Socially-Optimal and Truthful Online Spectrum Auction for Secondary Communication[*]

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ABSTRACT

Spectrum auctions are efficient mechanisms for licensed users to relinquish their under-utilized spectrum to secondary links for monetary remuneration. Truthfulness and social welfare maximization are two natural goals in such auctions, but cannot be achieved simultaneously with polynomial-time complexity by existing methods, even in a static network with fixed parameters. The challenge escalates in practical systems with QoS requirements and volatile traffic demands for secondary communication. Online, dynamic decisions are required for rate control, channel evaluation/bidding, and packet dropping at each secondary link, as well as for winner determination and pricing at the primary user. This work proposes an online spectrum auction framework with cross-layer decision making and *randomized* winner determination on the fly. The framework is truthful-in-expectation, and achieves close-to-offline-optimal time-averaged social welfare and individual utilities with polynomial time complexity. A new method is introduced for online channel evaluation in a stochastic setting. Simulation studies further verify the efficacy of the proposed auction in practical scenarios.

1. INTRODUCTION

As wireless devices and applications proliferate, static spectrum allocation (*e.g.*, by FCC in the U.S.A.) can no longer meet the dynamic demand for channels, resulting in congestion in unlicensed spectrum and under-utilization in the licensed counterparts [1]. Spectrum leasing [2] has been proposed to allow a primary user (licensed spectrum user) to lend its idle channel to secondary users with a monetary remuneration, for improved utilization of the spectrum resource.

Auctions are natural mechanisms for implementing spectrum leasing. Each secondary user can strategically bid for channels from the primary users to maximize its utility. Recent research in spectrum auction [3, 4, 5, 6, 7, 8, 9, 10] has mainly focused on the design of truthful and efficient auction mechanisms that can avoid market manipulation while boosting social welfare, *i.e.*, the overall utility of all participants in the auction. However, two fundamental issues are still not well addressed, when we consider repeated auctions in a long-run system: **Issue 1**, *how can each bidder decide its true value of the spectrum for its utility maximization over the long term?* And **Issue 2**, *how can the long-term social welfare be maximized by exploiting the spatial reuse of channels without transmission collisions?*

Before discussing the two issues, we first define the true value.

Definition 1 (True value). *The true value of a channel is one that satisfies the following condition: if a secondary link wins the channel by paying a price equal to the true value, then it ends up with the same time-averaged utility as when losing the channel.*

Issue 1. A bidder's true valuation of a channel is always assumed known in the current literature. This may be reasonable in the static case, where the true value is only related to the utility of one-time data delivery. However, in a dynamic network where data traffic from each secondary user varies unpredictably how to decide the true value of a channel that maximizes the *long-term* utility while maintaining network stability, is a non-trivial issue. Furthermore, we consider a practical quality-of-service goal for secondary communication, that there is a predefined maximum allowable delay for the delivery of each packet, by when it is either delivered or dropped. This goal further complicates channel evaluation. Besides, channel evaluation at one time is also closely connected with channel allocation and rate control in the subsequent time. How can a bidder calculate the impact of winning/losing the channel in time slot t on its long-term utility, without any future information?

The following example illustrates the challenge from issue 1. Consider a channel with unit capacity, *i.e.*, one packet can be transmitted in each time slot. Suppose a packet queue is maintained at each bidder, and one packet arrives at the queue in each time slot. The gain of getting one packet delivered is 5. Each packet should be delivered within 3 slots, or dropped at the penalty of 10 if not delivered.

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(a) Policy A

Queue	1	2	3	3
Bid price	5	5	15	15
Charged price	0	0	5	5
Action	None	None	Deliver	Deliver
Utility	0	0	0	0

(b) Policy B

Queue	1	2	2	2
Bid price	5	10	10	10
Charged price	0	4	4	4
Action	None	Deliver	Deliver	Deliver
Utility	0	1	1	1

Table 1: A motivating example.

The bidder, who knows nothing about future packet arrival, comes up with two policies for channel evaluation: i) policy A evaluates the channel based on the bidder's utility in the current time slot, and computes the value of the channel in current slot as $5 + \mathbf{1}_{drop} * 10$, where $\mathbf{1}_{drop}$ is an indicator function that equals 1 if there is any packet reaching its maximum allowable delay and 0 otherwise, given that the utility of the bidder is $5 - (5 + \mathbf{1}_{drop} * 10) = -\mathbf{1}_{drop} * 10$ if it wins the channel and is charged at the true value of $5+\mathbf{1}_{drop}*10$, which is equivalent to the utility of $-\mathbf{1}_{drop}*10$ if not getting the channel; and ii) policy B considers not only any to-be-outdated packet, but also the other packets in the queue, and derives the value of the channel in current slot as $5 * m + \mathbf{1}_{drop} * 10$, where m is the number of packets that have not reached their maximum allowable delays.

The bidder bids in each time slot with its true value of the channel. Suppose the bidder can always win the channel with any price higher than 9. The charge to the bidder is 4 (or 5) if it wins the channel with a bidding price 10 (or 15), and 0 if it loses. Table 1.a and 1.b present the bidder's queue length, channel evaluation (*i.e.*, bidding price), charged price, actions (deliver or drop a packet, or none) and utility in 4 consecutive time slots. The overall utility obtained using policy A and B is 0 and 3, respectively. Hence, deciding true values based only on utility in an individual time slot is not suitable for long-term utility maximization in a dynamic system. A more elaborated channel evaluation method is needed.

Issue 2. The second issue typically leads to an NP-hard problem, since collision-free channel allocation for social welfare maximization is equivalent to the weighted maximum independent set problem. Even for social welfare maximization in a static network, only heuristics are exploited to approximate the optimum [3, 4, 5, 6, 7, 8, 9, 10]. The problem becomes more difficult when we set to achieve *long-term* social welfare maximization in a dynamic system, together with guarantees of truthfulness in bids.

Our contributions. We propose a new, online auction framework to dynamically evaluate the true value of channels in each time slot, while maximizing the time-averaged individual utility and social welfare in the long run, under practical system dynamics. In the framework, each secondary link strategically decides its channel evaluation/bids, transmission rates and packet dropping in each time slot, through an online algorithm utilizing Lyapunov optimization [11]. Upon receiving the bids, a primary user selects a set of collision-free bids with maximum expected weights as the winners of the auction of its channel, based on a *random* access control protocol derived with Glauber dynamics [12]. Subsequently, the primary user charges each winner a tailored price, and the winning secondary links schedule their data transmissions on the obtained channels. Below we summarize the contributions of this work.

▷ To our knowledge, this work is the first to dynamically evaluate the true value of a channel in an online auction, instead of assuming it as *a priori* knowledge, for maximizing long-term averaged utility at each secondary link. Our main idea is that the true value of the channel at a bidder at each time should be proportional to the urgency level of delivering packets: higher when the cumulative delay of packets in the queue is large, *i.e.*, when packet dropping is a more imminent threat.

▷ As the current best result, our proposed randomized auction mechanism achieves the arbitrarily close-to-offline-optimal long-term *averaged* social welfare with *polynomial-time complexity* and the guarantee of *truthfulness in expectation*, *i.e.*, bidding with the true evaluation is the optimal strategy for the bidder in maximizing its expected time-averaged utility. Moreover, the long-term *averaged* utility of each individual secondary link is also arbitrarily close to its offline optimum. There is a tradeoff between how close these quantities are to their offline optima, and the maximum allowable delivery delay of packets, which we investigate in details.

In the rest of the paper, we discuss related literature in Sec. 2 and introduce the system model in Sec. 3. The auction framework and online algorithms are in Sec. 4. A benchmark algorithm is presented in Sec. 5 for comparison. We evaluate the efficacy of the proposed framework through theoretical analysis and simulation studies in Sec. 6 and Sec. 7, respectively. Sec. 8 concludes the paper.

2. RELATED WORK

Auctions have been widely studied for trading idle spectrum between a primary user and the secondary links. Truthfulness and social welfare optimality are two natural economic properties in spectrum auction design. The VCG mechanism, by Vickrey [13], Clarke [14] and Groves [15], is known to achieve both goals concurrently. However, the VCG mechanism can only be efficient when optimal solutions can be computed in polynomial time, while the collision-free channel allocation problem is typically NP-hard since it requires solving a maximum weight independent set problem.

Existing studies often focus on truthfulness while approximating the optimal social welfare using efficient algorithms. Zhou *et al.* [3] propose the first truthful spectrum auction with a monotonic allocation mechanism. A truthful double auction with multiple sellers is introduced by Zhou *et al.* [5]. The work of Jia *et al.* [4] maximizes the expected revenue of the spectrum seller, and approximately maximizes the social welfare with the guarantee of truthfulness. Wang *et al.* [6] present the first truthful *online* double auction for the spectrum market, but with a complete interference graph. A multi-unit double auction with truthfulness and asymptotic efficiency in social welfare is proposed by Xu *et al.* [7], without spacial spectrum reuse. Dong *et al.* [16] investigate the time-frequency flexibility using a combinatorial auction with both truthfulness and worst-case approximation of the social welfare.

With a natural extension of truthfulness into truthfulness in expectation, *randomized* algorithms are designed to approximate expected social welfare maximization in a spectrum auction. Gopinathan *et al.* [8] investigate truthful-in-expectation spectrum auction mechanisms with a balance between social welfare and fairness among secondary users. Hoefer *et al.* [10] present a novel linear program formulation using a non-standard parameter of the conflict graph, achieving an approximation of the social welfare with truthfulness in expectation. Zhu *et al.* [9] are first to study spectrum auctions for multi-hop communications, with truthfulness in expectation and provable approximation for optimal social welfare.

The above solutions either provide no efficiency guarantee or achieve only a fraction of the optimal social welfare, when truthfulness is guaranteed, with the assumption of known true evaluation of the spectrum. In contrast, this paper presents a spectrum auction mechanism which can arbitrarily closely approach the maximum social welfare in expectation with polynomial-time complexity, while guaranteeing truthfulness in expectation. A novel way to calculate the true evaluation of spectrum at each secondary link is also proposed.

3. PROBLEM MODEL

We consider a secondary network (v_p, V_s, E). v_p is the primary user of a licensed spectrum, which can be divided into C non-overlapping orthogonal channels for lease to a set of secondary links, V_s. E is a set characterizing the interference relation, and $<v_i, v_j> \in E$ indicates a collision between $v_i, v_j \in V_s$ if transmitting simultaneously on the same channel. Each sender/receiver of the secondary link has a half-duplex software-defined radio that can be tuned to any channel provided by the primary user. The network operates in a time-slotted fashion, such that each link can transmit on one channel only in each time slot, at a *unit capacity.*

Table 2: Important notations.

V_s	Set of secondary links	v_p	Primary user
E	Edges in conflict graph	C	# of channels
$U_i(\cdot)$	revenue func. of link v_i	$Q_i(t)$	data queue at link v_i
$\eta_i(t)$	Aux. var for $r_i(t)$	ϵ_i	constant for QoS at v_i
$A_i(t)$	Data arrival of secondary link v_i at slot t		
A_i^{max}	Maximum data arrival rate of secondary link v_i		
$\tilde{b}_i(t)$	true value for v_i to buy a channel at t		
$\hat{b}_i(t)$	Payment by secondary link v_i at slot t		
$b_i(t)$	bid of v_i for a channel at t		
$Y_i(t)$	Virtual queue for rate control at secondary link v_i		
$Z_i(t)$	Virtual queue for QoS at secondary link v_i		
$r_i(t)$	Admitted data at secondary link v_i at slot t		
$\mu_{ic}(t)$	Transmission variable: data delivered out of $Q_i(t)$		
$d_i(t)$	Dropped packets by secondary link v_i at slot t		
d_i^{max}	Maximum packet drop rate by secondary link v_i		
β_i	Penalty to drop one packet by secondary link v_i		

3.1 Data traffic model at secondary links

For each secondary link v_i, data packets arrive at its sender node with an ergodic process, with $A_i(t) \in [0, A_i^{max}]$ being the data arrival rate in t, upper-bounded by A_i^{max}. To maintain network stability, rate control is applied such that $r_i(t)$ packets in $A_i(t)$ are admitted into the transport layer with

$$r_i(t) \in [0, A_i(t)]. \tag{1}$$

Transport-layer queue: A data queue $Q_i(t)$ is maintained on the transport layer at each link v_i:

$$Q_i(t+1) = \max\{Q_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t), 0\} + r_i(t). \tag{2}$$

Here, $\mu_{ic}(t) \in \{0, 1\}$ is the channel allocation variable (decided by the primary user via auction) indicating whether v_i is transmitting on channel c in t, and $d_i(t)$ is the number of dropped packets that exceed their maximum allowable delays (see (6)) with

$$d_i(t) \in [0, d_i^{max}], \tag{3}$$

where d_i^{max} is the maximum dropping rate. A secondary link can transmit over one channel only at a given time, through the pair of half-duplex radios, *i.e.*,

$$\sum_{c\in[1,C]} \mu_{ic}(t) \leq 1, \ \forall v_i \in V_s. \tag{4}$$

To avoid interference, two mutually interfering links cannot be scheduled on the same channel at t:

$$\mu_{ic}(t) + \mu_{jc}(t) \leq 1, \ \forall v_i, v_j \in V_s, < v_i, v_j >\in E, c \in [1, C]. \tag{5}$$

3.2 Quality of service model

We consider the following transmission guarantee at each secondary link v_i:

A packet on link v_i is either delivered or dropped within D_i slots after entering the queue. (6)

Here, D_i is the maximum allowable delay for packets on link v_i. Naturally, a penalty β_i is incurred for dropping a packet at secondary link v_i.

