Some Analytical Results on Rate-Controlled Many-to-One Communication over Two Hops

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Abstract

We consider the problem of data-communication from the perimeter to the center of a circle. We present a class of transmission schedules that allow for greater energy-efficiency (as compared to direct communication) without degradation in throughput by exploiting the non-linear relationship between transmisson power and achievable rate. Some analytical results regarding the same are stated and proved. Besides, some observed trends for which we currently do not have a proof are also discussed.

Introduction

The many-to-one communication paradigm is extremely relevant in the emerging contexts of hybrid wireless networks as well as data gathering sensor networks. In this model of communication the point of data-convergence (base-station) becomes a bottleneck, thereby imposing a trivial upper bound of W on achievable aggregate capacity [1] where W is the maximum transmission rate possible. Some earlier work on many-to-one communication has considered hierarchical schemes wherein nodes are organized into clusters and intra-cluster communication occurs over a different channel than cluster-head to base-station (BS) communication [1], [2]. However we observe that even within the constraint of a single channel, it is possible to harness the potential for spatial re-use available in the network to obtain greater energy-efficiency while maintaining the same aggregate rate. We envisage a mechanism that exploits the non-linearity inherent in Shannon's equation: $W = B\log(1+SINR)$, and improves on energy consumption by replacing a single high rate transmission with many spatially spread out low rate ones.

We present a class of transmission schedules operating on a *single* channel that allow for more energy-efficient data-communication from the perimeter to the center of a circle (as compared to direct communication) without degradation in throughput. Some analytical results regarding the same are stated and proved. We also discuss some observed numerical trends.

Related Work

Upper bounds on the capacity of wireless networks have been proved first in [3] for ad-hoc networks, and thereafter in [4] and [5] for hybrid networks. In the context of networks where data converges at a single point, some analytical results based on the Protocol Model [3] have been reported in [1]. The trivial upper bound of W on aggregate capacity in a many-to-one network, where W is the maximum transmission rate possible, has been stated and proved. It has been shown that while W may be achievable in an arbitrary network with controlled node placement (when certain conditions on guard zone size are met), it is not achievable with high probability in a random network. However, $\Theta(W)$ aggregate capacity is proved to be achievable with high probability in a random network.

Work on energy-efficient communication, as in [6], has explored the energy gains obtainable by communicating via relays rather than directly. They develop a mathematical formulation for the *relay region* (if the receiver is located within this region relaying is certain to yield energy gains) while attempting to transmit information from one point to another. However, this work does not address the impact of relaying on capacity. It is a generally known fact that in the case of one-to-one communication between two points using a fixed rate, introduction of relays i.e. multiple hops reduces the attainable throughput for this single communication, as all links may not be simultaneously active. It is therefore relevant to consider capacity and energy in conjunction.

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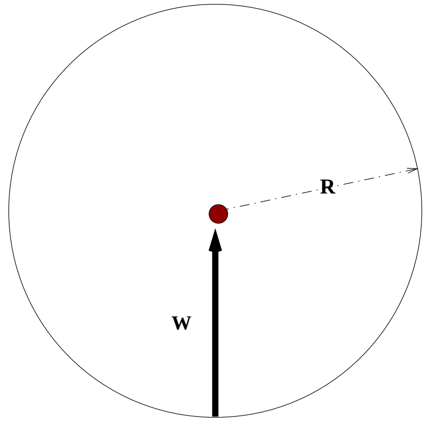

Fig. 1. Direct Communication Scheme

This paper proposes a mechanism that allows for greater energy-efficiency in many-to-one communication without any corresponding degradation in throughput.

COMMUNICATION MODEL

We consider a circular network of radius R. A BS is located at the center, and data is to be transferred from the perimeter of the circle to the BS. We seek to achieve a certain aggregate rate of W. Let us assume there is a very high linear density of nodes along the perimeter. Given a bandwidth B, and the desired aggregate throughput W, if the perimeter nodes were to communicate directly with the BS, we can visualize a TDMA schedule where in each slot exactly one perimeter node transmits (Fig. 1). We use the Physical Model [3] and assume that for successful communication at a rate W, the SINR at the receiver needs to be above a certain threshold β_W. To establish an upper bound, we obtain the value of β_W from Shannon's equation. We assume that transmitted and received power are related by the following expression: $P_{rx} = \frac{CP_{tx}}{d^\alpha}$ where P_{tx} is the transmitted power, P_{rx} is the received power, d is the distance between transmitter and receiver, and C is a constant. We assume that a source and destination are either separated by at least a minimum distance d_{min} such that $\frac{C}{{d_{min}}^\alpha} < 1$, else they are co-located (i.e. there is effectively no data communication happening). We assume $R > C^{\frac{1}{\alpha}}$ so that the minimum separation issue does not apply to the direct communication case, else it would not be of interest. Within this model, the SINR criterion for direct communication from perimeter to BS is formulated as:

$$\frac{\frac{CP_{direct}}{R^\alpha}}{N_t} \geq \beta_W$$

Thus the minimum power needed is given by:

$$P_{direct} = \frac{\beta_W N_t R^\alpha}{C}$$

where $\beta_W = 2^{W/B} - 1$, B is the available spectral bandwidth, α is path-loss exponent, N_t is the ambient thermal noise, and C is a constant depending on wavelength, antenna gain etc.

