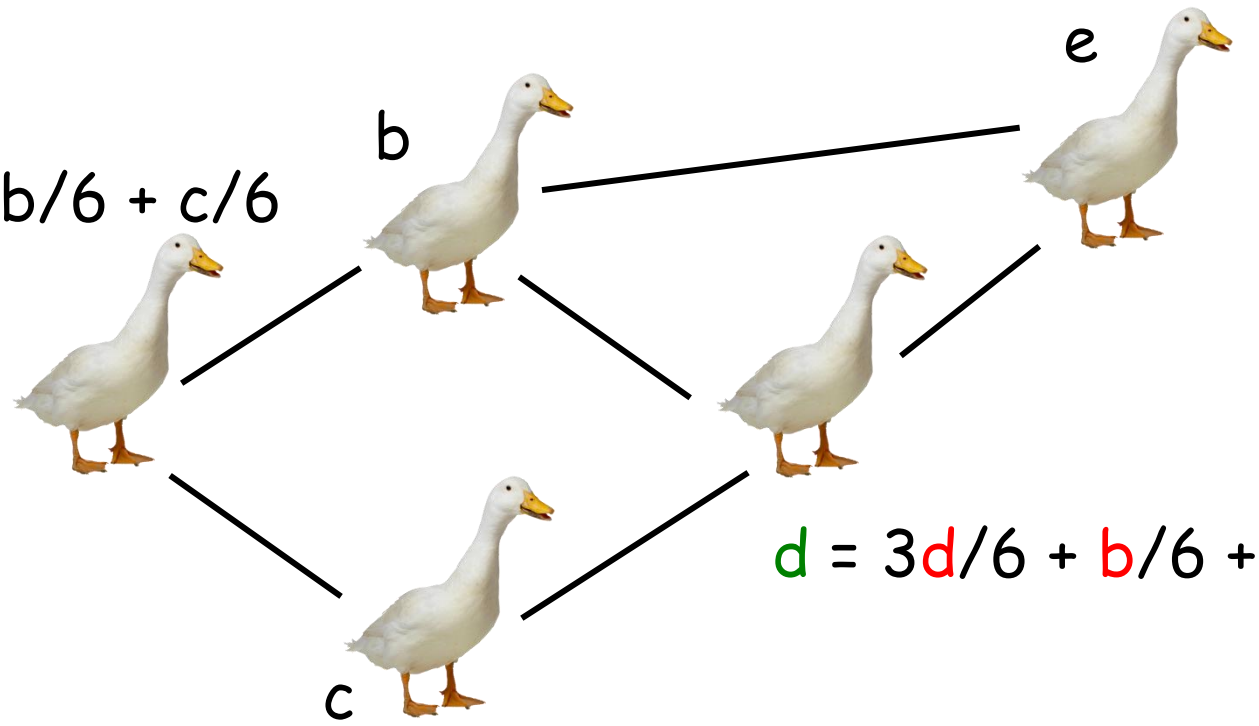


Iterative Average Consensus

a = 4a/6+ b/6 + c/6



Iterative Average Consensus

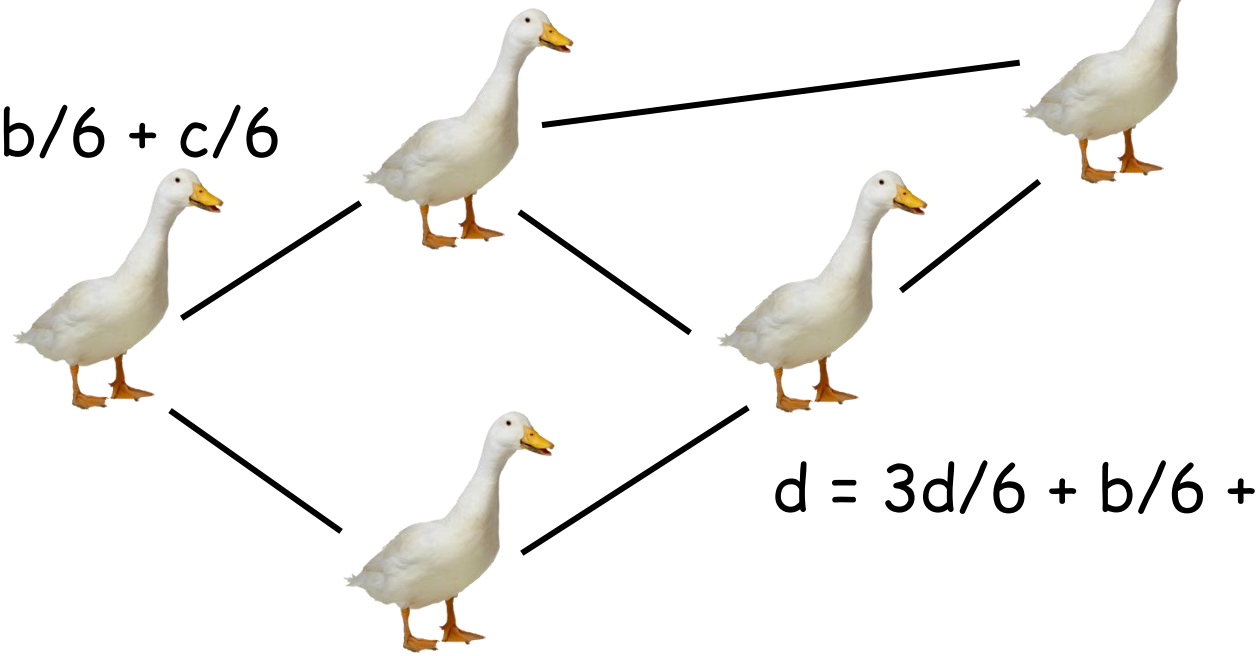


Iterative Average Consensus

b = 3b/6 + a/6 + d/6 + e/6

e = 3e/6 + b/6 + c/6 + d/6

a = 4a/6+ b/6 + c/6