3.3 Spectrum auction model

There are two types of entities in the spectrum auction: secondary links (bidders) and the primary user (auctioneer). The auction consists of three main steps:

Step 1: Each secondary link $v_i \in V_s$ computes true value $\tilde{b}_i(t)$ of obtaining one channel for transmission at t, and submits a bid $b_i(t)$ to the primary user. The secondary link aims to maximize its utility and could bid untruthfully, *i.e.*, $b_i(t) \neq \tilde{b}_i(t)$. We aim to design a strategy-proof spectrum auction where bidding truthfully is a dominant strategy for each secondary link.

Step 2: After collecting the bids from all secondary links, the primary user computes the channel allocation decisions $\mu_{ic}(t)$, indicating whether channel c is allocated to secondary link v_i at t. We consider the spatial reuse of channels such that a set of collision-free links can be concurrently scheduled on the same channel subject to constraints (4) and (5).

Step 3: The primary user decides the payment $\hat{b}_i(t)$ to be charged to each secondary link $v_i \in V_s$.

3.4 Economic properties of the auction

We now define the economic properties pursued in our design of the spectrum auction mechanism.

DEFINITION 2 (TRUTHFULNESS IN EXPECTATION). *An randomized auction is truthful in expectation if bidding the true* *value is a dominant strategy for each buyer,* i.e., *the bidder cannot gain a higher utility (in expectation) by unilaterally deviating from bidding true values, while other bidders' strategies remain the same.*

DEFINITION 3 (INDIVIDUAL RATIONALITY IN EXPECTATION). *An randomized auction is individually rational in expectation if each bidder ends up with non-negative* expected *utility.*

DEFINITION 4 (BUDGET BALANCE IN EXPECTATION). *The auctioneer's* expected *utility is non-negative,* i.e., *the total charge collected from the bidders is non-negative* in expectation.

3.5 Utility model at secondary links

Hereinafter, for any variable $\alpha(t)$, we denote its time-averaged value as $\bar{\alpha}$, *i.e.*, $\bar{\alpha} = \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}(\alpha(\tau))$, where $\mathbb{E}(\cdot)$ is the expectation.

We consider the selfishness of each secondary link $v_i \in V_s$, which aims to maximize its time-averaged utility φ_i. φ_i consists of three components: the revenue gain from data delivery, $U_i(\bar{r}_i)$, the cost of leasing spectrum, $\bar{\hat{b}}_i$, and the penalty for dropping packets, $\beta_i \cdot \bar{d}_i$:

$$\varphi_i = U_i(\bar{r}_i) - \bar{\hat{b}}_i - \beta_i \cdot \bar{d}_i.$$

Here, $U_i(\cdot)$ is a non-decreasing, concave and twice-differentiable revenue function for v_i. Hence, individual long-term utility maximization at secondary link $v_i \in V_s$ becomes:

$$\max \ \varphi_i \tag{7}$$
$$s.t. \ \text{Queue stability, and constraints (1), (3), (4), (5), (6) at } v_i.$$

3.6 Social welfare

The economic efficiency of an auction is measured in terms of its achieved social welfare, *i.e.*, the overall utility of all participants in the auction. The utility of each secondary link is its utility as discussed above, while the utility of the primary user is the overall payment from all the secondary links, $\sum_{v_i\in V_s} \bar{\hat{b}}_i$.

Cancelling payments made by links and revenue gleaned by the primary user, the social welfare, φ, becomes:

$$\varphi = \sum_{v_i\in V_s} [U_i(\bar{r}_i) - \beta_i \cdot \bar{d}_i].$$

The long-term-average social welfare maximization problem is:

$$\max \ \varphi \tag{8}$$
$$s.t. \ \text{Queue stability and constraints (1), (3), (4), (5), (6) at each link.}$$

Our objective includes for each secondary link to maximize its time-averaged utility, *i.e.*, optimization problem (7), and for the network to maximize its social welfare at the same time.

4. ALGORITHM DESIGN

In this section, we present our spectrum auction framework and the algorithms designed for both the secondary links and the primary user. Fig. 1 outlines the sketch of the spectrum auction between secondary links and the primary user.

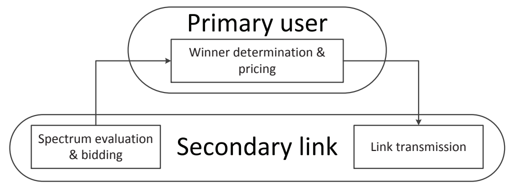


Figure 1: The modules of spectrum auction.

4.1 Spectrum Evaluation and Bidding at Secondary Link

A secondary link dynamically decides its channel evaluation and bids, as well as associated decisions on allocation of acquired channels for packet transmission, rate control and packet dropping, in each time slot, with the goal of maximizing its time-averaged utility in (7). We transform (7) into a sequence of one-shot optimization problems, and solve them respectively to derive the online algorithm based on the Lyapunov optimization technique [11]. To achieve that, apart from the data packet queues defined in Eqn. (2), each secondary link $v_i \in V_s$ also maintains two types of virtual queues.

Virtual queue for rate control: To deal with non-linear revenue functions $U_i(\cdot)$ [11], each secondary link v_i has the following virtual queue for its rate control:

$$Y_i(t+1) = \max\{Y_i(t) - r_i(t), 0\} + \eta_i(t). \tag{9}$$

Here $\eta_i(t)$ is an auxiliary variable for rate control at secondary link v_i with

$$\eta_i(t) \in [0, A_i^{max}]. \tag{10}$$

If virtual queue $Y_i(t)$ is kept stable, $\bar{\eta}_i \le \bar{r}_i$, *i.e.*, the time-averaged value of auxiliary variable $\eta_i(t)$ constitutes a lower bound for the time-averaged throughput. We will show that maximizing the utility of $\bar{\eta}_i$ can approximately maximize the utility on average throughput $\bar{r}_i$.

Virtual queue for QoS guarantee: We apply the $\epsilon-$persistence queue [17] technique to guarantee the QoS goal in (6). Each link v_i maintains the following virtual queue

$$Z_i(t+1) = \max\{Z_i(t) + \mathbf{1}_{\{Q_i(t)>0\}} \cdot (\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t)) - d_i(t) - \mathbf{1}_{\{Q_i(t)=0\}}, 0\}. \tag{11}$$

Here, $\mathbf{1}_{\{\cdot\}}$ is a binary indicator function. ϵ_i is a positive constant. The virtual queue $Z_i(t)$ approximately keeps track of the delay in data packet queue $Q_i(t)$. A longer virtual queue $Z_i(t)$ represents a larger cumulative queuing delay of packets in $Q_i(t)$. In Sec. 6, we demonstrate that, with the aid of this virtual queue, our algorithm can provide worst-case delay guarantee for each packet.

Hence, the sender of each secondary link v_i maintains a set of queues $\Theta_i(t) = \{Q_i(t), Y_i(t), Z_i(t)\}$ at each time t. We define a Lyapunov function as follows:

$$L(\Theta_i(t)) = \frac{1}{2}[[Q_i(t)]^2 + [Y_i(t)]^2 + [Z_i(t)]^2].$$

The one-slot conditional Lyapunov drift is:

$$\Delta(\Theta_i(t)) = L(\Theta_i(t+1)) - L(\Theta_i(t)).$$

The *drift-plus-penalty* is (equivalent to *drift-minus-utility* here; derivation details are in Appendix A),

$$\Delta(\Theta_i(t)) - V \cdot [U_i(\eta_i(t)) - \hat{b}_i(t) - \beta_i \cdot d_i(t)]$$
$$\le B_i - \Phi_i^{(1)}(t) - \Phi_i^{(2)}(t) - \Phi_i^{(3)}(t) - \Phi_i^{(4)}(t) + \epsilon_i \cdot Z_i(t). \tag{12}$$

Here, $V > 0$ is a user-defined parameter for gauging the optimality of time-averaged utility. $B_i = \frac{1}{2}[[\epsilon_i]^2 + 3[A_i^{max}]^2 + 2[1 + d_i^{max}]^2]$ is a constant value. $\Phi_i^{(1)}(t)$, $\Phi_i^{(2)}(t)$, $\Phi_i^{(3)}(t)$ and $\Phi_i^{(4)}(t)$ are related to the auxiliary variable $\eta_i(t)$, the rate control variable $r_i(t)$, channel allocation & charge variable $\mu_{ic}(t)$ and $\hat{b}_{ic}(t)$, and packet dropping variable $d_i(t)$, respectively:

$$\Phi_i^{(1)}(t) = V \cdot U_i(\eta_i(t)) - \eta_i(t) \cdot Y_i(t), \tag{13}$$

$$\Phi_i^{(2)}(t) = r_i(t) \cdot [Y_i(t) - Q_i(t)], \tag{14}$$

$$\Phi_i^{(3)}(t) = \sum_{c \in [1,C]} [\mu_{ic}(t) \cdot [Q_i(t) + Z_i(t)] - V \cdot \hat{b}_i(t), \tag{15}$$

$$\Phi_i^{(4)}(t) = d_i(t) \cdot [Q_i(t) + Z_i(t) - V \cdot \beta_i]. \tag{16}$$

According to the Lyapunov optimization theory [11], we can maximize a lower bound of the time-averaged utility for v_i and find optimal solutions to the rate control, channel evaluation & bidding, and packet dropping variables by minimizing the RHS of the *drift-plus-penalty* equality (12), observing the queue lengths $\Theta_i(t)$ and the packet arrival $A_i(t)$ in each time slot t. Hence, we can derive an online algorithm to solve (7), that solves the one-shot optimization problem in each time slot t as follows:

$$\max \ \Phi_i^{(1)}(t) + \Phi_i^{(2)}(t) + \Phi_i^{(3)}(t) + \Phi_i^{(4)}(t) \tag{17}$$
$$s.t. \text{ Constraints (1), (3), (4), (5) and (10) at } v_i.$$

The maximization problem in (17) can be decoupled into four independent optimization problems:

$$\max \ \Phi_i^{(3)}(t) \tag{18}$$

which is related to the optimal channel evaluation & bidding decisions, with $\tilde{b}_i(t)$ and $b_i(t)$, which also determine the channel allocation decisions with $\mu_{ic}(t)$, $\forall c \in [1, C]$ and channel charge decisions with $\hat{b}_i(t)$ after the auction by primary user (the interference constraints (4) and (5) are satisfied by getting channel allocation decisions from the spectrum auction mechanism, to be introduced in Sec. 4.2); and

$$\max \ \Phi_i^{(1)}(t) \tag{19}$$
$$s.t. \text{ Constraint (10)},$$

which is related to the optimal decision on the auxiliary variable $\eta_i(t)$; and

$$\max \ \Phi_i^{(2)}(t) \tag{20}$$
$$s.t. \text{ Constraint (1)},$$

which is related to the optimal decision on the rate control variable $r_i(t)$; and

$$\max \ \Phi_i^{(4)}(t) \tag{21}$$
$$s.t. \text{ Constraint (3)},$$

which is related to the optimal decision on packet dropping with $d_i(t)$. The following is our algorithm to solve the four one-shot optimization problems. The detailed derivation is given in [18]

Channel evaluation and bidding: We seek to design a truthful auction (in Sec. 4.2) where each secondary link v_i bids its true valuation of the channel, *i.e.*, $b_i(t) = \tilde{b}_i(t)$. According to the definition in Sec. 1, the true value of a bidder is the highest price it is willing to pay, charged with which (*i.e.*, $\hat{b}_i(t) = \tilde{b}_i(t)$) its utility in (18) if one channel is allocated to link v_i, *i.e.*, $\exists c$, $\mu_{ic}(t) = 1$, is exactly the same as if losing the auction. Following this argument, each secondary link v_i evaluates a channel based on its queue lengths in each time slot as follows:

$$\tilde{b}_i(t) = \frac{Q_i(t) + Z_i(t)}{V}. \tag{22}$$

The rationale is that the true value for v_i to buy one channel is determined by its level of traffic congestion and cumulative delay (or data transmission urgency), *i.e.*, $Q_i(t) + Z_i(t)$. A large value of $Q_i(t)$ implies high congestion (or transmission urgency), while a large value of $Z_i(t)$ indicates an urgency in dropping packets.