Another relation obtained from Shannon's equation that shall be extensively used in our analysis is the following:

$$\beta_{W/k} = (1 + \beta_W)^{1/k} - 1$$

where $\beta_{W/k}$ is the SINR threshold required for rate W/k.

TWO-HOP COMMUNICATION

We consider alternatives to the direct communication schedule. We analyze three different classes of transmission schedules that operate over 2 hops i.e. use relay nodes located at intermediate locations (radial distance xR) between the perimeter and BS. x varies within the range allowable by the minimum separation criterion stated earlier, else lapses to 0 or 1 respectively. The angular position θ of concurrent outer-ring transmissions is also a variable parameter in these schedule classes. We observe that in each case the throughput per ring is W and that gives us the required aggregate throughput of W. We formulate equations for required SINR at each receiver and obtain expressions for total power consumed. We obtain the ratio $\frac{P_{total}^{min}}{P_{direct}} = \iota$ for each schedule, where we term ι as the best-possible *improvement factor*. $\iota > 1$ implies degradation in energy-efficiency.

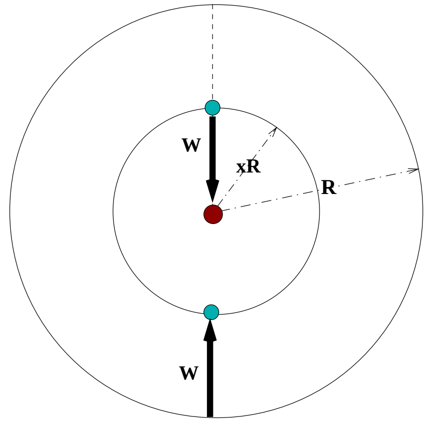

Fig. 2. 1-Relay 1-Rate Schedules

1-Relay 1-Rate Schedules

Fig. 2 illustrates a schedule in which each slot has two simultaneous transmissions ongoing viz. a perimeter node transmitting to a relay and a relay diametrically opposite transmitting to the BS. All transmissions are at rate W. The powers used by these two transmitting nodes are P_{in} and P_{out} respectively.

The SINR equations are:

At the BS:

$$\frac{\frac{CP_{in}}{x^\alpha R^\alpha}}{N_t + \frac{CP_{out}}{R^\alpha}} \geq \beta_W$$

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + \beta_W x^\alpha P_{out} \tag{1}$$

At the receiving relay:

$$\frac{\frac{CP_{out}}{(1-x)^\alpha R^\alpha}}{N_t + \frac{CP_{in}}{2^\alpha x^\alpha R^\alpha}} \geq \beta_W$$

$$P_{out} \geq \frac{\beta_W N_t R^\alpha}{C}(1-x)^\alpha + \beta_W \frac{(1-x)^\alpha}{2^\alpha x^\alpha} P_{in} \tag{2}$$

Back-substituting in Eqn. 1:

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + \frac{\beta_W x^\alpha \beta_W N_t R^\alpha}{C}(1-x)^\alpha + {\beta_W}^2 \frac{(1-x)^\alpha}{2^\alpha} P_{in}$$

$$P_{in}\left[1 - {\beta_W}^2 \frac{(1-x)^\alpha}{2^\alpha}\right] \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha \left[1 + \beta_W (1-x)^\alpha\right]$$

Since R.H.S of the above equation is positive and so will P_{in} be, we obtain the following feasibility condition:

$$1 - {\beta_W}^2 \frac{(1-x)^\alpha}{2^\alpha} > 0 \tag{3}$$

The following expression is thus obtained for P_{in}:

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha 2^\alpha \frac{1 + \beta_W (1-x)^\alpha}{2^\alpha - {\beta_W}^2 (1-x)^\alpha} \tag{4}$$

Substituting in Eqn. 2, we obtain:

$$P_{out} \geq \frac{\beta_W N_t R^\alpha}{C}\left[(1-x)^\alpha + \beta_W (1-x)^\alpha \frac{1 + \beta_W (1-x)^\alpha}{2^\alpha - {\beta_W}^2 (1-x)^\alpha}\right] \tag{5}$$

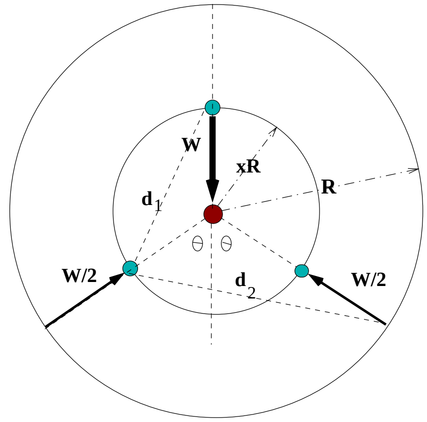

Fig. 3. 2-Relay 2-Rate Schedules

Thus the total power consumed in this schedule is:

$$P_{total} = P_{in} + P_{out} \geq \frac{\beta_W N_t R^\alpha}{C}\left[(1-x)^\alpha + \frac{([1+\beta_W(1-x)^\alpha][2^\alpha x^\alpha + \beta_W(1-x)^\alpha]}{2^\alpha - {\beta_W}^2(1-x)^\alpha}\right]$$

$$= P_{direct}\left[(1-x)^\alpha + \frac{([1+\beta_W(1-x)^\alpha][2^\alpha x^\alpha + \beta_W(1-x)^\alpha]}{2^\alpha - {\beta_W}^2(1-x)^\alpha}\right] \quad (6)$$

$$\iota = \left[(1-x)^\alpha + \frac{([1+\beta_W(1-x)^\alpha][2^\alpha x^\alpha + \beta_W(1-x)^\alpha]}{2^\alpha - {\beta_W}^2(1-x)^\alpha}\right] \quad (7)$$

2-Relay 2-Rate Schedules

Fig. 3 depicts a schedule in which the single communication in the outer ring is replaced by two transmissions at rate $W/2$. We denote the transmission power used by each perimeter node as P_{out} and that used by the transmitting relay as P_{in}.

Using the cosine formula, we obtain the values of d_1 and d_2 (see figure) as:

$$d_1 = 2xRcos\frac{\theta}{2} \quad (8)$$

$$d_2 = R\sqrt{(x^2 - 2xcos(2\theta) + 1)} \quad (9)$$

The SINR equations are:

At the BS:

$$\frac{\frac{CP_{in}}{x^\alpha R^\alpha}}{N_t + 2\frac{CP_{out}}{R^\alpha}} \geq \beta_W \quad (10)$$

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + 2\beta_W x^\alpha P_{out} \quad (11)$$

At the receiving relays:

$$\frac{\frac{CP_{out}}{(1-x)^\alpha R^\alpha}}{N_t + \frac{CP_{in}}{{d_1}^\alpha} + \frac{CP_{out}}{{d_2}^\alpha}} \geq \beta_{W/2} \quad (12)$$

This simplifies to:

$$P_{out} \geq \frac{\beta_{W/2} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha} P_{in} + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_2}^\alpha} P_{out} \quad (13)$$

which may be re-written as:

$$P_{out}\overbrace{\left[1 - \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_2}^\alpha}\right]}^{E_1} \geq \frac{\beta_{W/2} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha} P_{in} \quad (14)$$

Since the R.H.S. of the above equation is positive and P_{out} is also always positive, we thus obtain a feasibility condition:

$$E_1 > 0 \tag{15}$$

Therefore:

$$P_{out} \geq \frac{\beta_{W/2} N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha E_1} P_{in} \tag{16}$$

Back-substituting in the earlier expression for P_{in} (Eqn. 11):

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + 2x^\alpha \beta_W \left[\frac{\beta_{W/2} N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha E_1} P_{in} \right]$$

$$P_{in} \overbrace{\left[1 - 2\frac{\beta_{W/2} \beta_W R^\alpha x^\alpha (1-x)^\alpha}{{d_1}^\alpha \cdot E_1} \right]}^{E_2} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + 2\frac{\beta_{W/2} \beta_W N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha x^\alpha$$

This yields another feasibility condition:

$$E_2 > 0 \tag{17}$$

$$\begin{aligned} P_{in} \cdot E_2 &\geq \frac{\beta_W N_t R^\alpha}{C} \left[x^\alpha + \frac{2\beta_{W/2}(1-x)^\alpha x^\alpha}{E_1} \right] \\ &= P_{direct} \left[x^\alpha + \frac{2\beta_{W/2}(1-x)^\alpha x^\alpha}{E_1} \right] \end{aligned}$$

We thus obtain the expression:

$$P_{in} \geq P_{direct} \frac{1 + \frac{2\beta_{W/2}(1-x)^\alpha}{E_1}}{E_2} x^\alpha \tag{18}$$

We now substitute in Eqn. 16 to obtain:

$$P_{out} \geq \frac{\beta_{W/2} N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha + \left(\frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha \cdot E_1} \right) \left(\frac{1 + \frac{2\beta_{W/2}(1-x)^\alpha}{E_1}}{E_2} \right) x^\alpha P_{direct} \tag{19}$$

This simplifies further to:

$$P_{out} \geq P_{direct} \left[\frac{\beta_{W/2}(1-x)^\alpha}{\beta_W E_1} + \frac{\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha \cdot E_1} \frac{1 + \frac{2\beta_{W/2}(1-x)^\alpha}{E_1}}{E_2} x^\alpha \right] \tag{20}$$

The total power consumed by this schedule is give by:

$$P_{total} = P_{in} + 2P_{out} \geq P_{direct} \left[\left(\frac{1 + \frac{2\beta_{W/2}(1-x)^\alpha}{E_1}}{E_2} \right) \left(1 + \frac{2\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha \cdot E_1} \right) x^\alpha + \left(\frac{2\beta_{W/2}}{\beta_W \cdot E_1} \right) (1-x)^\alpha \right] \tag{21}$$

$$\iota = \left[\left(\frac{1 + \frac{2\beta_{W/2}(1-x)^\alpha}{E_1}}{E_2} \right) \left(1 + \frac{2\beta_{W/2} R^\alpha (1-x)^\alpha}{{d_1}^\alpha \cdot E_1} \right) x^\alpha + \left(\frac{2\beta_{W/2}}{\beta_W \cdot E_1} \right) (1-x)^\alpha \right] \tag{22}$$

3-Relay 2-Rate Schedules

Fig. 4 depicts yet another schedule wherein there are three simultaneous transmissions occurring at rate $W/3$ in the outer ring. We denote the power used by the transmitting relay as P_{in}, while the perimeter node communicating with relay C uses P_1 and nodes communicating with relays B and D use power P_2.