d = 3d/6 + b/6 + c/6 + e/6

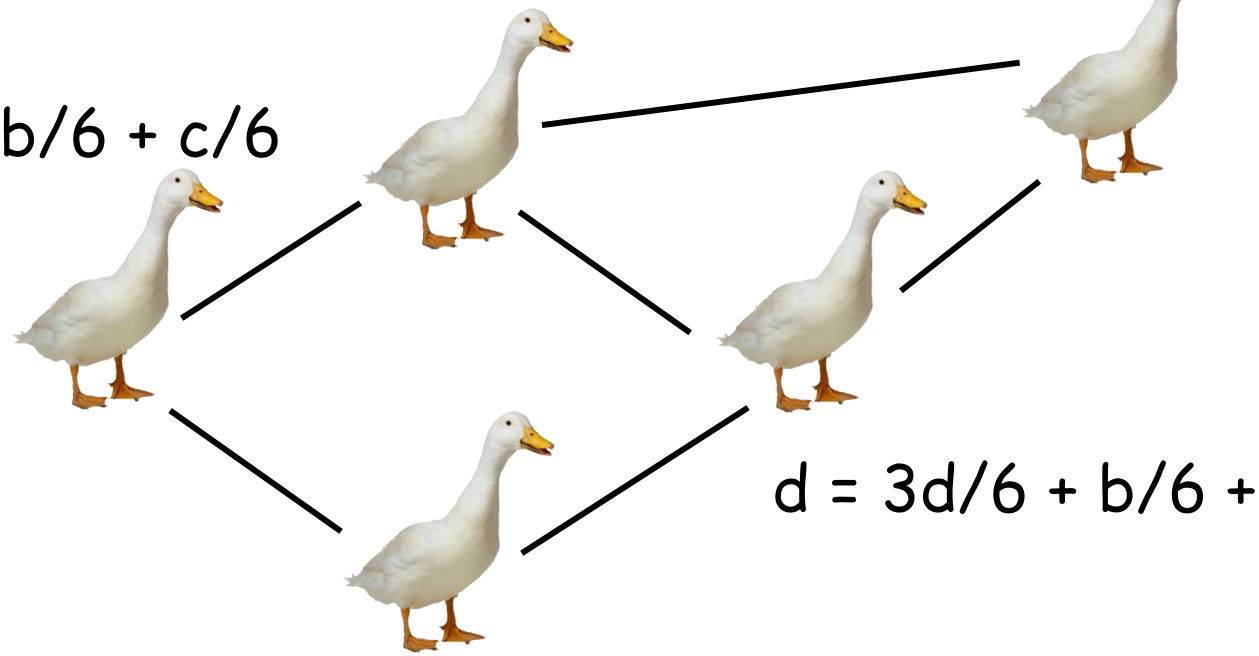
c = 4c/6 + a/6 + d/6

Iterative Average Consensus

b = 3b/6 + a/6 + d/6 + e/6

e = 3e/6 + b/6 + c/6 + d/6

a = 4a/6+ b/6 + c/6



d = 3d/6 + b/6 + c/6 + e/6

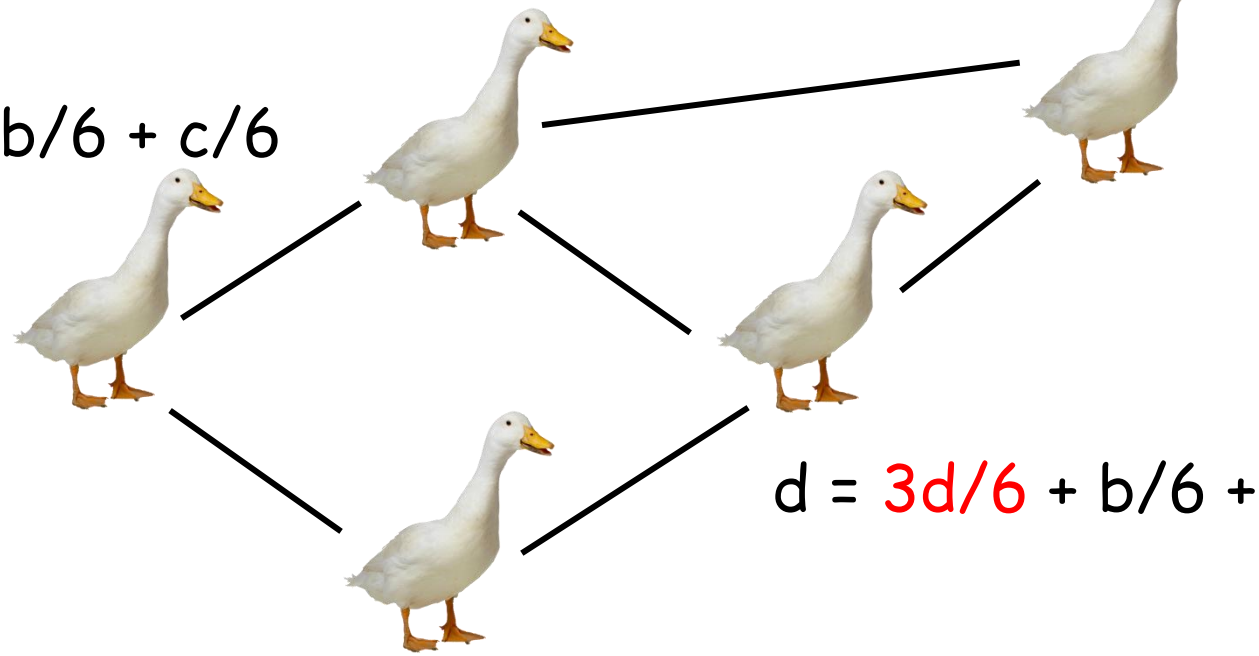