Rate control: Each secondary link computes assignments to the auxiliary variable and the rate control variable by solving the one-shot optimization problems (19) and (20) respectively,

$$\eta_i(t) = \max\{\min\{U_i'^{-1}(\frac{Y_i(t)}{V}), A_i^{max}\}, 0\}, \tag{23}$$

where $U_i'^{-1}(\cdot)$ is the inverse function of the first-order derivative of $U_i(\cdot)$, and

$$r_i(t) = \begin{cases} A_i(t) & \text{if } Y_i(t) > Q_i(t) \\ 0 & \text{Otherwise} \end{cases}. \tag{24}$$

Note, each secondary link only needs local information, *i.e.*, revenue function $U_i(\cdot)$ and queue lengths. Virtual queue $Y_i(t)$ can be regarded as the unused tokens for data admission. A large value for $Y_i(t)$ indicates an adequate number of available tokens, which results in fewer new tokens (*i.e.*, $\eta_i(t)$) to be added in this time slot. Meanwhile, $Q_i(t)$ reflects the congestion level on the link. $Y_i(t) - Q_i(t) > 0$ means that we have enough tokens while relatively low congestion. Thus, we admit all the arrived jobs. Otherwise, no job is admitted into the network.

Packet dropping: We decide the number of packets to drop by solving optimization (21) at each t:

$$d_i(t) = \begin{cases} d_i^{max} & \text{if } Q_i(t) + Z_i(t) > V \cdot \beta_i \\ 0 & \text{Otherwise.} \end{cases} \tag{25}$$

The rationale is that $Q_i(t) + Z_i(t)$ represents the urgency level to schedule/drop packets. If the scheduling/dropping urgency outweights the weighted dropping penalty $V \cdot \beta_i$, packets are dropped at the maximum rate; otherwise no packets are dropped. That is, each link is reluctant to drop packets unless the queue lengths exceed certain thresholds, above which packets are suffering long delays.

Algorithm 1 Dynamic Utility Maximization Algorithm at Secondary Link v_i in Time Slot t

Input: $A_i(t)$, A_i^{max}, $Y_i(t)$, $Q_i(t)$, $Z_i(t)$, β_i, d_i^{max}, $U_i(\cdot)$ and V.
Output: $\eta_i(t)$, $r_i(t)$, $\hat{b}_i(t)$, $\tilde{b}_i(t)$, $b_i(t)$, $d_i(t)$ and $\mu_{ic}(t)$, $\forall c \in [1, C]$.

1: **Rate control**: Decide $\eta_i(t)$ and $r_i(t)$ with Eqn. (23) and (24);
2: **Channel valuation and bid**: Decide $\tilde{b}_i(t)$ and $b_i(t)$ with Eqn. (22);
3: **Channel allocation and payment**: Get decisions on $\hat{b}_i(t)$ and μ_{ic}, $\forall c \in [1, C]$, from the auction;
4: **Packet dropping**: Decide $d_i(t)$ with Eqn. (25);
5: Update $Q_i(t)$, $Y_i(t)$ and $Z_i(t)$ with Eqn. (2) (9) and (11).

4.2 Auction at Primary User

After collecting all the bids from secondary links, the primary user executes a *randomized* auction mechanism, which is truthful, individual rational and budget balanced, all in expectation. This randomized auction has two modules: *winner determination* and *channel pricing*.

Winner determination: This module randomly decides a subset of secondary links in V_s, each winning one of the C channels; other secondary links are not allocated with a channel. Equivalently, the auctioneer finds a collision-free channel allocation strategy $\chi(t) = \{\mu_{ic}(t) \in \{0,1\} | \forall v_i \in V_s, c \in [1, C]\}$ in each time slot, such that constraints (4) and (5) are satisfied. Glauber dynamics are utilized in the algorithm design. Especially, the winners in each time slot are selected randomly based on i) the bidding prices, ii) channel allocation in the previous time slot, and iii) interference constraints. There are two steps of the algorithm at each t:

Step 1: The primary user uniformly randomly selects a set of collision-free channel allocation variables, $m(t)$ (referred to as the *decision set*). For each channel allocation variable not included in the decision set, *i.e.*, $\mu_{ic}(t) \notin m(t)$, it sets $\mu_{ic}(t) = \mu_{ic}(t-1)$.

In practical implementation, we can associate a timer, which is uniformly randomly set with a value from a range $[0, W]$ ($W > 0$), with each channel allocation variable $\mu_{ic}(t)$, $\forall v_i \in V_s, c \in [1, C]$. If the timer of $\mu_{ic}(t)$ expires before that of any of its mutually-interfering allocation variables, *i.e.*, $\mu_{jc}(t)$ with $<v_i, v_j> \in E$ and $\mu_{ic'}(t)$ with $c' \neq c$, $\mu_{ic}(t)$ is included in the decision set $m(t)$; otherwise, $\mu_{ic}(t)$ is not in the set $m(t)$ and let $\mu_{ic}(t) = \mu_{ic}(t-1)$.

Step 2: For each channel allocation variable $\mu_{ic}(t)$ in the decision set $m(t)$, do the following:

- If any mutual-interfering allocation variable of $\mu_{ic}(t)$ is included in the decision set in a previous time slot, *i.e.*, $\exists <v_i, v_j> \in E$ with $\mu_{jc}(t-1) = 1$ or $\exists c' \neq c$ with $\mu_{ic'}(t-1) = 1$, variable $\mu_{ic}(t)$ will not be included in the decision set in the current time slot by setting $\mu_{ic}(t) = 0$;
- Otherwise, $\mu_{ic}(t)$ is included with probability p_i, *i.e.*,

$$\mu_{ic}(t) = 1 \text{ with probability } p_i = \frac{e^{V \cdot b_i(t)}}{1 + e^{V \cdot b_i(t)}},$$

and not included with probability $1 - p_i$, *i.e.*,

$$\mu_{ic}(t) = 0 \text{ with probability } 1 - p_i = \frac{1}{1 + e^{V \cdot b_i(t)}}.$$

Step 3: If $\mu_{ic}(t) = 1$, channel c is allocated to secondary link v_i for data transmission.

Remarks: The rationale of the winner determination module is that: i) in step 1, we provide equal chance for each link to change its status, *i.e.*, winning or losing the auction; and ii) in step 2, we give preference to those links with higher bidding price, *i.e.*, $b_i(t)$.

Channel Pricing: For each link $v_i \in V_s$, its payment to the primary user in time slot t is calculated as:

$$\hat{b}_i(t) = \sum_{v_j \in V_s, v_j \neq v_i} b_j(t) \cdot \sum_{c \in [1,C]} [\mu_{jc}^{(i)}(t) - \mu_{jc}(t)]. \qquad (26)$$

Here, $\mu_{jc}^{(i)}(t)$ is the channel allocation decision made by the winner determination algorithm stated above, with $b_i(t) = 0$ and unchanged bids from other links.

4.3 Computation complexity

We show that, in each time slot, the computation complexity of our auction framework, *i.e.*, Alg. 1 and 2, is in a polynomial order of the total network size and number of channels, *i.e.*, $|V_s|$ and C.

Algorithm 2 Spectrum Auction at Primary User v_p in Time Slot t

Input: $b_i(t)$, $\mu_{ic}(t-1)$, and E, $\forall v_i \in V_s, c \in [1, C]$.
Output: $\hat{b}_i(t)$, and $\mu_{ic}(t)$, $\forall v_i \in V_s, c \in [1, C]$.

Module 1: **Winner determination**

1: *Step 1*: Uniformly randomly select a decision set $m(t)$;
2: *Step 2*: For each channel allocation variable $\mu_{ic}(t)$ ($\forall v_i \in V_s, c \in [1, C]$):
– If $\mu_{ic}(t) \notin m(t)$, set $\mu_{ic}(t) = \mu_{ic}(t-1)$;
– Otherwise,
–If $\exists v_j \in V_s, < v_i, v_j > \in E$ with $\mu_{jc}(t-1) = 1$ or $\exists c' \neq c$ with $\mu_{ic'}(t-1) = 1$, set $\mu_{ic}(t) = 0$;
–Otherwise, set $\mu_{ic}(t) = 1$ with probability $p_i = \frac{e^{V \cdot b_i(t)}}{1+e^{V \cdot b_i(t)}}$ while $\mu_{ic}(t) = 0$ with probability $1 - p_i = \frac{1}{1+e^{V \cdot b_i(t)}}$
3: *Step 3*: If $\mu_{ic}(t) = 1$, channel c is allocated to secondary link v_i.

Module 2: **Channel Pricing**

1: The payment of each secondary link $v_i \in V_s$ is calculated with Eqn. (26).

For each secondary link $v_i \in V_s$, Algorithm 1 decides the rate control, channel evaluation/bidding, and packet dropping in constant complexity with Eqn. (23), (24), (22) and (25). Thus, the overall complexity to run Algorithm 1 in the network is in the order of the secondary network size $|V_s|$.

In each time slot, Algorithm 2 decides the decision set with a complexity in the order of total number of channel allocation variables, *i.e.*, $|V_s| \cdot C$, by keeping one timer for each of them. In the next step, for each channel allocation variable in the decision set, the allocation decision in previous slot for each of its mutually-interfering variable is checked. In the worst case, the size of the decision set is in $O(|V_s|)$ while the interference degree of one channel allocation variable is in $O(C + |V_s|)$. Hence, the complexity in this step is $O(|V_s|(C + |V_s|))$. Since the primary user also runs the same winner determination module (for the sake of channel pricing) for each secondary user v_i by setting $b_i(t) = 0$ at each time t, the overall complexity for the winner determination module is $O(|V_s|^2(C + |V_s|))$. The channel allocation and pricing decisions are then computed in constant complexity for each allocation variable. Therefore, the overall complexity of Algorithm 2 is $O(|V_s|^2(C + |V_s|))$.

In summary, our auction framework has a computation complexity of $O(|V_s|^2(C + |V_s|))$.

5. SOCIAL WELFARE MAXIMIZATION

We next propose a benchmark algorithm for evaluating the efficiency of our spectrum auction mechanism in social welfare. In this benchmark algorithm, each participant in the network, including each secondary link and the primary user, is altruistic. There is no more auction, but a centralized decision maker to decide channel allocation, rate control, link scheduling, and packet dropping in each time slot, to maximize the time-averaged social welfare of the entire network as defined in (8). A set of queues $\Theta(t) = \{Q_i(t), Y_i(t), Z_i(t) | \forall v_i \in V_s\}$ are maintained over time. To solve (8), a Lyapunov function is defined as follows:

$$L(\Theta(t)) = \frac{1}{2} \sum_{v_i \in V_s} [[Q_i(t)]^2 + [Y_i(t)]^2 + [Z_i(t)]^2].$$

The one-slot conditional Lyapunov drift is:

$$\Delta(\Theta(t)) = L(\Theta(t+1)) - L(\Theta(t)).$$

The *drift-plus-penalty* (equivalent to *drift-minus-utility* here; derivation details can be found in Appendix B) is:

$$\begin{aligned}&\Delta(\Theta(t)) - V\cdot\sum_{v_i\in V_s}[U_i(\eta_i(t)) - \hat{b}_i(t) - \beta_i\cdot d_i(t)]\\ \le& B - \sum_{v_i\in V_s}[\Phi_i^{(1)}(t) + \Phi_i^{(2)}(t) + \Phi_i^{(4)}(t)] - \Psi(t). \qquad (27)\end{aligned}$$

Here $V>0$ is a user-defined parameter for gauging the optimality of time-averaged social welfare. $B = \frac{1}{2}\sum_{v_i\in V_s}[[\epsilon_i]^2 + 3[A_i^{max}]^2 + 2[1+d_i^{max}]^2]$ is a constant value. $\Phi_i^{(1)}(t)$, $\Phi_i^{(2)}(t)$, and $\Phi_i^{(4)}(t)$, $\forall v_i\in V_s$, are defined in Eqn. (13), (14) and (16).