Using the cosine formula, we obtain expressions for distances d_1, d_2, d_3 and d_4 (see figure):

$$d_1 = 2xRcos\frac{\theta}{2} \tag{23}$$

$$d_2 = d_3 = R\sqrt{(x^2 - 2xcos\theta + 1)} \tag{24}$$

$$d_4 = R\sqrt{(x^2 - 2xcos(2\theta) + 1)} \tag{25}$$

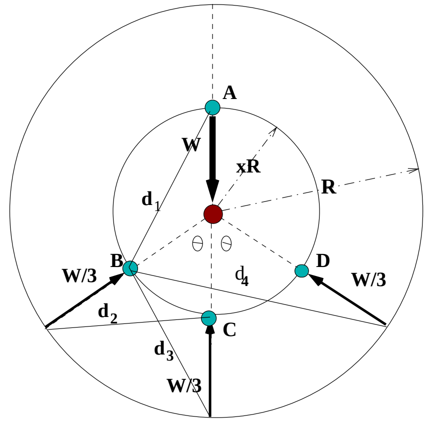

Fig. 4. 3-Relay 2-Rate Schedules

The SINR equations are:

At the BS:

$$\frac{\frac{CP_{in}}{x^\alpha R^\alpha}}{N_t + \frac{CP_1}{R^\alpha} + 2\frac{CP_2}{R^\alpha}} \geq \beta_W \tag{26}$$

$$P_{in} \geq \frac{\beta_W N_t R^\alpha}{C} x^\alpha + \beta_W x^\alpha P_1 + 2\beta_W x^\alpha P_2 \tag{27}$$

At the relay C:

$$\frac{\frac{CP_1}{(1-x)^\alpha R^\alpha}}{N_t + \frac{CP_{in}}{2x^\alpha R^\alpha} + \frac{2CP_2}{{d_2}^\alpha}} \geq \beta_{W/3} \tag{28}$$

$$P_1 \geq \frac{\beta_{W/3} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/3}(1-x)^\alpha}{2^\alpha x^\alpha} P_{in} + 2\frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{{d_2}^\alpha} P_2 \tag{29}$$

At relays B and D:

$$\frac{\frac{CP_2}{(1-x)^\alpha R^\alpha}}{N_t + \frac{CP_{in}}{{d_1}^\alpha} + \frac{CP_1}{{d_3}^\alpha} + \frac{CP_2}{{d_4}^\alpha}} \geq \beta_{W/3} \tag{30}$$

$$P_2 \geq \frac{\beta_{W/3} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/3}(1-x)^\alpha}{d_1^\alpha} P_{in} + \frac{\beta_{W/3}(1-x)^\alpha}{d_3^\alpha} P_1 + \frac{\beta_{W/3}(1-x)^\alpha}{d_4^\alpha} P_2 \tag{31}$$

$$P_2 \overbrace{\left[1 - \frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{d_4^\alpha}\right]}^{E_1} \geq \frac{\beta_{W/3} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{d_1^\alpha} P_{in} + \frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{d_3^\alpha} P_1$$

This yields the feasibility condition:

$$E_1 > 0 \tag{32}$$

Continuing:

$$P_2 \geq \frac{\beta_{W/3} N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha + \frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{d_1^\alpha \cdot E_1} P_{in} + \frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{{d_3}^\alpha \cdot E_1} P_1 \tag{33}$$

Substituting in Eqn. 29 :

$$P_1 \geq \frac{\beta_{W/3} N_t R^\alpha}{C}(1-x)^\alpha + \frac{\beta_{W/3}(1-x)^\alpha}{2^\alpha x^\alpha} P_{in} + 2\frac{\beta_{W/3}(1-x)^\alpha R^\alpha}{{d_2}^\alpha}\left[\frac{\beta_{W/3} N_t R^\alpha}{C \cdot E_1}(1-x)^\alpha\right] \tag{34}$$

$$+\frac{2{\beta_{W/3}}^2(1-x)^{2\alpha} R^{2\alpha}}{{d_1}^\alpha \cdot {d_2}^\alpha \cdot E_1} P_{in} + \frac{2{\beta_{W/3}}^2(1-x)^{2\alpha} R^{2\alpha}}{{d_2}^\alpha \cdot {d_3}^\alpha \cdot E_1} P_1 \tag{35}$$