c = 4c/6 + a/6 + d/6

Iterative Average Consensus

b = 3b/6 + a/6 + d/6 + e/6

e = 3e/6 + b/6 + c/6 + d/6

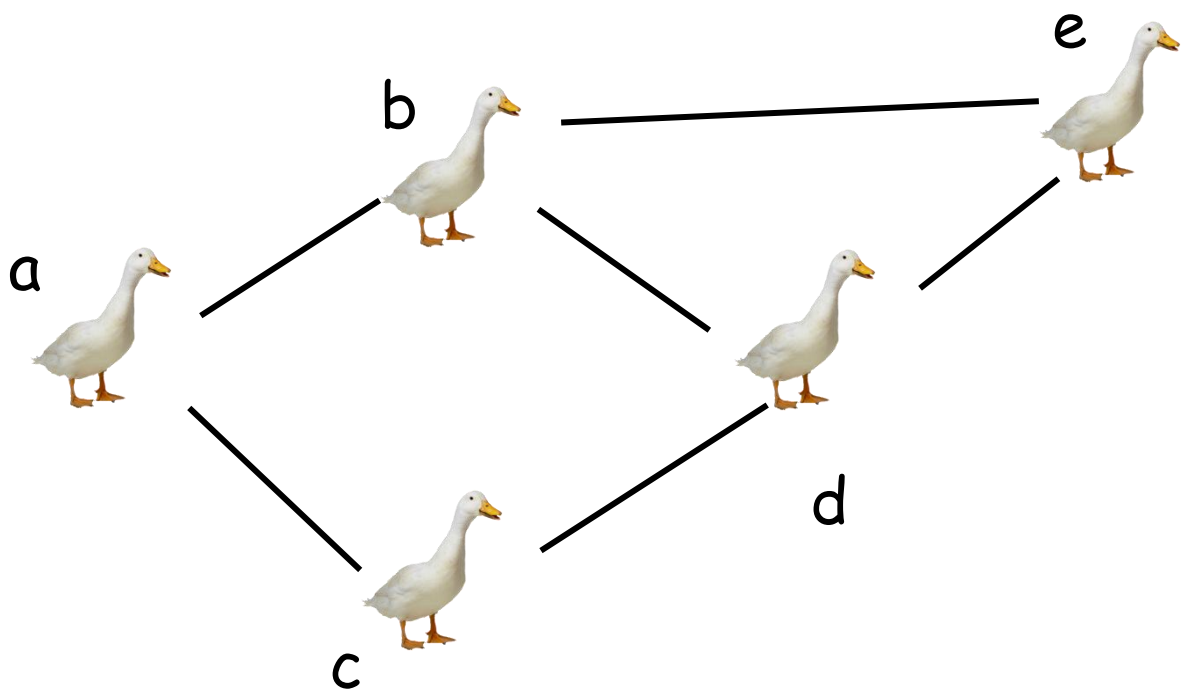
a = 4a/6+ b/6 + c/6



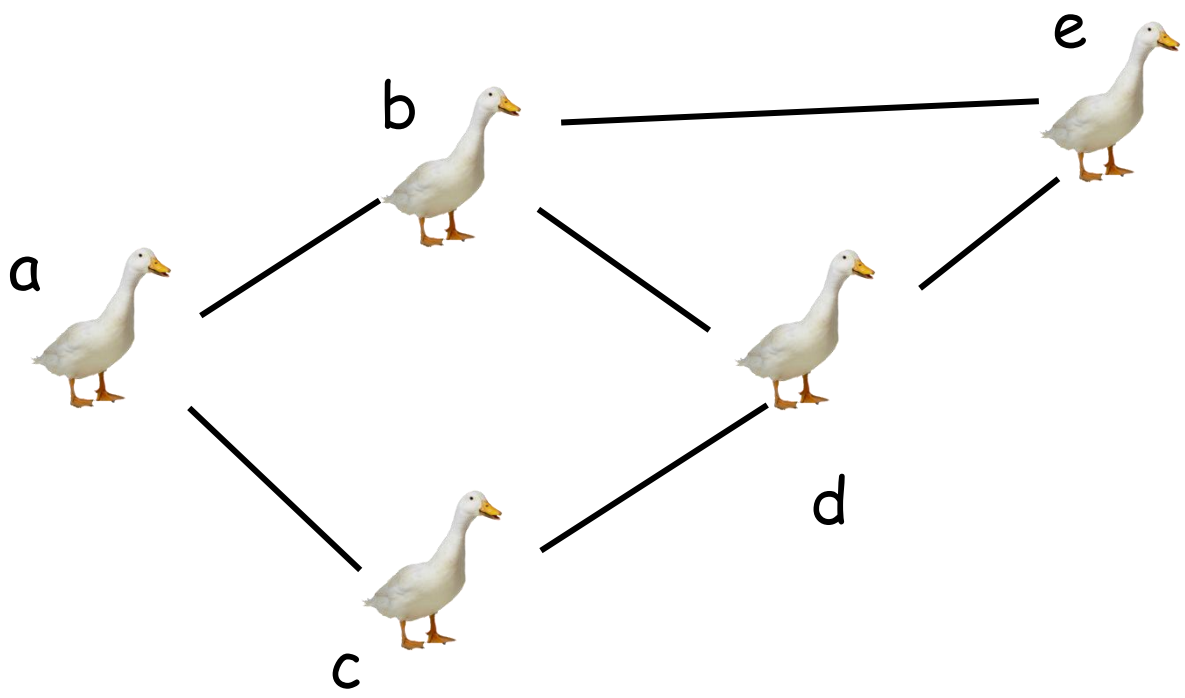
d = 3d/6 + b/6 + c/6 + e/6

c = 4c/6 + a/6 + d/6