$\Psi(t)$ is related to the channel allocation variables $\mu_{ic}(t)$, $\forall c\in[1,C], v_i\in V_s$:

$$\Psi(t) = \sum_{v_i\in V_s}[Q_i(t) + Z_i(t)]\sum_{c\in[1,C]}\mu_{ic}(t). \qquad (28)$$

Similarly, we can maximize a lower bound of the time-averaged social welfare and find optimal solutions to the rate control, channel allocation and packet dropping variables by minimizing the RHS of (27), observing the queue lengths $\Theta(t)$ and the packet arrivals $A_i(t)$, $\forall v_i\in V_s$, in each time slot t. An online algorithm is hence derived for solving (8), that solves the one-shot optimization problem in each time slot t as follows:

$$\max \sum_{v_i\in V_s}[\Phi_i^{(1)}(t) + \Phi_i^{(2)}(t) + \Phi_i^{(4)}(t)] + \Psi(t) \qquad (29)$$

s.t. Constraints (1), (3), (4), (5) and (10) at each link.

The maximization problem in (29) can be decoupled into four independent optimization problems: i) problem (19) for each $v_i\in V_s$; ii) problem (20) for each $v_i\in V_s$; iii) problem (21) for each $v_i\in V_s$; and we make the channel allocation decisions by solving optimization problem (30),

$$\max \ \Psi(t) = \sum_{v_i\in V_s}[Q_i(t) + Z_i(t)]\sum_{c\in[1,C]}\mu_{ic}(t) \qquad (30)$$

s.t. Interference constraints Eqn. (4) and (5), $\forall v_i\in V_s$.

(30) is a *maximum weight scheduling* problem, which is NP-hard since computing a maximum weighted independent set is required. A centralized *branch-and-bound* algorithm can be implemented to approximate $1-\delta$ ($\delta\in[0,1]$) fraction of the maximum $\Psi(t)$ [19]. The benchmark algorithm is summarized in Alg. 3.

Algorithm 3 Dynamic Social Welfare Maximization Algorithm in Time Slot t

Input: $A_i(t)$, A_i^{max}, $Y_i(t)$, $Q_i(t)$, $Z_i(t)$, β_i, d_i^{max}, $U_i(\cdot)$ and V, $\forall v_i\in V_s$.
Output: $\eta_i(t)$, $r_i(t)$, $d_i(t)$ and $\mu_{ic}(t)$, $\forall c\in[1,C], v_i\in V_s$.

1: **Rate control**: Decide $\eta_i(t)$ and $r_i(t)$, $\forall v_i\in V_s$, with Eqn. (23) and (24).
2: **Channel allocation**: Decide $\mu_{ic}(t)$, $\forall c\in[1,C], v_i\in V_s$, by solving optimization problem Eqn. (30) with branch-and-bound algorithm [19].
3: **Packet dropping**: Decide $d_i(t)$, $\forall v_i\in V_s$, with Eqn. (25);
4: Update $Q_i(t)$, $Y_i(t)$ and $Z_i(t)$, $\forall v_i\in V_s$, with Eqn. (2) (9) and (11).

6. THEORETICAL ANALYSIS

We present theoretical analysis of our proposed spectrum auction framework and dynamic algorithms in this section. All detailed proofs are included in the Appendix.

6.1 QoS guarantee

Lemma 1 (Bounded queues). Let Y_i^{max}, Q_i^{max} and Z_i^{max} be defined as follows,

$$\begin{aligned}Y_i^{max} =& V\cdot U_i'(0) + A_i^{max},\ \forall v_i\in V_s,\\ Q_i^{max} =& V\cdot U_i'(0) + 2A_i^{max},\ \forall v_i\in V_s,\\ Z_i^{max} =& V\cdot\beta_i + \epsilon_i,\ \forall v_i\in V_s.\end{aligned}$$

For each $v_i\in V_s$, if $d_i^{max}\ge\max\{A_i^{max},\epsilon_i\}$, the transport layer data queue $Q_i(t)$, and the virtual queues $Y_i(t)$ and $Z_i(t)$ are bounded for each slot t as follows,

$$Y_i(t)\le Y_i^{max}, \quad Q_i(t)\le Q_i^{max}, \quad Z_i(t)\le Z_i^{max}.$$

This lemma is proved by induction based on Algorithm 1 and the queueing laws (2), (9) and (11).

Theorem 1 (QoS guarantee). Each packet on secondary link $v_i\in V$ is either delivered or dropped with Algorithm 1 before its maximum delay D_i, if we set $\epsilon_i = \frac{Q_i^{max}+Z_i^{max}}{D_i}$ and $d_i^{max}\ge\max\{A_i^{max},\epsilon_i\}$.

This theorem can be proved based on Lemma 1 and the $\epsilon-$persistence queue technique [17]. The condition on ϵ_i is to ensure that the queue lengths can grow to satisfy the job drop condition, *i.e.*, $Q_i(t)+Z_i(t) > V\cdot\beta_i$, if some packets remain undelivered in the last D_i slots.

6.2 Economic Properties of the Auction

Theorem 2 (Optimal Winner Determination). The winner determination in Algorithm 2 computes collision-free channel allocations that maximize the expectation of $\Psi(t)$ as defined in (30), if each secondary link bids truthfully and $V\to\infty$.

The correctness of the collision-free channel allocations can be proved by contradiction, while the maximization of $\Psi(t)$ in expectation is based on the Glauber dynamics to find a stationary distribution (which is converged when $V\to\infty$) for each feasible channel allocation decision. This theorem is utilized in the proof for the truthfulness, individual rationality, budget balance and optimality in social welfare.

Theorem 3 (True Evaluation). The channel valuations in (22), $\forall v_i\in V_s$, are true values.

This theorem is proved based on the definition of the true values and that (17) is solved in each time slot by each secondary link.

Theorem 4 (Truthfulness in Expectation). Bidding truthfully is the dominant strategy of each secondary link in the auction in Algorithm 2, *i.e.*, no secondary link can achieve a higher utility in expectation in terms of the one-shot optimization problem (17), by bidding with values other than its true values in Eqn. (22), if $V\to\infty$.

We prove this theorem by contradiction and show that, in all cases, no secondary link can do better with one-shot optimization problem (17) by bidding untruthfully.

Theorem 5 (Individual Rationality). No winning secondary link pays, in expectation, more than its bidding price, *i.e.*, $\mathbb{E}\{\hat{b}_i(t)\}\le b_i(t)$, $\forall v_i\in V_s$, if $V\to\infty$.

This theorem can be proved based on the winner determination and pricing schemes in our auction mechanism, together with Theorem 2.

Theorem 6 (Budget Balance at Primary User). At the primary user, the total payment-in-expectation collected from the secondary links is non-negative, *i.e.*, $\sum_{v_i \in V_s} \mathbb{E}\{\hat{b}_i(t)\} \geq 0$, if $V \to \infty$.

This theorem is proved with the pricing mechanism and Theorem 2.

6.3 Optimality of Individual Utility and Social Welfare

Theorem 7 (Individual Utility Maximization). Let Ω_i^* be the offline optimum of time-averaged utility of secondary link $v_i \in V_s$, obtained in a truthful-in-expectation, individual-rational-in-expectation and budget-balanced spectrum auction, with complete information on its data arrivals and channel availability in the entire time span [0, T-1]. The online Algorithm 1 can achieve a time-averaged utility Ω_i for secondary link v_i within a constant gap B_i/V to Ω_i^*, *i.e.*,

$$\Omega_i \geq \Omega_i^* - B_i/V,$$

where $V > 0$ and $B_i = \frac{1}{2}[[\epsilon_i]^2 + 3[A_i^{max}]^2 + 2[1 + d_i^{max}]^2]$ is a constant.

The proof to Theorem 7 is rooted in Lyapunov optimization theory [11]. The gap B_i/V can be arbitrarily close to zero by increasing V.

Theorem 8 (Social Welfare Optimality). Let Π^* be the offline optimum of the time-averaged social welfare in (8), obtained with full information of the network over the entire time span [0, T-1]. The time-averaged social welfare Π_{12} and Π_3, achieved by running Alg. 1 & 2 and Alg. 3, respectively, approach the offline-optimal social welfare Π^* with a constant gap B/V, *i.e.*,

$$\Pi_{12} \geq \Pi^* - B/V, \quad \Pi_3 \geq \Pi^* - B/V,$$

where $V > 0$ and $B = \frac{1}{2}\sum_{v_i \in V_s}[[\epsilon_i]^2 + 3[A_i^{max}]^2 + 2[1 + d_i^{max}]^2]$.

We prove this theorem by first showing that the dynamic decisions made by Alg. 1 & 2 have the same expected values as that by Alg. 3 according to their Algorithm definitions and Theorem 2, which means they have the same expected social welfare in a long run. Next, we prove their social welfare optimality based on Lyapunov optimization theory [11]. The gap B/V can be arbitrarily close to zero by increasing V.

With Theorem 7 and 8, we see that both the individual utility of each secondary link and the social welfare of the network can be made arbitrarily close to their optima by setting $V \to \infty$. However, by Lemma 1 and Theorem 1, the maximum allowable delay D_i is also proportionally increasing with V if ϵ_i is a constant. Hence, there is a tradeoff, adjusted by V, between the maximum allowable delay and the optimality of individual utility and social welfare.

7. PERFORMANCE EVALUATION

7.1 Simulation Setup

We consider 16 secondary links[1] uniformly randomly distributed in a network with an average interference degree of 4. The primary user has 4 orthogonal channels for sale. Each link has a data arrival rate per time slot uniformly distributed between 0 and 0.4, with an average of 0.2 (data units). The revenue function for an average throughput $\bar{r}_i$ is $\log(1+\bar{r}_i)$. The penalty to drop one unit of data is $\beta_i = 1.0$, $\forall v_i \in V_s$. The constant value of ϵ_i, $\forall v_i \in V_s$ is fixed at 1.0. The maximum packet drop rate d_i^{max} is also 1.0 such that $d_i^{max} \geq \max\{A_i^{max}, \epsilon_i\}$.

For the benchmark algorithm, its channel allocation decisions are derived in each time slot by solving problem (30) with `glpk` [20]. Each experiment is executed for 100,000 time slots, and each datum is the average of 100 trials.

7.2 Social Welfare

Since it is not computationally feasible to derive the offline-optimal long-term-average social welfare, we compare the social welfare achieved by our auction framework, *i.e.*, Algorithms 1 and 2, with that of the benchmark Algorithm 3, which is proven to be arbitrarily close to the offline optimum long-term-average social welfare (in Theorem 8). Fig. 2 shows that, when V is larger, the social welfare obtained by our auction framework is even better, and is mostly within 10.1% of that by the benchmark algorithm. Hence, our auction framework achieves a social welfare closer to its offline optimum when V scales up, validating the result in Theorem 8.

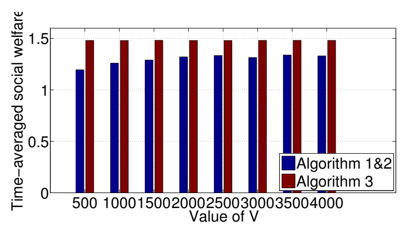


Figure 2: Comparisons of social welfare.

7.3 Average delay and packet drop rate

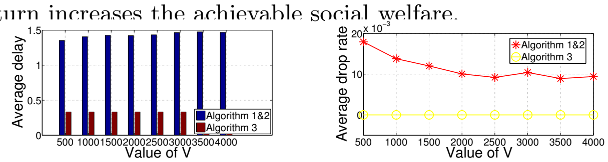
From Lemma 1 and Theorem 1, we see that the maximum allowable delay is proportional to the value of V. Thus, approaching the optimal social welfare by scaling up V will inevitably lead to an increased delay. We next examine the performance of average delay and packet drop rate by our auction framework and the benchmark algorithm with different values of V.

A nice observation in Fig. 3(a) is that, although the maximum allowable delay grows proportionally to V, the average delay that packets actually experience increases slowly with V, implying that our auction framework can approach the offline optimal social welfare without significantly sacrificing the average delay.

We also study the average number of admitted packets in the entire secondary network that are eventually dropped, in our auction framework and in the benchmark algorithm, respectively. Fig. 3(b) reveals that the average drop rate decreases quickly as V grows, and drops to a level close to that of the benchmark algorithm when $V > 2500$. Intuitively, with a larger V, less packets are dropped in order to

[1]While our auction is efficient, the benchmark algorithm needs to solve an integer program in each time slot, limiting the network size in the simulation.