$$P_1 \overbrace{\left[1 - \frac{2{\beta_{W/3}}^2 (1-x)^{2\alpha} R^{2\alpha}}{{d_2}^{\alpha} \cdot {d_3}^{\alpha} \cdot E_1}\right]}^{E_2} \geq \frac{\beta_{W/3} N_t R^{\alpha}}{C}(1-x)^{\alpha} + \frac{2{\beta_{W/3}}^2 N_t (1-x)^{2\alpha} R^{2\alpha}}{C \cdot {d_2}^{\alpha} \cdot E_1} \tag{36}$$

$$+ \overbrace{\left[\frac{\beta_{W/3}(1-x)^{\alpha}}{2^{\alpha} x^{\alpha}} + \frac{2{\beta_{W/3}}^2 (1-x)^{2\alpha} R^{2\alpha}}{{d_1}^{\alpha} \cdot {d_2}^{\alpha} \cdot E_1}\right]}^{C_1} P_{in} \tag{37}$$

This yields another feasibility condition:

$$E_2 > 0 \tag{38}$$

Proceeding further:

$$\begin{aligned}
P_{in} \geq \frac{\beta_W N_t R^{\alpha}}{C} x^{\alpha} + \beta_W x^{\alpha} P_1 + 2\beta_W x^{\alpha} \left[\frac{\beta_{W/3} N_t R^{\alpha}}{C \cdot E_1}(1-x)^{\alpha} + \frac{\beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_1}^{\alpha} \cdot E_1} P_{in} + \frac{\beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_3}^{\alpha} \cdot E_1} P_1\right] \\
= \frac{\beta_W N_t R^{\alpha}}{C} x^{\alpha} + \frac{2\beta_W x^{\alpha} \beta_{W/3} N_t R^{\alpha}}{C \cdot E_1}(1-x)^{\alpha} + \frac{2\beta_W x^{\alpha} \beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_1}^{\alpha} \cdot E_1} P_{in} \\
+ \overbrace{\left[\beta_W + 2\beta_W \frac{\beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_3}^{\alpha} \cdot E_1}\right] x^{\alpha}}^{C_2} P_1 \\
P_{in} \geq \frac{\beta_W N_t R^{\alpha}}{C} x^{\alpha} \left[1 + \frac{2\beta_{W/3}(1-x)^{\alpha}}{E_1}\right] + 2\beta_W x^{\alpha} \frac{\beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_1}^{\alpha} \cdot E_1} P_{in}
\end{aligned}$$

$$\begin{aligned}
+ C_2 \cdot \overbrace{\left[\frac{\beta_{W/3} N_t R^{\alpha}}{C \cdot E_2}(1-x)^{\alpha} + \frac{2\beta_{W/3}^2 N_t (1-x)^{2\alpha} R^{2\alpha}}{C \cdot {d_2}^{\alpha} \cdot E_1 \cdot E_2}\right]}^{C_3} + \frac{C_1 \cdot C_2}{E_2} P_{in} \\
P_{in} \overbrace{\left[1 - \frac{2\beta_W x^{\alpha} \beta_{W/3}(1-x)^{\alpha} R^{\alpha}}{{d_1}^{\alpha} \cdot E_1} - \frac{C_1 \cdot C_2}{E_2}\right]}^{E_3} \geq \frac{\beta_W N_t R^{\alpha}}{C} x^{\alpha} \left[1 + \frac{2\beta_{W/3}(1-x)^{\alpha}}{E_1}\right] + C_2 \cdot C_3
\end{aligned}$$

$$\begin{aligned}
P_{in} \geq \frac{P_{direct}}{E_3} x^{\alpha} \left[1 + \frac{2\beta_{W/3}(1-x)^{\alpha}}{E_1}\right] + \frac{C_2 \cdot C_3}{E_3} \\
= \frac{P_{direct}}{E_3} \left[x^{\alpha}\left(1 + \frac{2\beta_{W/3}(1-x)^{\alpha}}{E_1}\right) + C2 \cdot \left[\frac{\beta_{W/3}(1-x)^{\alpha}}{\beta_W \cdot E_2} + \frac{2\beta_{W/3}^2 R^{\alpha}(1-x)^{2\alpha}}{{d_2}^{\alpha} \cdot E_1 \cdot E_2}\right]\right]
\end{aligned}$$

We also obtain a third feasibility condition:

$$E_3 > 0 \tag{39}$$

Back-substitution yields values of P_1 and P_2, and then we can obtain P_{total} as:

$$P_{total} = P_{in} + P_1 + 2P_2 \tag{40}$$

We do not reproduce the entire expression here.

Some observations on $\lim_{W \to 0} x_{opt}(W)$

We define $x_{opt}(W)$ as the lowest value of x in which an optimal (minimum) power consumption is obtained while obtaining rate W. The stipulation of *lowest* value is required since we have no means of claiming that the optimal power consumption shall be achieved at a unique value of X.