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$

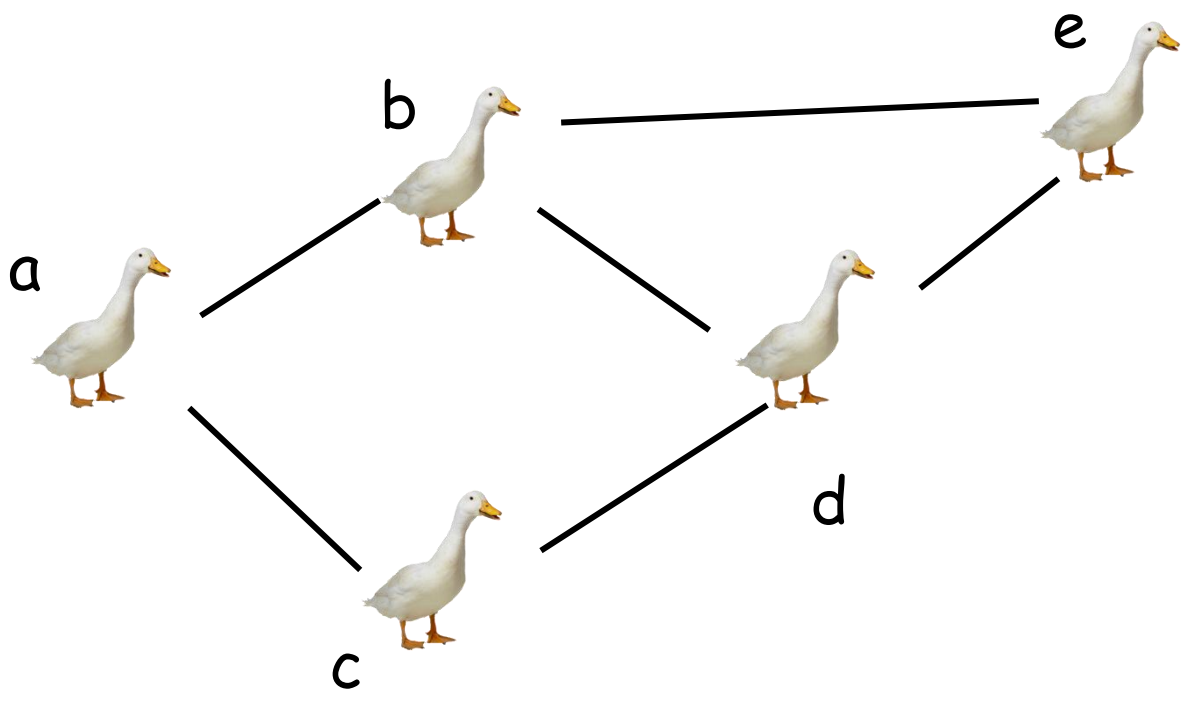


$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$



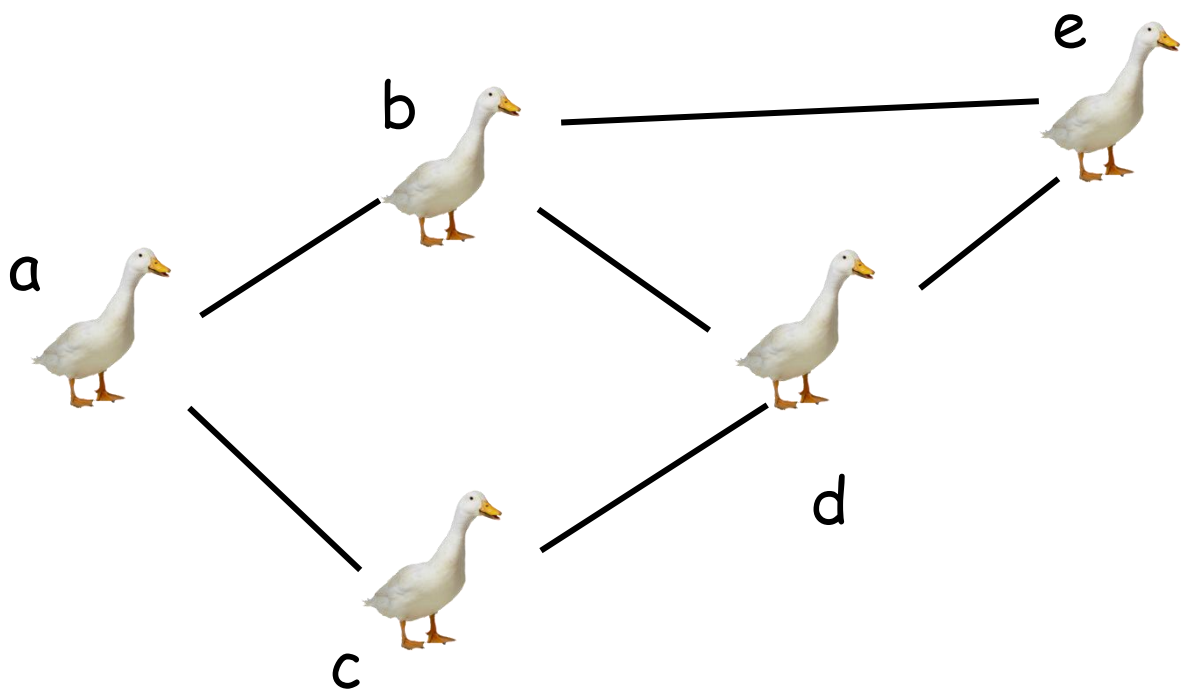
$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$