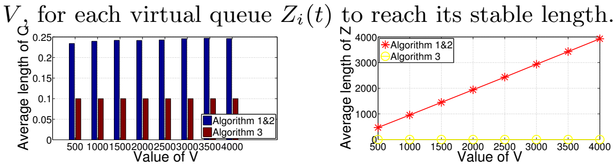
decrease the penalty incurred by packet dropping, which in turn increases the achievable social welfare.



(a) Average delay (b) Average drop rate

Figure 3: Average delay and drop rate.

For reasons behind such low average delay and its slow scaling with V, we compare the average lengths of the packet queues and virtual $\epsilon-$persistence queues in Fig. 4. Fig. 4(a) shows that the average packet queue length is a small value within 0.25, so packets are promptly delivered/dropped without being accumulated, consistent with the low and slow scaling average delay in Fig. 3(a). Similar result is also found for the benchmark algorithm. However, in Fig. 4(b), the average lengths of the virtual queue Z have clear differences in our auction framework and in the benchmark algorithm: the former is large and grows quickly with V while the latter is mostly close to zero. In each time slot, the channel allocation decisions in Algorithm 3 are derived by solving a max-weight scheduling problem. As long as the scheduling weight for the allocation variable $\mu_{ic}(t)$, *i.e.*, $Q_i(t) + Z_i(t)$, is positive, link v_i has a chance to be scheduled. Thus, the virtual queue is neither necessary, since the packet queue length is already positive, nor possible to accumulate to a long length, since the transmission opportunities are readily obtained. To the contrast, our auction framework requires a higher value of $Q_i(t) + Z_i(t)$ at a link to get a larger chance of being allocated a channel, according to the definition of probability p_i in Algorithm 2 and the bidding price $b_i(t)$ with its true value in Eqn. (22). However, a nice property of our auction is that, packet queues are not necessarily long since the lengths of virtual queue are already large enough, leading to a high chance of channel allocation and a short delay. The only cost lies in convergence time, scaling with V, for each virtual queue $Z_i(t)$ to reach its stable length.



(a) Average length of packet queue Q (b) Average length of virtual queue Z

Figure 4: Average lengths of queues.

8. CONCLUSION

We investigated online auction design for maximization of long-term-averaged social welfare in a network of secondary links, and of long-term-averaged utility at each secondary link, under QoS requirements and volatile traffic demands. The goals are truthfulness and computational efficiency. A novel, online spectrum auction framework was proposed to dynamically decide the rate control, channel evaluation/bidding and packet dropping at each secondary link, as well as the winner determination and pricing at the primary user, achieving the above goals simultaneously.

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APPENDIX

A. DERIVATION OF THE ONE-SHOT OPTIMIZATION PROBLEM FOR INDIVIDUAL UTILITY MAXIMIZATION AT SECONDARY LINKS

Squaring the queueing laws (2), (9), and (11), we can derive the following inequality,

$$
\begin{aligned}
\Delta(\Theta_i(t)) \leq &\frac{1}{2}[[\eta_i(t)]^2 + [r_i(t)]^2 + 2Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
&+ [r_i(t)]^2 + [\sum_{c\in[1,C]} \mu_{ic}(t) + d_i(t)]^2 \\
&+ 2Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
&+ [\epsilon_i]^2 + [\sum_{c\in[1,C]} \mu_{ic}(t) + d_i(t)]^2 \\
&+ 2Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)]] \\
\leq &\frac{1}{2}[[A_i^{max}]^2 + [A_i^{max}]^2 + 2Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
&+ [A_i^{max}]^2 + [1 + d_i^{max}]^2 \\
&+ 2Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
&+ [\epsilon_i]^2 + [1 + d_i^{max}]^2 \\
&+ 2Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)]] \\
= &B_i + Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
&+ Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
&+ Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)],
\end{aligned}
$$

where $B_i = \frac{1}{2}[[\epsilon_i]^2 + 3[A_i^{max}]^2 + 2[1 + d_i^{max}]^2]$. The first inequality is based on the queueing laws (2), (9), and (11), and the fact that: if $a, b, c \geq 0$, we have that $(a+b-c)^2 - a^2 \leq b^2 + c^2 + 2a(b-c)$. The second inequality is derived based on the conditions that $\eta_i(t) \leq A_i^{max}]$, $r \leq A_i^{max}$, $d_i(t) \leq d_i^{max}$ and $\sum_{c\in[1,C]} \mu_{ic}(t) \leq 1$.

By applying the drift-plus-penalty framework (or equivalently, drift-minus-utility here), we subtract the weighted one-shot individual utility of secondary link v_i in time slot t, *i.e.*, $V \cdot [U_i(\eta_i(t)) - \hat{b}_i(t) - \beta_i \cdot d_i(t)]$, on both sides of the above inequality. Hence, we have the following inequality,

$$
\begin{aligned}
&\Delta(\Theta_i) - V \cdot [U_i(\eta_i(t)) - \hat{b}_i(t) - \beta_i \cdot d_i(t)] \\
\leq &B_i - \Phi_i^{(1)}(t) - \Phi_i^{(2)}(t) - \Phi_i^{(3)}(t) - \Phi_i^{(4)}(t).
\end{aligned}
$$

$V > 0$ is a user-defined positive constant that can be understood as the weight of utility in the expression. Here, $\Phi_i^{(1)}(t)$, $\Phi_i^{(2)}(t)$, $\Phi_i^{(3)}(t)$ and $\Phi_i^{(4)}(t)$ are defined as in Eqn. (13), (14), (15) and (16), respectively.

□

B. DERIVATION OF THE ONE-SHOT OPTIMIZATION PROBLEM FOR SOCIAL WELFARE

Squaring the queueing laws (2), (9), and (11), $\forall v_i \in V_s$, we can derive the following inequality,

$$
\begin{aligned}
\Delta(\Theta(t)) \leq & \frac{1}{2} \sum_{v_i \in V_s} [[\eta_i(t)]^2 + [r_i(t)]^2 + 2Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
& + [r_i(t)]^2 + [\sum_{c\in[1,C]} \mu_{ic}(t) + d_i(t)]^2 \\
& + 2Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
& + [\epsilon_i]^2 + [\sum_{c\in[1,C]} \mu_{ic}(t) + d_i(t)]^2 \\
& + 2Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)]] \\
\leq & \frac{1}{2} \sum_{v_i \in V_s} [[A_i^{max}]^2 + [A_i^{max}]^2 + 2Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
& + [A_i^{max}]^2 + [1 + d_i^{max}]^2 \\
& + 2Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
& + [\epsilon_i]^2 + [1 + d_i^{max}]^2 \\
& + 2Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)]] \\
= & B + \sum_{v_i \in V_s} Y_i(t) \cdot [\eta_i(t) - r_i(t)] \\
& + \sum_{v_i \in V_s} Q_i(t) \cdot [r_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)] \\
& + \sum_{v_i \in V_s} Z_i(t) \cdot [\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t)],
\end{aligned}
$$

where $B = \sum_{v_i \in V_s} B_i$.

By applying the drift-plus-penalty framework (or equivalently, drift-minus-utility here), we subtract the weighted one-shot social welfare in time slot t, *i.e.*, $V \cdot \sum_{v_i \in V_s} [U_i(\eta_i(t)) - \beta_i \cdot d_i(t)]$, on both sides of the above inequality. Hence, we have the following inequality,

$$
\begin{aligned}
& \Delta(\Theta(t)) - V \cdot \sum_{v_i \in V_s} [U_i(\eta_i(t)) - \beta_i \cdot d_i(t)] \\
\leq & B - \sum_{v_i \in V_s} [\Phi_i^{(1)}(t) - \Phi_i^{(2)}(t) - \Phi_i^{(4)}(t)] - \Psi(t).
\end{aligned}
$$

$V > 0$ is a user-defined positive constant that can be understood as the weight of social welfare in the expression. Here, $\Phi_i^{(1)}(t)$, $\Phi_i^{(2)}(t)$, $\Phi_i^{(4)}(t)$ and $\Psi(t)$ are defined as in Eqn. (13), (14), (16) and (28), respectively.

□

C. PROOF TO LEMMA 1

We prove this lemma by induction.

Induction basis: At time slot 0, each queue in the network is zero. Hence, we have that

$$
\begin{aligned}
Y_i(0) \leq & Y_i^{max}, \\
Q_i(0) \leq & Q_i^{max}, \\
Z_i(0) \leq & Z_i^{max}.
\end{aligned}
$$

Induction steps: We assume that, at time slot t, each queue is bounded as

$$
\begin{aligned}
Y_i(t) \leq & Y_i^{max}, \\
Q_i(t) \leq & Q_i^{max}, \\
Z_i(t) \leq & Z_i^{max}.
\end{aligned}
$$

Next, we analyze the queue lengths at time slot $t+1$. We start with the virtual queue $Y_i(t)$, which has the following two possible cases.

Case 1: $Y_i(t) \leq V \cdot U_i'(0)$. For this case, we have that

$$
\begin{aligned}
\eta_i(t) = & \max\{\min\{U_i'^{-1}(\frac{Y_i(t)}{V}), A_i^{max}\}, 0\} \\
> & \max\{\min\{U_i'^{-1}(\frac{VU_i'(0)}{V}), A_i^{max}\}, 0\} \\
= & \max\{\min\{U_i'^{-1}(U_i'(0)), A_i^{max}\}, 0\} = 0,
\end{aligned}
$$

according to Eqn. (23). Note that the inequality is because that $U_i(\cdot)$ is differential and concave, which means $U_i'^{-1}(\cdot)$ is a decreasing function.

Hence, in this case, $0 < \eta_i(t) \leq A_i^{max}$. We further have that

$$
\begin{aligned}
Y_i(t+1) = & \max\{Y_i(t) - r_i(t), 0\} + \eta_i(t) \\
\leq & \max\{V \cdot U_i'(0), 0\} + A_i^{max} \\
\leq & Y_i^{max}.
\end{aligned}
$$

The first inequality is based on the fact that $r_i(t) \in [0, A_i^{max}]$ and $\eta_i(t) \in [0, A_i^{max}]$.

Case 2: $V \cdot U_i'(0) < Y_i(t) \leq V \cdot U_i'(0) + A_i^{max}$. For this case, we have that

$$
\begin{aligned}
\eta_i(t) = & \max\{\min\{U_i'^{-1}(\frac{Y_i(t)}{V}), A_i^{max}\}, 0\} \\
\leq & \max\{\min\{U_i'^{-1}(\frac{VU_i'(0)}{V}), A_i^{max}\}, 0\} \\
= & \max\{\min\{U_i'^{-1}(U_i'(0)), A_i^{max}\}, 0\} = 0,
\end{aligned}
$$

according to Eqn. (23).

Hence, we further have that

$$
\begin{aligned}
Y_i(t+1) = & \max\{Y_i(t) - r_i(t), 0\} + \eta_i(t) \\
\leq & \max\{V \cdot U_i'(0) + A_i^{max}, 0\} \\
\leq & Y_i^{max}.
\end{aligned}
$$

The first inequality is based on the fact that $r_i(t) \in [0, A_i^{max}]$.

Up to now, $Y_i(t) \leq Y_i^{max}$, $\forall v_i \neq V_s$ for each time slot t is proved.

We next analyze the queue length of $Q_i(t+1)$. We have the following two possible cases.

Case 1: $Q_i(t) \leq V \cdot U_i'(0) + A_i^{max}$. For this case, we have that

$$
\begin{aligned}
Q_i(t+1) = & \max\{Q_i(t) - \sum_{c\in[1,C]\mu_{ic}(t) - d_i(t)}, 0\} + r_i(t) \\
\leq & \max\{V \cdot U_i'(0) + A_i^{max}, 0\} + A_i^{max} \\
\leq & Q_i^{max}.
\end{aligned}
$$

The first inequality is based on the fact that $r_i(t) \in [0, A_i^{max}]$.

Case 2: $V \cdot U_i'(0) + A_i^{max} < Q_i(t) \leq V \cdot U_i'(0) + 2A_i^{max}$. For this case, we have that

$$
\begin{aligned}
Q_i(t+1) = & \max\{Q_i(t) - \sum_{c\in[1,C]} \mu_{ic}(t) - d_i(t), 0\} + r_i(t) \\
\leq & \max\{V \cdot U_i'(0) + 2A_i^{max}, 0\} \\
\leq & Q_i^{max}.
\end{aligned}
$$

The first inequality is based on the fact that $r_i(t) = 0$ with Eqn. (24) and $Q_i(t) > V \cdot U_i'(0) + A_i^{max} = Y_i^{max} \geq Y_i(t)$.