Node B sends fractions of its mass to neighbors



$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$

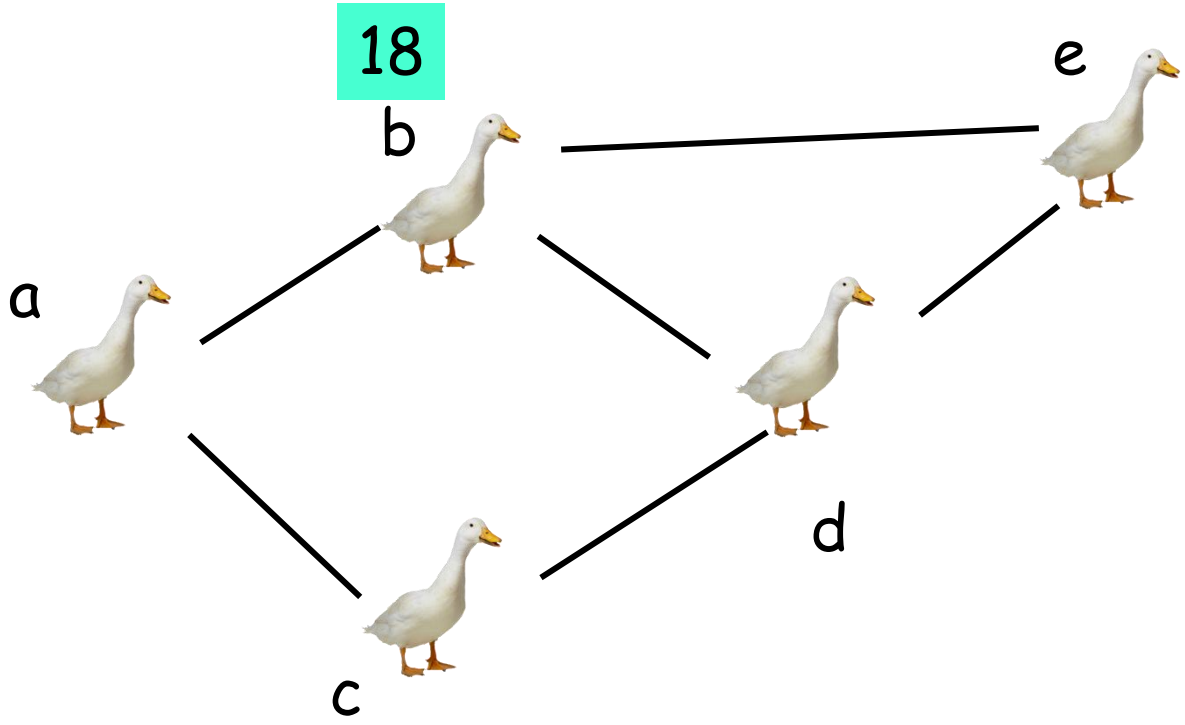
Node B accumulates mass sent by neighbors



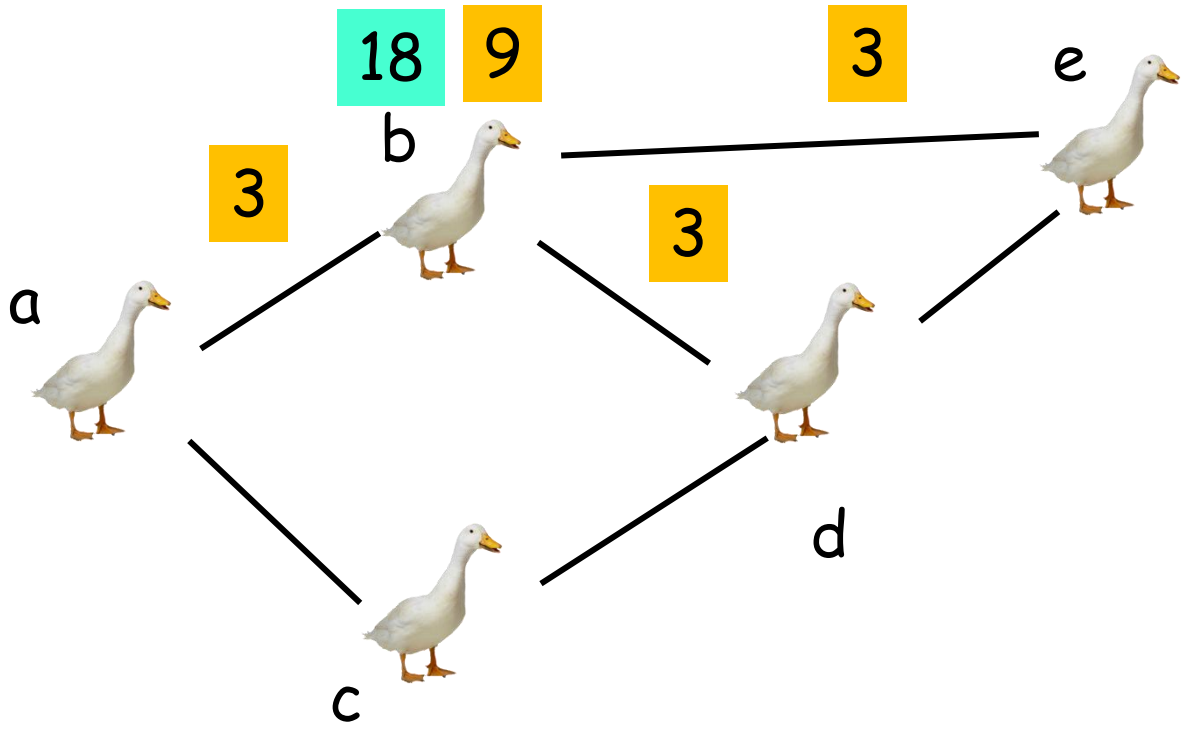
Well-Known Result

- State of the nodes converges to average
- Results assuming loss-less links

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$



$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 0 & 1/6 & 1/6 \\ 1/6 & 0 & 4/6 & 1/6 & 0 \\ 0 & 1/6 & 1/6 & 3/6 & 1/6 \\ 0 & 1/6 & 0 & 1/6 & 4/6 \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ d \\ e \end{bmatrix}$$



Wireless Network Model

- Time varying topology
  … mobility of nodes, links breaking, etc.

- Algorithm converges to average
  if available links are reliable
  and the topology is connected over time

More Accurate Model ?

- Unreliable transmissions
- "Mass transfer" needs to be reliable for the algorithm to work

- B should know that A has received mass
- A should know that B knows that A has received mass
- …
- Common knowledge required

Unreliable Links

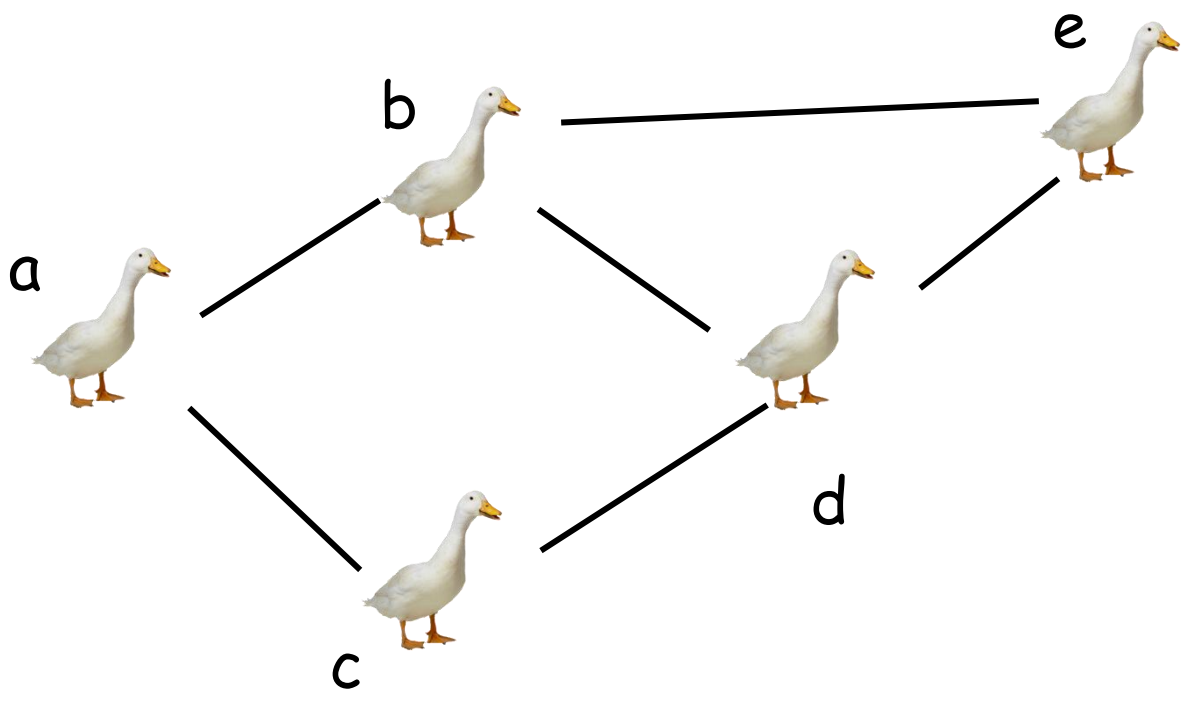
- How to design iterative algorithms in presence of unreliable links

- Changes the problem & solution approach significantly

- Possible to converge to average

Lossy Links

- Node B may not be able to reliably transfer mass to a neighbor

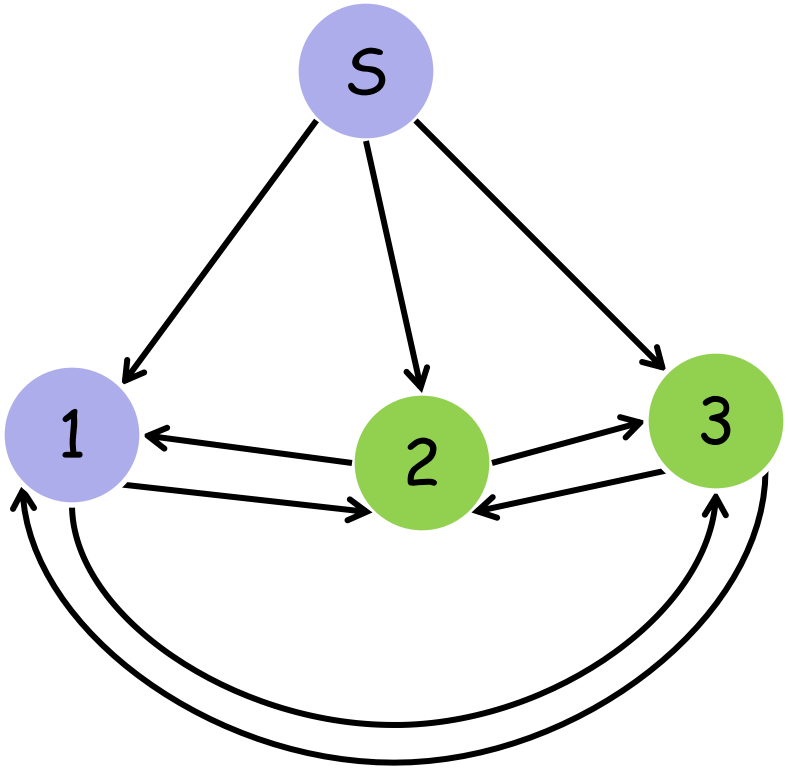


Thanks!