Up to now, $Q_i(t) \leq Q_i^{max}$, $\forall v_i \neq V_s$ for each time slot t is proved.

Then, we analyze the queue length of $Z_i(t)$. We can have the following two cases.

Case 1: $Z_i(t) \leq V \cdot \beta_i$. In this case, we have that

$$\begin{aligned} Z_i(t+1) =& \max\{Z_i(t) + \mathbf{1}_{\{Q_i(t)>0\}} \cdot (\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t)) - d_i(t) \\ & - \mathbf{1}_{\{Q_i(t)=0\}}, 0\} \\ \leq& \max\{V \cdot \beta_i + \epsilon_i, 0\} \\ \leq& Z_i^{max}. \end{aligned}$$

Case 2: $V \cdot \beta_i < Z_i(t) \leq V \cdot \beta_i + \epsilon_i$. For this case, we can have that

$$\begin{aligned} Z_i(t+1) =& \max\{Z_i(t) + \mathbf{1}_{\{Q_i(t)>0\}} \cdot (\epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(t)) - d_i(t) \\ & - \mathbf{1}_{\{Q_i(t)=0\}}, 0\} \\ \leq& \max\{V \cdot \beta_i + \epsilon_i, 0\} \\ \leq& Z_i^{max}. \end{aligned}$$

The first inequality is based on the fact that $d_i(t) = d_i^{max}$ with Eqn. (25) and $d_i^{max} \geq \epsilon_i$.

Up to now, $Z_i(t) \leq Z_i^{max}$, $\forall v_i \neq V_s$ for each time slot t is proved.

In conclusion, Lemma 1 is proved.

□

D. PROOF TO THEOREM 1

We prove this theorem by contradiction.

For each secondary link $v_i \in V$, the admitted data packets at time slot $t \geq 0$ is $r_i(t)$ and the earliest time they can depart the queue $Q_i(t)$ is $t + 1$. We show that all these packets depart (by being either delivered or dropped) on or before $t + D_i$.

Suppose this is not true, we will come to a contradiction. We must have that $Q_i(\tau) > 0$ for all $\tau \in [t + 1, \ldots, t + D_i]$ (otherwise, all the packets have departed by time $t + D_i$). With the queueing law in Eqn. (11), we have that

$$\begin{aligned} Z_i(\tau+1) =& \max\{Z_i(\tau) + \epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(\tau) - d_i(\tau), 0\} \\ \geq& Z_i(\tau) + \epsilon_i - \sum_{c\in[1,C]} \mu_{ic}(\tau) - d_i(\tau). \end{aligned}$$

Summing the above over $\tau \in [t + 1, \ldots, t + D_i]$, we have that

$$Z_i(t + D_i + 1) - Z_i(t + 1) \geq \epsilon_i \cdot D_i - \sum_{\tau=t+1}^{t+D_i} [\sum_{c\in[1,C]} \mu_{ic}(\tau) + d_i(\tau)].$$

Rearranging the above inequality and using the fact that $Z_i(t + D_i + 1) \leq Z_i^{max}$ and $Z_i(t + 1) \geq 0$, we have that

$$\epsilon_i \cdot D_i - Z_i^{max} \leq \sum_{\tau=t+1}^{t+D_i} [\sum_{c\in[1,C]} \mu_{ic}(\tau) + d_i(\tau)]. \quad (31)$$

Since the packets are departing in a FIFO fashion, the packets $r_i(t)$, which arrive and are admitted at slot t, are placed at the end of the queue at slot $t + 1$, and should be fully cleared when all the packets backlogged in $Q_i(t + 1)$ have departed. That is, the last job of $r_i(t)$ departs on slot $t + T$ with $T > 0$ as the smallest integer satisfying $\sum_{\tau=t+1}^{t+T} [\sum_{c\in[1,C]} \mu_{ic}(\tau) + d_i(\tau)] \geq Q_i(t + 1)$. Based on our assumption that not all of the $r_i(t)$ packets depart by time $t + D_i$, we must have that

$$\sum_{\tau=t+1}^{t+D_i} [\sum_{c\in[1,C]} \mu_{ic}(\tau) + d_i(\tau)] < Q_i(t + 1) \leq Q_i^{max}. \quad (32)$$

Combining Eqn. (31) and (32), we have that

$$\begin{aligned} & \epsilon_i \cdot D_i - Z_i^{max} < Q_i^{max} \\ \Rightarrow & \epsilon_i < \frac{Q_i^{max} + Z_i^{max}}{D_i}. \end{aligned}$$

This contradicts with the given fact that $\epsilon_i = \frac{Q_i^{max}+Z_i^{max}}{D_i}$. Hence, we have proved that each packet on secondary link v_i is either scheduled or dropped with Algorithm 1 before its maximum delay D_i, if we set $\epsilon_i = \frac{Q_i^{max}+Z_i^{max}}{D_i}$.

□

E. PROOF TO THEOREM 2

We first prove the correctness of Algorithm 2 by showing that the generated channel allocations are collision-free. We prove the optimality by modeling the channel allocation decisions as a Discrete-Time Markov Chain (DTMC), which is next proved to be reversible. Hence, we derive the stationary distribution for each collision-free channel allocation decision, based on which we evaluate the achievable value for $\Psi(t)$ in expectation with Algorithm 2.

Note that, in the following proof, we make the time separation assumption, whereby the CSMA Markov chain converges to its steady-state distribution instantaneously compared to the timescale of adaptation of the CSMA parameters. That is commonly assumed in [12] and references therein, and justified by [21, 22].

Let $\chi(t) = \{\mu_{ic}(t)|\mu_{ic}(t) = 1, \forall v_i \in V_s, c \in [1, C]\}$ be a channel allocation decision in time slot t, and Λ be the set of all collision-free channel allocation decisions. Denote $\mathcal{M}$ be the set of all possible decision sets of $m(t)$. It is clear that $\mathcal{M} \subseteq \Lambda$, since the inclusion of allocation variable $\mu_{ic}(t)$ into $m(t)$ means no interfering allocation variable of $\mu_{ic}(t)$ times out before it, and $\mu_{ic}(t)$ will generate a 'INTENT' message that blocks its interfering allocation variables from joining $m(t)$.

Let $\rho(m(t)) > 0$ be the probability of selecting $m(t)$ as the decision set. We have that $\sum_{m(t)\in\mathcal{M}} \rho(m(t)) = 1$.

We denote $\mathcal{C}(\mu_{ic}(t))$ as the set of interfering channel allocation variables of $\mu_{ic}(t)$ and $\mathcal{C}(\chi(t))$ as the set of interfering channel allocation variables of channel allocation decision $\chi(t)$.

The correctness can be proven by induction. The induction basis is trivial since, at the beginning of the system (time 0), no channel is allocated, *i.e.*, $\chi(0) = \emptyset$, which is naturally a collision-free decision. The induction steps can be proved with the following lemma.

LEMMA 2. *If the channel allocation in time slot* $t-1$ *and the decision set in time slot* t *are both collision free,* i.e., $\chi(t-1) \in \Lambda$ *and* $m(t) \in \Lambda$, *we have that the channel allocation in time slot* t *with Algorithm 2 is also collision free,* i.e., $\chi(t) \in \Lambda$.

PROOF. A channel allocation decision $\chi(t)$ is collision-free if and only if $\forall \mu_{ic}(t) \in \chi(t)$, we have $\mu_{jk}(t) = 0$, $\forall \mu_{jk}(t) \in \mathcal{C}(\mu_{ic}(t))$.

Consider any $\mu_{ic}(t) \in \chi(t)$. If $\mu_{ic}(t) \notin m(t)$, we have that $\mu_{ic}(t-1) = \mu_{ic}(t) = 1$ based on Step 2 in the winner determination module of Algorithm 2, which means $\mu_{ic}(t-1) \in \chi(t-1)$. Since $\chi(t-1)$ is collision-free, we know that $\mu_{jk}(t-1) = 0$, $\forall \mu_{jk}(t-1) \in \mathcal{C}(\mu_{jk}(t-1))$. Then, we can discuss the value of $\mu_{jk}(t)$ as follows.

- If $\mu_{jk}(t) \notin m(t)$, we know that $\mu_{jk}(t) = \mu_{jk}(t-1) = 0$ based on Step 2 in the winner determination module of Algorithm 2.
- If $\mu_{jk}(t) \notin m(t)$, we have $\mu_{jk}(t) = 0$ since $\mu_{ic}(t-1) \in \chi(t-1)$ and $\mu_{ic}(t) \in \mathcal{C}(\mu_{jk}(t))$.

If $\mu_{ic}(t) \in m(t)$, we have that $\mu_{ic}(t) \in \chi(t)$ only if $\mu_{jk}(t-1) = 0, \forall \mu_{jk}(t-1) \in \mathcal{C}(\mu_{ic}(t-1))$. Since $\mu_{ic}(t) \in m(t)$ and $m(t)$ is collision-free, we know that $\mathcal{C}(\mu_{ic}(t)) \cap m(t) = \emptyset$. Hence, $\mu_{jk}(t) = \mu_{jk}(t-1) = 0$.

Thus, we prove this lemma by showing that $\forall \mu_{ic}(t) \in \chi(t)$, we have $\mu_{jk}(t) = 0, \forall \mu_{jk}(t) \in \mathcal{C}(\mu_{ic}(t))$.

□

To sum up the above, we have proved the correctness of Algorithm 2 to generate collision-free channel allocation decisions. Next, we prove the optimality on the basis of the following two lemmas.

LEMMA 3. *A channel allocation decision $\chi \in \Lambda$ can transit to a channel allocation decision $\chi' \in \Lambda$ if and only if $\chi \cup \chi' \in \Lambda$ and there exists a decision set $m \in \mathcal{M}$ such that*

$$\chi \triangle \chi' = (\chi/\chi') \cup (\chi'/\chi) \subseteq m,$$

and the transition probability from χ to χ' is

$$P(\chi, \chi') = \sum_{m \in \mathcal{M}: \chi \triangle \chi' \subseteq m} \rho(m) \left(\prod_{\mu_{ic} \in \chi/\chi'} 1 - p_i \right) \left(\prod_{\mu_{ic} \in \chi'/\chi} p_i \right) \left(\prod_{\mu_{ic} \in m \cap (\chi \cap \chi')} p_i \right) \left(\prod_{\mu_{ic} \in m/(\chi \cup \chi')/\mathcal{C}(\chi \cup \chi')} 1 - p_i \right) \tag{33}$$

PROOF. we first prove the necessity and then the sufficiency.
Necessity: Suppose μ is the current decision in time slot t and χ' is the next decision in slot $t+1$. $\chi/\chi' = \{\mu_{ic} | \mu_{ic}(t) = 1, \mu_{ic}(t+1) = 0\}$ is the set of channel allocation variables that change their state from 1 to 0. $\chi'/\chi = \{\mu_{ic} | \mu_{ic}(t) = 0, \mu_{ic}(t+1) = 1\}$ is the set of channel allocation variables that change their state from 0 to 1.

Based on Algorithm 2, we have that a channel allocation variable can change its state only if it is included in the decision set $m(t)$ (m in the proof). Therefore, χ can transit to χ' only if there exists a decision set $m \in \Lambda$ such that the symmetric difference $\chi \triangle \chi' = (\chi/\chi') \cup (\chi'/\chi) \subseteq m$. In addition, we have $\chi \cup \chi' = (\chi/\chi') \cup (\chi'/\chi) \cup (\chi \cap \chi') \in \Lambda$, since $(\chi \cap \chi') \cup (\chi/\chi') = \chi \in \Lambda$, $(\chi \cap \chi') \cup (\chi'/\chi) = \chi' \in \Lambda$, and $(\chi/\chi') \cup (\chi'/\chi) = \chi \triangle \chi'$.
Sufficiency: Suppose $\chi \cup \chi' \in \Lambda$ and there is an $m \in \Lambda$ such that $\chi \triangle \chi' \subseteq m$. Given m is selected randomly, we can calculate the probability for χ to transit to χ' by dividing the variables in m 5 cases as follows.