Asymmetric Networks

- Upper bound 1 on throughput

min-cut(S,X | f peers removed)

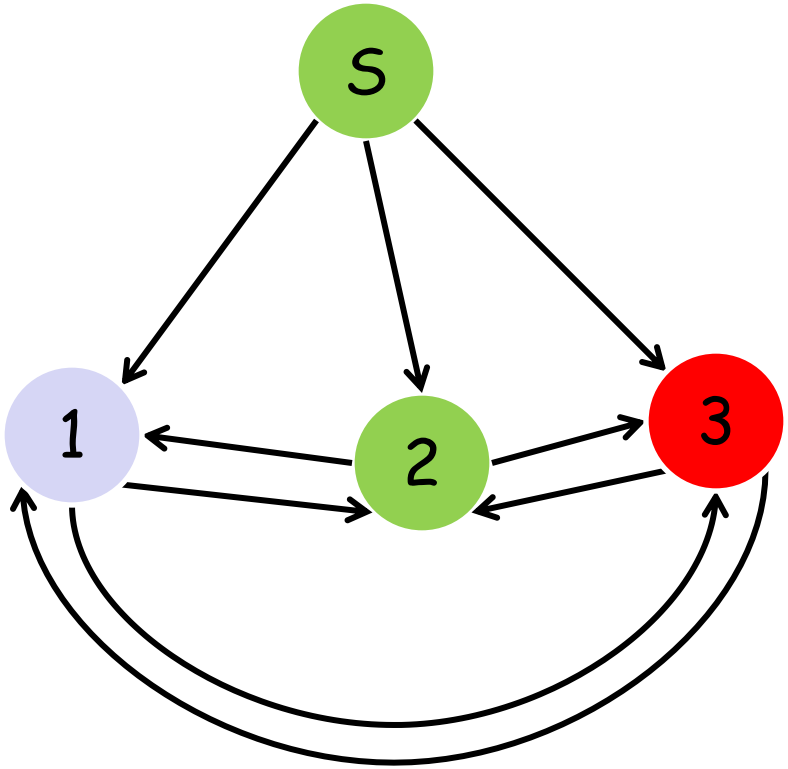


f = 1
X = 1

Asymmetric Networks

- Upper bound 1 on throughput

  min-cut(S,X | f peers removed)



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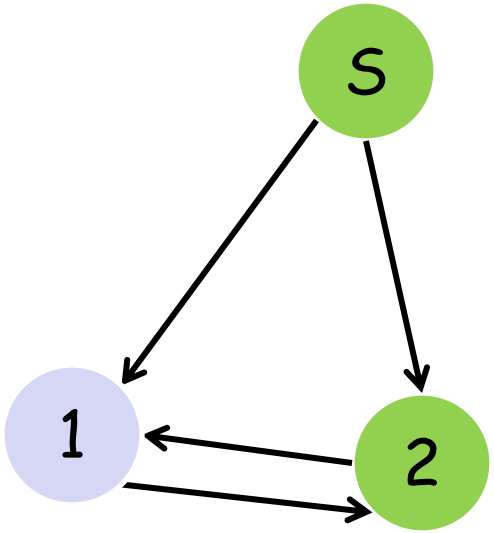
X = 1

Asymmetric Networks

- Upper bound 1 on throughput

min-cut(S,X | f peers removed)

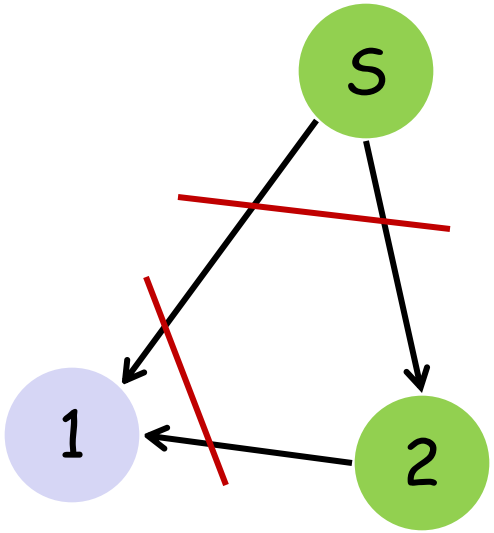


f = 1
X = 1

Asymmetric Networks

- Upper bound 1 on throughput

min-cut(S,X | f peers removed)

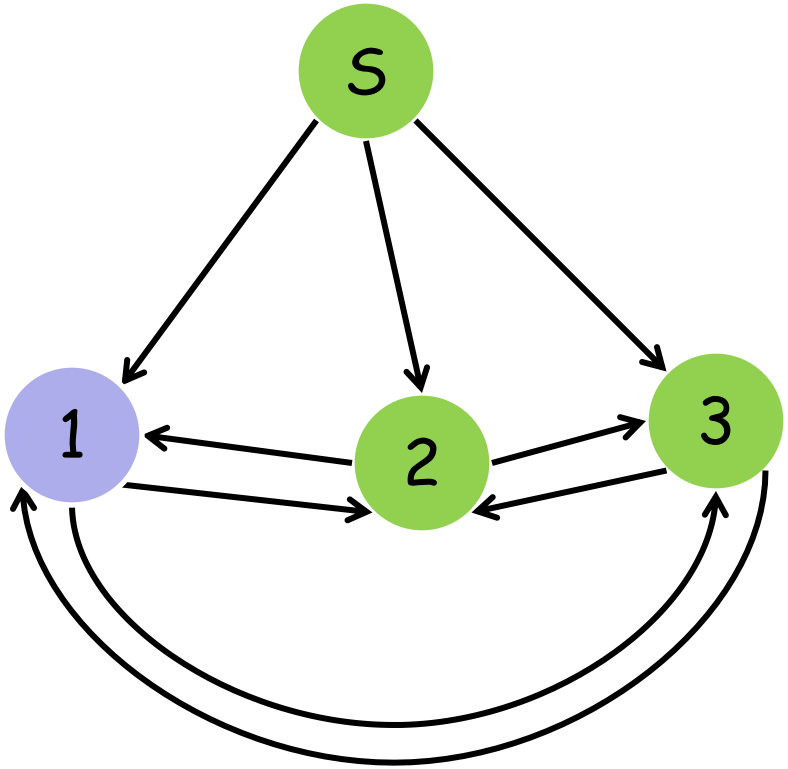


f = 1
X = 1

Asymmetric Networks

- Upper bound 2 on throughput

incoming(X | f nodes removed)