- $\mu_{ic}(t) \in \chi/\chi'$: Variable $\mu_{ic}(t)$ is decided to change its state from 1 to 0, which happens with probability $1-p_i$ with Algorithm 2.
- $\mu_{ic}(t) \in \chi'/\chi$: Variable $\mu_{ic}(t)$ is decided to change its state from 0 to 1, which occurs with probability p_i.
- $\mu_{ic}(t) \in m \cap (\chi \cap \chi')$: Variable $\mu_{ic}(t)$ is decided to keep the sate 1, which occurs with probability p_i.
- $\mu_{ic}(t) \in m \cap \mathcal{C}(\chi)$: Variable $\mu_{ic}(t)$ has to keep its state 0. This occurs with probability 1.
- $\mu_{ic}(t) \in m/(\chi \cup \chi')/\mathcal{C}(\chi)$: Variable $\mu_{ic}(t)$ decides to keep its state 0, which happens with probability with $1 - p_i$

Note that $m \cap \mathcal{C}(\chi'/\chi) = \emptyset$ since $\chi'/\chi \subseteq m$, we have that $m/(\chi \cup \chi')/\mathcal{C}(\chi) = m/(\chi \cup \chi')/\mathcal{C}(\chi \cup \chi')$. As each variable in m is decided independently of each other, we can multiply these probabilities together. Summing over all possible decision sets, we can get the overall transition probability from χ to χ' as in Eqn. (33). □

LEMMA 4. *A necessary and sufficient condition for the DTMC of the channel allocation decisions to be irreducible and aperiodic is*

$$\bigcup_{m(t) \in \mathcal{M}} m(t) = \{\mu_{ic} | \forall v_i \in V_s, c \in [1, C]\},$$

and in this case DTMC is reversible and has the following stationary distribution,

$$\pi(\chi) = \frac{1}{H} \prod_{\mu_{ic} \in \chi} \frac{p_i}{1 - p_i}, \tag{34}$$

$$H = \sum_{\chi \in \Lambda} \prod_{\mu_{ic} \in \chi} \frac{p_i}{1 - p_i},. \tag{35}$$

PROOF. We first prove the necessity and sufficiency condition for the DTMC to be irreducible and aperiodic, and next verify the reversibility and stationary distribution.
Necessity: Suppose $\bigcup_{m(t) \in \mathcal{M}} m(t) \neq \{\mu_{ic} | \forall v_i \in V_s, c \in [1, C]\}$. Let $\mu_{jk}(t) \notin \bigcup_{m(t) \in \mathcal{M}} m(t)$. Then, we have that, from the initial state of channel allocation decision, *i.e.*, $\emptyset$, the DMTC will never reach a collision-free decision including $\mu_{jk}(t)$. The necessity is proved.
Sufficiency: If $\bigcup_{m(t) \in \mathcal{M}} m(t) = \{\mu_{ic} | \forall v_i \in V_s, c \in [1, C]\}$, with Lemma 3, we have that the initial decision $\emptyset$ can reach any other collision-free decision $\chi \in \Lambda$ with positive probability in a finite number of steps, and vice versa. To sum up, the DTMC is irreducible and aperiodic.

If allocation decision χ can transit to decision χ', we can verify that Eqn. (34) satisfies the balance equation,

$$\begin{aligned} \pi(\chi)P(\chi, \chi') =& \frac{1}{H} \sum_{m \in \mathcal{M}: \chi \triangle \chi' \subseteq m} \rho(m) \left(\frac{\prod_{\mu_{ic} \in \chi \cup \chi'} p_i}{\prod_{\mu_{jk} \in \chi \cap \chi'} 1 - p_j} \right) \\ & \times \left(\prod_{\mu_{ic} \in m \cap (\chi \cap \chi')} p_i \right) \left(\prod_{\mu_{ic} \in m/(\chi \cup \chi')/\mathcal{C}(\chi \cup \chi')} 1 - p_i \right) \\ =& \pi(\chi')P(\chi', \chi). \end{aligned} \tag{36}$$

Therefor, the DTMC is reversible and Eqn. (34) is the stationary distribution [23].

□

Finally, we prove Theorem 2 based on the above lemmas.

PROOF. Given any δ and θ with $0 < \delta, \theta < 1$. Let $\Psi^*(t) = \max_{\chi \in \Lambda} \Psi(t)$. We define

$$\xi = \{\chi \in \Lambda | \Psi(t) < (1 - \delta)\Psi^*(t)\}.$$

As the DTMC has the stationary distribution in Eqn. (34), we have that

$$\begin{aligned} \pi(\xi) = \sum_{\chi \in \xi} \pi(\chi) = \sum_{\chi \in \xi} \frac{e^{\sum_{\mu_{ic}(t) \in \chi} V \cdot b_i}}{H} \\ \leq \frac{|\xi| e^{1-\delta} \Psi^*(t)}{H} < \frac{2^{|V_s| \cdot C}}{e^{\delta \Psi^*(t)}}, \end{aligned} \tag{37}$$

where the secondary inequality comes from the fact that $|\xi| \le |\Lambda| \le 2^{|V_s|\cdot C}$ (Here, $|V_s|\cdot C$ is the total number of channel allocation variables), and $H > e^{\max_{\chi\in\Lambda}\sum_{\mu_{ic}\in\chi} V\cdot b_i(t)} = e^{\Psi^*(t)}$. Hence, if

$$\Psi^*(t) > \frac{1}{\delta}\left(|V_s|\cdot C\log 2 + \log\frac{1}{\theta}\right), \tag{38}$$

we could have that $\pi(\xi) < \theta$. Since $\Psi^*(t)$ is a continuous, nondecreasing function of the packet queues and QoS virtual queues, *i.e.*, $\Gamma(t) = \{Q_i(t), Z_i(t)|\forall v_i \in V_s\}$, we can further have that, with $\lim_{\|\Gamma(t)\|\to\infty}\Psi^*(t) = \infty$, there exists a constant value B_Γ such that inequality (38) holds so that $\pi(\xi) < \delta$ whenever $\|\Gamma(t)\| > B$.

According to Lemma 1, the value of $\Gamma(t)$ is proportional to V. By scaling up $V \to \infty$, we have that $\Psi^*(t) \to \infty$ such that $\delta \to 0$ and $\theta \to 0$ with Eqn. (37) and (38). Since the DTMC generate a value of $\Psi(t)$ in $(1-\delta)\Psi^*(t)$ with a probability of at least $1-\theta$, we have that, with $V \to \infty$, Algorithm 2 results in an expected value of $\Psi(t)$ arbitrarliy close to $\Psi^*(t)$.

□

F. PROOF TO THEOREM 3

PROOF. The maximization of (17) is decoupled into four independent one-shot optimizations as in (19), (20), (18) and (21). Since (19), (20), and (21) are independent of the spectrum auction, and already optimized with Algorithm 1. We just need to prove that, bidding truthfully can optimize the one-shot optimization problem in (18).

The definition of the bud-bid's true evaluation is the value, charged above which the bidder will have negative utility gain from the auction. According to the definition, we know that the true value of $\tilde{b}_i(t)$ should be $\frac{Q_i(t)+Z_i(t)}{V}$ as defined in Eqn. (22), since: i) if $\hat{b}_i(t) > \frac{Q_i(t)+Z_i(t)}{V}$, the utility in (18) is negative for secondary link v_i; ii) if $\hat{b}_i(t) < \frac{Q_i(t)+Z_i(t)}{V}$, the utility in (18) is positive; and iii) if $\hat{b}_i(t) = \frac{Q_i(t)+Z_i(t)}{V}$, the utility in (18) is zero for secondary link v_i.

□

G. PROOF TO THEOREM 4

We prove this theorem to show that any secondary link $v_i \in V_s$ cannot obtain higher utility gain in expectation by bidding untruthfully, *i.e.*, $b_i(t) \ne \tilde{b}_i(t)$, by analyzing all possible auction results.

The expected utility gain by bidding with $b_i(t)$ is

$$\begin{aligned}
&\sum_{c\in[1,C]}[\pi_{ic}\cdot[Q_i(t)+Z_i(t)]] - V\cdot\mathbb{E}\{\hat{b}_i(t)\}\\
=&V\cdot\sum_{c\in[1,C]}[\pi_{ic}\cdot\tilde{b}_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi_{jc} - \sum_{j\ne i} b_j(t)\cdot\pi'_{jc}]\\
\ge&V\cdot\sum_{c\in[1,C]}[\pi''_{ic}\cdot\tilde{b}_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi''_{jc} - \sum_{j\ne i} b_j(t)\cdot\pi'_{jc}].
\end{aligned}$$

Here, π'_{jc} denotes the stationary distribution for $\mu_{jc}(t) = 1$, $\forall v_j \in V_s, c \in [1,C]$, when $b_i(t) = 0$; and π''_{jc} denotes the stationary distribution for $\mu_{jc}(t) = 1$, $\forall v_j \in V_s, c \in [1,C]$, when $b_i(t) \ne \tilde{b}_i(t)$. The inequality is based on Theorem 2 that bidding truthfully results in maximized value of $\mathbb{E}\{\Psi(t)\}$, which equals to $\sum_{c\in[1,C]}[\pi_{ic}\cdot b_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi_{jc}]$.

□

H. PROOF TO THEOREM 5

We prove this theorem by showing that, the expected utility gain of each secondary link v_i is non-negative.

The expected utility gain by bidding with $\tilde{b}_i(t)$ is

$$\begin{aligned}
&\sum_{c\in[1,C]}[\pi_{ic}\cdot[Q_i(t)+Z_i(t)]] - V\cdot\mathbb{E}\{\hat{b}_i(t)\}\\
=&V\cdot\sum_{c\in[1,C]}[\pi_{ic}\cdot\tilde{b}_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi_{jc} - \sum_{j\ne i} b_j(t)\cdot\pi'_{jc}]\\
\ge&V\cdot\sum_{c\in[1,C]}[\pi_{ic}\cdot\tilde{b}_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi_{jc} - \sum_{j\ne i} b_j(t)\cdot\pi'_{jc} - \pi'_{ic}\cdot\tilde{b}_i(t)]\\
\ge&0.
\end{aligned}$$

The first inequality comes from the fact that $\pi'_{ic}\cdot\tilde{b}_i(t) \ge 0$. And the second inequality is based on Theorem 2 that bidding truthfully results in maximized value of $\mathbb{E}\{\Psi(t)\}$, which equals to $\sum_{c\in[1,C]}[\pi_{ic}\cdot\tilde{b}_i(t) + \sum_{j\ne i} b_j(t)\cdot\pi_{jc}]$.

Hence, each secondary link is individually rational in expectation.

□

I. PROOF TO THEOREM 6

We prove this theorem by showing that, the expected payment from each secondary link v_i is non-negative. Hence, the cumulative payment from all secondary links is also non-negative.

The expected payment from secondary link v_i is as follows,

$$\begin{aligned}
\mathbb{E}\{\hat{b}_i(t)\} =&\mathbb{E}\{\sum_{j\ne i} b_j(t)\cdot[\mu_j^{(i)}(t) - \mu_j(t)]\}\\
=&\sum_{j\ne i} b_j(t)\cdot[\pi'_j - \pi_j].
\end{aligned}$$

Since $\sum_{j\ne i} b_j(t)\cdot\pi''_j + 0\cdot\pi''_i = \sum_{j\ne i} b_j(t)\cdot\pi''_j$, the winner determination in Algorithm 2 maximizes $\sum_{j\ne i} b_j(t)\cdot\pi''_j$ only with $b_i(t) = 0$ and $\pi''_j = \pi'_j$. Hence, we have that

$$\sum_{j\ne i} b_j(t)\cdot[\pi'_j - \pi_j] \ge 0$$

Hence, the expected income of primary user is non-negative.