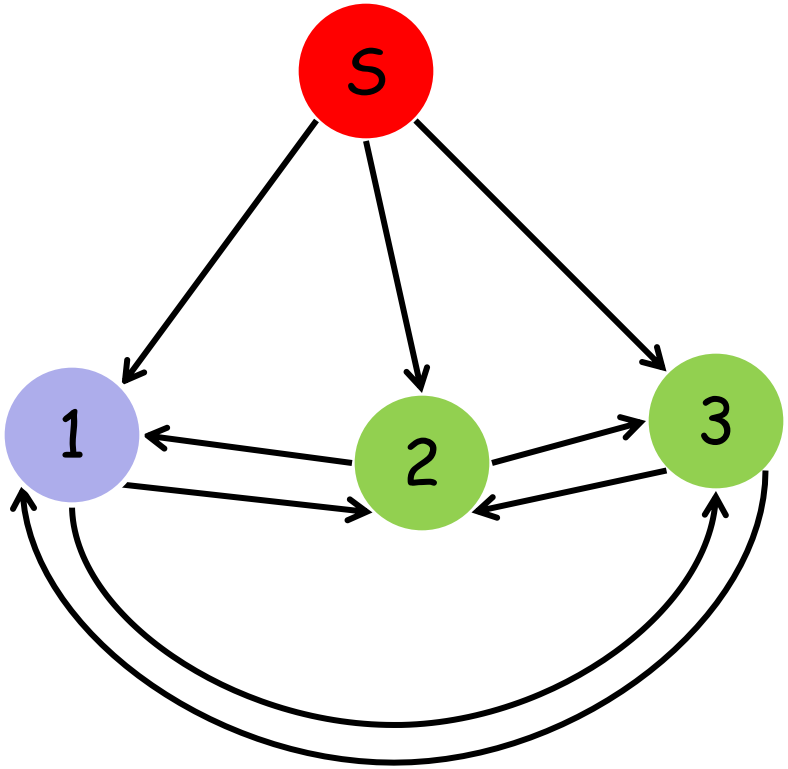


f = 1
X = 1

Asymmetric Networks

- Upper bound 2 on throughput

incoming(X | f nodes removed)

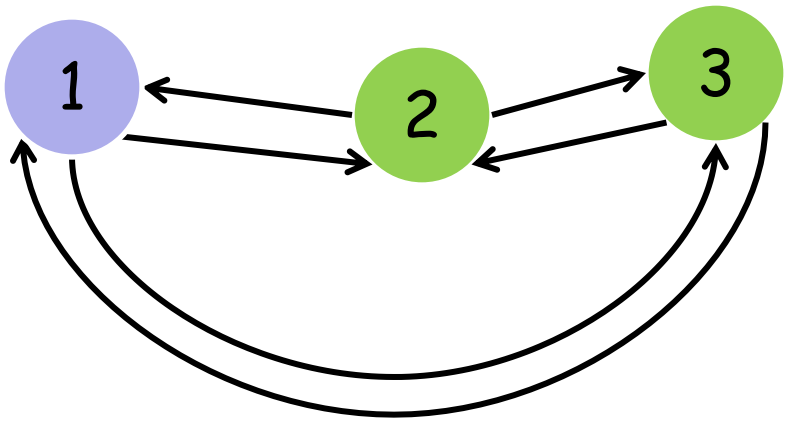


f = 1
X = 1

Asymmetric Networks

- Upper bound 2 on throughput

incoming(X | f nodes removed)

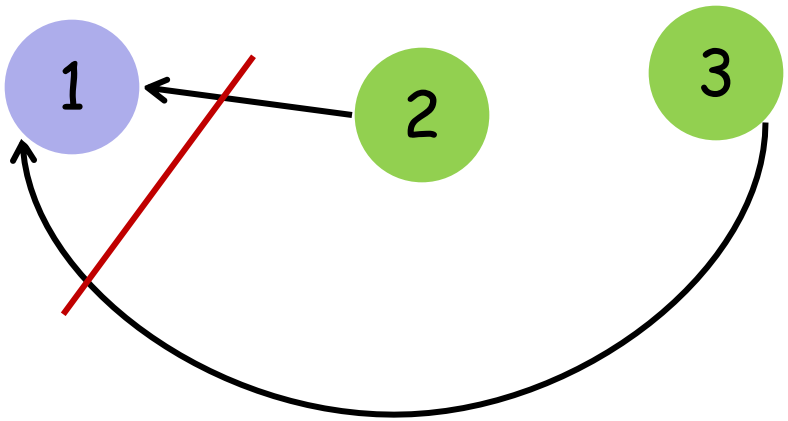


f = 1
X = 1

Asymmetric Networks

- Upper bound 2 on throughput

incoming(X | f nodes removed)



f = 1
X = 1

4-Node Networks

- Our approach using

  capacity-dependent coding

  optimal

Arbitrary Networks

Reduction

Consensus with Byzantine fault tolerance

- ➔ Consensus with Byzantine fault detection
- ➔ Multi-party equality (with local communication)

Local Coding

- No forwarding of packets
- Code and check locally
- Desirable property when using in Byzantine broadcast … faulty nodes cannot tamper packets, if they don't forward anything

Claims

- Bad nodes cannot tamper someone else's packets
- If no good node finds inconsistency,

  their values are identical

- This equality checking helps

  achieve Byzantine broadcast within

  constant fraction of optimal

After Failure Identified