□

J. PROOF TO THEOREM 7

We prove this theorem based on the Lyapunov optimization theory. Since the data arrivals and channel availabilities follow ergodic processes at each secondary link, we know that there exits a stationary randomized algorithm, which dynamically decides the rate control (with $\eta_i^*(t)$ and $r_i^*(t)$), channel evaluation & bidding (with $\hat{b}_i^*(t)$ and $b_i^*(t)$), and packet dropping (with $d_i^*(t)$) such that the offline optimal utility Ω_i^* can be achieved at secondary link $v_i \in V_s$, together with $\bar{\eta}_i^* \le \bar{r}_i^*$, $\bar{r}_i^* \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$, and $\epsilon_i \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$. [2]

Based on the derivations of the optimization problem (17) and its solution in Sec. 4, we know that the Algorithm 1 minimizes the right-hand-side of the drift-plus-penalty (drift-minus-utility) inequality (12), in expectation, at time slot t, with individual utility maximization as the utility, overall all possible algorithms. Hence, we can have that

[2] The assignment to $\mu_{ic}^*(t)$ is totally determined by the bidding price $b_i^*(t)$ and the auction mechanism. The stationary randomized algorithm does not decide $\mu_{ic}^*(t)$ directly.

$$
\begin{aligned}
&\mathbb{E}\{\Delta(\Theta_i(t))\} - V\cdot[\mathbb{E}\{U_i(\eta_i(t))\} - \mathbb{E}\{\hat{b}_i(t)\} - \beta_i\cdot\mathbb{E}\{d_i(t)\}] \\
\le& B_i + \epsilon_i\cdot Z_i(t) - V\cdot\mathbb{E}\{U_i(\eta_i(t))\} + \mathbb{E}\{\eta_i(t)\}\cdot Y_i(t) \\
&- \mathbb{E}\{r_i(t)\}\cdot[Y_i(t) - Q_i(t)] \\
&- \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}(t)\}\cdot[Q_i(t)+Z_i(t)] + V\cdot\mathbb{E}\{\hat{b}_i(t)\} \\
&- \mathbb{E}\{d_i(t)\}\cdot[Q_i(t)+Z_i(t)-V\cdot\beta_i] \\
\le& B_i + \epsilon_i\cdot Z_i(t) - V\cdot\mathbb{E}\{U_i(\eta_i^*(t))\} + \mathbb{E}\{\eta_i^*(t)\}\cdot Y_i(t) \\
&- \mathbb{E}\{r_i^*(t)\}\cdot[Y_i(t) - Q_i(t)] \\
&- \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\}\cdot[Q_i(t)+Z_i(t)] + V\cdot\mathbb{E}\{\hat{b}_i^*(t)\} \\
&- \mathbb{E}\{d_i^*(t)\}\cdot[Q_i(t)+Z_i(t)-V\cdot\beta_i] \\
=& B_i - V\cdot[\mathbb{E}\{U_i(\eta_i^*(t))\} - \mathbb{E}\{\hat{b}_i^*(t)\} - \beta_i\cdot\mathbb{E}\{d_i^*(t)\}] \\
&+ Y_i(t)\cdot[\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}] \\
&+ Q_i(t)\cdot[\mathbb{E}\{r_i^*(t)\} - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}] \\
&+ Z_i(t)\cdot[\epsilon_i - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}].
\end{aligned}
$$

By summing over the T slots on both sides of the inequality, we have that

$$
\begin{aligned}
&\mathbb{E}\{L(\Theta_i(T))\} - \mathbb{E}\{L(\Theta_i(0))\} - V\cdot\sum_{t=0}^{T-1}[\mathbb{E}\{U_i(\eta_i(t))\} \\
&- \mathbb{E}\{\hat{b}_i(t)\} - \beta_i\cdot\mathbb{E}\{d_i(t)\}] \\
\le& T\cdot B_i - \sum_{t=0}^{T-1}V\cdot[\mathbb{E}\{U_i(\eta_i^*(t))\} - \mathbb{E}\{\hat{b}_i^*(t)\} - \beta_i\cdot\mathbb{E}\{d_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Y_i(t)\cdot[\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Q_i(t)\cdot[\mathbb{E}\{r_i^*(t)\} - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Z_i(t)\cdot[\epsilon_i - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}].
\end{aligned}
$$

Since $\mathbb{E}\{L(\Theta_i(T))\} \ge 0$ and $\mathbb{E}\{L(\Theta_i(0))\} = 0$ according to the definition of the Lyapunov function, we have that

$$
\begin{aligned}
&- V\cdot\sum_{t=0}^{T-1}[\mathbb{E}\{U_i(\eta_i(t))\} - \mathbb{E}\{\hat{b}_i^*(t)\} - \beta_i\cdot\mathbb{E}\{d_i^*(t)\}] \\
\le& T\cdot B_i - \sum_{t=0}^{T-1}V\cdot[\mathbb{E}\{U_i(\eta_i^*(t))\} - \mathbb{E}\{\hat{b}_i^*(t)\} - \beta_i\cdot\mathbb{E}\{d_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Y_i(t)\cdot[\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Q_i(t)\cdot[\mathbb{E}\{r_i^*(t)\} - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}] \\
&+ \sum_{t=0}^{T-1}Z_i(t)\cdot[\epsilon_i - \sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{d_i^*(t)\}].
\end{aligned}
$$

Dividing $T\cdot V$ on both sides of the above inequality and taking limitation on T to infinity, we have that

$$
\begin{aligned}
-\Omega_i \le& B_i/V - \Omega_i^* + \bar{Y}_i\cdot[\bar{\eta}_i^* - \bar{r}_i^*] \\
&+ \bar{Q}_i\cdot[\bar{r}_i^* - \sum_{c\in[1,C]}\bar{\mu}_{ic}^* - \bar{d}_i^*(t)] \\
&+ \bar{Z}_i\cdot[\epsilon_i - \sum_{c\in[1,C]}\bar{\mu}_{ic}^* - \bar{d}_i^*(t)] \\
\le& B_i/V - \Omega_i^*.
\end{aligned}
$$

The second inequality comes from the fact that $\bar{\eta}_i^* \le \bar{r}_i^*$, $\bar{r}_i^* \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$, and $\epsilon_i \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$. Rearranging the two sides, we have that

$$
\Omega_i \ge \Omega_i^* - B_i/V.
$$

□

K. PROOF TO THEOREM 8

Our proof is based on the Lyapunov optimization theory. Since the data arrivals and channel availabilities follow ergodic processes at each secondary link, we know that there exits a stationary randomized algorithm, which dynamically decides the rate control (with $\eta_i^*(t)$ and $r_i^*(t)$), channel allocation (with $\mu_{ic}^*(t)$), and packet dropping (with $d_i^*(t)$), such that the offline optimal social welfare Π^* can be achieved, together with $\bar{\eta}_i^* \le \bar{r}_i^*$, $\bar{r}_i^* \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$, and $\epsilon_i \le \sum_{c\in[1,C]}\bar{\mu}_{ic}^* + \bar{d}_i^*$, $\forall v_i \in V_s$.

Based on the derivations of the optimization problem (29) and its solution in Sec. 5, we know that the Algorithm 3 minimizes the expected right-hand-side of the drift-plus-penalty (drift-minus-utility) inequality (27) at each time slot t, with social welfare maximization as the utility, overall all possible algorithms. Comparing Algorithm 3 and the combination of Algorithm 1 and 2, we have that they share the same decisions on the auxiliary variables, rate control and packet dropping. Also, as proved in Theorem 2, we have that the expectation of $\Psi(t)$ as defined in Eqn. (28) is maximized with Algorithm 1 and 2 if each secondary link bids truthfully. Hence, the expectation of $\Psi(t)$ by Algorithm 3 and the combination of Algorithm 1 and 2 are the same. Then, we have that Algorithm 1 and 2 also minimizes the expected right-hand-side of the drift-plus-penalty (drift-minus-utility) inequality (27) at each time slot t. Therefore, Algorithm 3 should achieve the same social welfare in expectation with that by Algorithm 1 and 2.

We can have that

$$
\begin{aligned}
&\mathbb{E}\{\Delta(\Theta(t))\} - V\cdot\sum_{v_i\in V_s}[\mathbb{E}\{U_i(\eta_i(t))\} - \beta_i\cdot\mathbb{E}\{d_i(t)\}] \\
\le& B + \sum_{v_i\in V_s}\epsilon_i\cdot Z_i(t) - V\cdot\sum_{v_i\in V_s}\mathbb{E}\{U_i(\eta_i(t))\} \\
&+ \sum_{v_i\in V_s}\mathbb{E}\{\eta_i(t)\}\cdot Y_i(t) - \sum_{v_i\in V_s}\mathbb{E}\{r_i(t)\}\cdot[Y_i(t)-Q_i(t)] \\
&- \sum_{v_i\in V_s}\sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}(t)\}\cdot[Q_i(t)+Z_i(t)] \\
&- \sum_{v_i\in V_s}\mathbb{E}\{d_i(t)\}\cdot[Q_i(t)+Z_i(t)-V\cdot\beta_i] \\
\le& B + \sum_{v_i\in V_s}\epsilon_i\cdot Z_i(t) - V\cdot\sum_{v_i\in V_s}\mathbb{E}\{U_i(\eta_i^*(t))\} \\
&+ \sum_{v_i\in V_s}\mathbb{E}\{\eta_i^*(t)\}\cdot Y_i(t) - \sum_{v_i\in V_s}\mathbb{E}\{r_i^*(t)\}\cdot[Y_i(t)-Q_i(t)] \\
&- \sum_{v_i\in V_s}\sum_{c\in[1,C]}\mathbb{E}\{\mu_{ic}^*(t)\}\cdot[Q_i(t)+Z_i(t)] \\
&- \sum_{v_i\in V_s}\mathbb{E}\{d_i^*(t)\}\cdot[Q_i(t)+Z_i(t)-V\cdot\beta_i]
\end{aligned}
$$

$$=B - V \cdot \sum_{v_i \in V_s} [\mathbb{E}\{U_i(\eta_i^*(t))\} - \beta_i \cdot \mathbb{E}\{\{d_i^*(t)\}]$$
$$+ \sum_{v_i \in V_s} Y_i(t) \cdot [\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}]$$
$$+ \sum_{v_i \in V_s} Q_i(t) \cdot [\mathbb{E}\{r_i^*(t)\} - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{\hat{b}_i^*(t)\}]$$
$$+ \sum_{v_i \in V_s} Z_i(t) \cdot [\epsilon_i - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{d_i^*(t)\}].$$

By summing over the T slots on both sides of the inequality, we have that

$$\mathbb{E}\{L(\Theta(T))\} - \mathbb{E}\{L(\Theta(0))\} - V \cdot \sum_{t=0}^{T-1} \sum_{v_i \in V_s} [\mathbb{E}\{U_i(\eta_i(t))\} - \beta_i \cdot \mathbb{E}\{d_i(t)\}]$$
$$\leq T \cdot B - \sum_{t=0}^{T-1} V \cdot \sum_{v_i \in V_s} [\mathbb{E}\{U_i(\eta_i^*(t))\} - \beta_i \cdot \mathbb{E}\{\{d_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Y_i(t) \cdot [\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Q_i(t) \cdot [\mathbb{E}\{r_i^*(t)\} - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{\hat{b}_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Z_i(t) \cdot [\epsilon_i - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{d_i^*(t)\}].$$

Since $\mathbb{E}\{L(\Theta(T))\} \geq 0$ and $\mathbb{E}\{L(\Theta(0))\} = 0$ according to the definition of the Lyapunov function, we have that

$$- V \cdot \sum_{t=0}^{T-1} \sum_{v_i \in V_s} [\mathbb{E}\{U_i(\eta_i(t))\} - \beta_i \cdot \mathbb{E}\{d_i(t)\}]$$
$$\leq T \cdot B - \sum_{t=0}^{T-1} V \cdot \sum_{v_i \in V_s} [\mathbb{E}\{U_i(\eta_i^*(t))\} - \beta_i \cdot \mathbb{E}\{\{d_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Y_i(t) \cdot [\mathbb{E}\{\eta_i^*(t)\} - \mathbb{E}\{r_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Q_i(t) \cdot [\mathbb{E}\{r_i^*(t)\} - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{\hat{b}_i^*(t)\}]$$
$$+ \sum_{t=0}^{T-1} \sum_{v_i \in V_s} Z_i(t) \cdot [\epsilon_i - \sum_{c \in [1,C]} \mathbb{E}\{\mu_{ic}^*(t)\} - \mathbb{E}\{\{d_i^*(t)\}].$$

Dividing $T \cdot V$ on both sides of the above inequality and taking limitation on T to infinity, we have that

$$\begin{aligned} -\Pi_{12} \leq & B/V - \Pi^* + \sum_{v_i \in V_s} \bar{Y}_i \cdot [\bar{\eta}_i^* - \bar{r}_i^*] \\ & + \sum_{v_i \in V_s} \bar{Q}_i \cdot [\bar{r}_i^* - \sum_{c \in [1,C]} \bar{\mu}_{ic}^* - \bar{d}_i^*(t)] \\ & + \sum_{v_i \in V_s} \bar{Z}_i \cdot [\epsilon_i - \sum_{c \in [1,C]} \bar{\mu}_{ic}^* - \bar{d}_i^*(t)] \\ \leq & B/V - \Pi^*. \end{aligned}$$

The second inequality comes from the fact that $\bar{\eta}_i^* \leq \bar{r}_i^*$, $\bar{r}_i^* \leq \sum_{c \in [1,C]} \bar{\mu}_{ic}^* + \bar{d}_i^*$, and $\epsilon_i \leq \sum_{c \in [1,C]} \bar{\mu}_{ic}^* + \bar{d}_i^*$. Rearranging the two sides, we have that

$$\Pi_{12} \geq \Pi^* - B/V.$$

As discussed at the beginning of the proof, we can also have that

$$\Pi_3 \geq \Pi^* - B/V.$$

□