We now observe that in all the three schedules discussed above, the following can be shown to hold:

$$\lim_{W \to 0} P_{total} \geq P_{direct}\left[x^{\alpha} + (1-x)^{\alpha}\right] \tag{41}$$

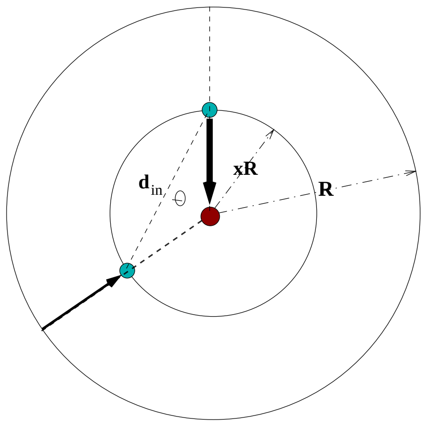

Fig. 5. Distance of Interfering Relay

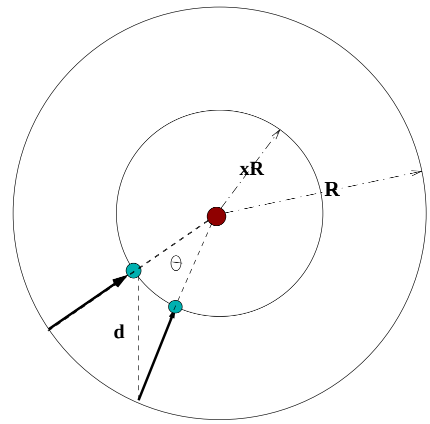

Fig. 6. Distance of Interfering Perimeter Transmitter

This is minimized at $x = 0.5$. Thus it proceeds that:

$$\lim_{W \to 0} x_{opt}(W) = 0.5 \tag{42}$$

where $x_{opt}(W)$ is the optimal radial relay placement position. This is intuitive, as at very low W, path loss becomes the dominant controllable factor (mutual interference is very low and plays a negligible role). Hence midway relays are the best option.

GENERALIZED n-RELAY 2-RATE SCHEDULES

We consider the general case in which one relay transmits to the BS at rate W while n perimeter nodes transmit to their respective relays concurrently ($n \geq 1$) at rate W/n. We do not impose any constraints on the transmission power or angular position of the concurrently transmitting nodes, other than the SINR requirement which is to be met. We obtain some lower bounds on feasible relay positions using an upper bound argument for achievable SINR.

Prior to that we present proofs for certain trends.

Claim 1: $x_1 < x_2$ and W feasible at $x_1 \longrightarrow W$ feasible at x_2

Proof: Suppose $x_1 < x_2$, W feasible at x_1. Now consider a configuration C with $x = x_1$ that achieves rate W. Such a C exists by definition. Then in this configuration C:

SINR at BS $\geq \beta_W$

SINR at each receiving relay $\geq \beta_{W/n}$

Let us denote the power used by the transmitting relay as P_{in} and the total power used by all perimeter nodes as P_{out}. Now consider configuration $C^{'}$ which is otherwise identical to C (in terms of position of transmitting nodes on perimeter) except that the relays are shifted from x_1 to $x_2 > x_1$. Now consider increasing P_{in} to $P_{in} \cdot \left(\frac{x_2}{x_1}\right)^{\alpha}$ while keeping all other transmit powers the same.

We observe here (Fig. 5) that the distance of the transmitting relay to any receiving relay (at angular position θ w.r.t. the transmitting relay) is given by:
$d_{in}(x) = 2xRsin(\theta/2)$ where $\theta =$ angle subtended by the sector they make on the circle of radius xR

We also observe (Fig. 6)that the distance between a receiving relay and any transmitting perimeter node is governed by the formula:

$d(x) = R\sqrt{(x^2 - 2xcos\theta + 1)}$ where $\theta =$ angle subtended between them at the center

Consider $D = d^2$. Then:

$$D(x) = R^2(x^2 - 2xcos\theta + 1)$$

$$\frac{D(x_2)}{D(x_1)} = \frac{{x_2}^2 - 2x_2cos(\theta) + 1}{{x_1}^2 - 2x_1cos(\theta) + 1} = \frac{(1-x_2)^2 + 2x_2(1-cos(\theta))}{(1-x_1)^2 + 2x_1(1-cos(\theta))}$$

$$x_2 > x_1 \longrightarrow x_2 > \left(\frac{1-x_2}{1-x_1}\right)^2 x_1$$

$$\longrightarrow \frac{D(x_2)}{D(x_1)} = \frac{(1-x_2)^2 + 2x_2(1-cos(\theta))}{(1-x_1)^2 + 2x_1(1-cos(\theta))}$$

$$> \frac{(1-x_2)^2 + 2x_1\left(\frac{1-x_2}{1-x_1}\right)^2(1-cos(\theta))}{(1-x_1)^2 + 2x_1(1-cos(\theta))}$$

$$= \frac{(1-x_2)^2(1-x_1)^2 + 2x_1(1-x_2)^2(1-cos(\theta))}{(1-x_1)^2(1-x_1)^2 + 2x_1(1-x_1)^2(1-cos(\theta))}$$

$$= \frac{(1-x_2)^2\left((1-x_1)^2 + 2x_1(1-cos(\theta))\right)}{(1-x_1)^2\left((1-x_1)^2 + 2x_1(1-cos(\theta))\right)}$$

$$= \frac{(1-x_2)^2}{(1-x_1)^2}$$

$$\longrightarrow \frac{d(x_2)}{d(x_1)} > \frac{1-x_2}{1-x_1}$$

Hence the maximum decrease in interferer distance possible is by a multiplicative factor of $\frac{1-x_2}{1-x_1}$. It follows that in the configurations under consideration:
SINR at BS:

$SINR_{old} = \frac{\frac{CP_{in}}{{x_1}^\alpha R^\alpha}}{N_t + \frac{CP_{out}}{R^\alpha}}$
$SINR_{new} = SINR_{old} * \left(\frac{x_2}{x_1}\right)^\alpha * \left(\frac{x_1}{x_2}\right)^\alpha = SINR_{old} = \beta_W$

SINR at each receiving relay:

$SINR_{old} = \frac{Signal_{old}}{N_t + I_{in} + I_{perimeter}}$
where I_{in} denotes interference posed by the transmitting relay, and $I_{perimeter}$ denotes interference posed by the transmitting perimeter nodes.

$SINR_{new} \geq \frac{Signal_{old} * \left(\frac{1-x_1}{1-x_2}\right)^\alpha}{N_t + \left(I_{in} * \left(\frac{x_2}{x_1}\right)^\alpha \left(\frac{x_1}{x_2}\right)^\alpha\right) + \left(I_{perimeter} * \left(\frac{1-x_1}{1-x_2}\right)^\alpha\right)} \geq \frac{Signal_{old} * \left(\frac{1-x_1}{1-x_2}\right)^\alpha}{(N_t + I_{in} + I_{perimeter}) * \left(\frac{1-x_1}{1-x_2}\right)^\alpha} \geq SINR_{old}$
Intuitively, the distance of the interfering relay increases by the same factor as its transmit power increases ($\left(\frac{x_2}{x_1}\right)^\alpha$),

while the interference from other transmitting perimeter nodes increases by less than a factor of $\left(\frac{1-x_1}{1-x_2}\right)^{\alpha}$ (in fact, it might even decrease for some nodes) and this increase gets more than compensated by the increase in signal by a factor of $\left(\frac{1-x_1}{1-x_2}\right)$ due to decrease in source-destination distance.

Thus W is also feasible at $x = x_2$. The claim thus stands proven. ∎

Claim 2: Let $x_{min}(W)$ = minimum value of x at which an aggregate rate of W is feasible. Then:

$$W_1 < W_2 \longrightarrow x_{min}(W_1) \leq x_{min}(W_2)$$

Proof:

$$\text{Consider } W_1, W_2 \text{ s.t. } W_1 < W_2 \text{ and } x_{min}(W_2) = x_2$$
$$W_1 < W_2 \rightarrow \beta_{W1} < \beta_{W2}$$

Now consider a configuration C that achieves rate W_2 at $x = x_{min}(W_2)$. Such a C exists by definition. Then in this configuration C:

SINR at BS $\geq \beta_{W_2}$

SINR at each receiving relay $\geq \beta_{W_2/n}$

$\rightarrow$ SINR at BS $\geq \beta_{W_1}$

SINR at each receiving relay $\geq \beta_{W_1/n}$

$\rightarrow$ configuration C can also support aggregate rate W_1

$\rightarrow x_{min}(W_1) \leq x_{min}(W_2)$

Therefore, the minimum feasible x is a monotonic non-decreasing function of W. ∎

Claim 3: $\lim_{W \longrightarrow \infty} x_{min}(W) \longrightarrow 1$

Proof: To prove this claim, we formulate some upper bound equations on SINR in the general case.

Let the power consumed by the relay transmitting to the BS be P_{in}. Let the total power consumed by the transmitting perimeter nodes be P_{out}. Then $\exists$ at least one perimeter node (call it A) such that A transmits at a power $P_A \leq \frac{P_{out}}{n}$. Besides, we shall continue to enforce the minimum separation criterion introduced earlier.

The SINR criterion at the BS:

$$\frac{\frac{CP_{in}}{x^{\alpha}R^{\alpha}}}{N_t + \frac{CP_{out}}{R^{\alpha}}} \geq \beta_W$$

$$P_{in} \geq \frac{\beta_W N_t R^{\alpha}}{C} x^{\alpha} + \beta_W x^{\alpha} P_{out} \tag{43}$$

We formulate a similar equation for the destination relay of the perimeter node A which is transmitting at $P_A \leq \frac{P_{out}}{n}$:

$$\frac{\frac{CP_A}{(1-x)^{\alpha}R^{\alpha}}}{N_t + \frac{CP_{in}}{d_{in}{}^{\alpha}} + \sum_{i=1}^{n-1} \frac{CP_i}{d_i{}^{\alpha}}} \geq \beta_{W/n}$$

where d_{in} = distance between A's destination and the transmitting relay, and d_i is the distance between A's destination and the ith transmitting perimeter node. We observe that $d_{in} \leq 2xR$ and $\sum_{i=1}^{n-1} \frac{CP_i}{d_i{}^{\alpha}} \geq 0$. A better bound may be obtained by observing that $d_i \leq 2R, \forall i$. However, as of now, we shall keep matters simple by using the former. Therefore:

$$\frac{\frac{CP_{out}}{n(1-x)^{\alpha}R^{\alpha}}}{N_t + \frac{CP_{in}}{2^{\alpha}x^{\alpha}R^{\alpha}}} \geq \frac{\frac{CP_A}{(1-x)^{\alpha}R^{\alpha}}}{N_t + \frac{CP_{in}}{d_{in}{}^{\alpha}} + \sum_{i=1}^{n-1}\frac{CP_i}{d_i{}^{\alpha}}} \geq \beta_{W/n}$$

$$P_{out} \geq \frac{(n\beta_{W/n})N_t R^{\alpha}}{C}(1-x)^{\alpha} + \frac{(n\beta_{W/n})(1-x)^{\alpha}}{2^{\alpha}x^{\alpha}}P_{in} \tag{44}$$

Substituting in Eqn. 43 we obtain:

$$P_{in}\left[1 - \frac{\beta_W(n\beta_{W/n})(1-x)^{\alpha}}{2^{\alpha}}\right] \geq \frac{\beta_W N_t R^{\alpha}}{C}x^{\alpha}\left[1 + (n\beta_{W/n})(1-x)^{\alpha}\right]$$

This yields the following feasibility condition:

$$1 - \frac{\beta_W(n\beta_{W/n})(1-x)^{\alpha}}{2^{\alpha}} > 0 \tag{45}$$

and may be re-stated as :

$$x > 1 - \frac{2}{\beta_W{}^{\frac{1}{\alpha}}(n\beta_{W/n})^{\frac{1}{\alpha}}} \tag{46}$$

it proceeds that $x_{min}(W)$ is bound by the above feasibility condition where the R.H.S. of the above equation is an increasing function of W. We shall use the above condition to get a *lenient* estimate of the feasible relay placement region for a given W. We observe that:

$$\lim_{W\to\infty} 1 - \frac{2}{\beta_W{}^{\frac{1}{\alpha}}(n\beta_{W/n})^{\frac{1}{\alpha}}} = 1$$

Thus, it proceeds that:

$$\lim_{W\to\infty} x_{min}(W) = 1 \tag{47}$$

We explicitly note here that $x_{min}(W)$ shall take the value 1 much earlier than indicated by the above limit since by our minimum separation criterion, once $(1-x)R < d_{min}$, x lapses to 1. ■

Some observations on $\lim_{n\to\infty} x_{min}(W)$

We observe that $n\beta_{W/n}$ is a decreasing function of n for $n \geq 1$. Besides:

$$\lim_{n\to\infty} n\beta_{W/n} = \ln(1+\beta_W) \tag{48}$$

Thus we can conclude that for a given W, the derived feasibility condition becomes less strict as n increases. However, when $n \to \infty$:

$$x \geq 1 - \frac{2}{\beta_W{}^{\frac{1}{\alpha}}[\ln(1+\beta_W)]^{\frac{1}{\alpha}}} \tag{49}$$

The existence of this limit indicates an intrinsic limitation on feasible relay placement for a given W i.e. the size of the feasible *x-region* is upper-bounded by the above limit, regardless of the value of n and the schedule employed.

A Loose Bound on Power Consumption

By solving the above presented upper-bound formulation, we can get a loose lower bound on total power consumed in the generalized n-relay 2-rate schedules. This bound is a decreasing function of n and hence attains its lower bound when n tends to infinity. Thus the limiting value represents the lowest (and unachievable) bound on total power consumption over all n. The expression obtained is:

$$\lim_{n\to\infty}\frac{P_{total}}{P_{direct}} = \left(x^{\alpha} + \frac{\ln(1+\beta_W)}{2^{\alpha}}(1-x)^{\alpha}\right)\left(\frac{1 + \ln(1+\beta_W)(1-x)^{\alpha}}{1 - \frac{\beta_W\ln(1+\beta_W)(1-x)^{\alpha}}{2^{\alpha}}}\right) + \frac{\ln(1+\beta_W)}{\beta_W}(1-x)^{\alpha} \tag{50}$$

A somewhat tighter bound would be possible using the earlier made observation on mutual interference of perimeter nodes.

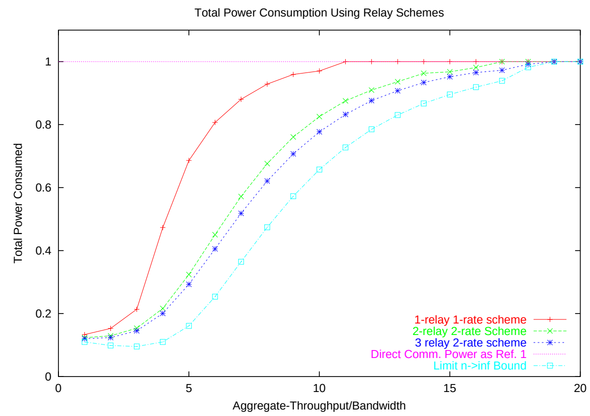

Fig. 7. Minimum Power Consumption for Various Schedules

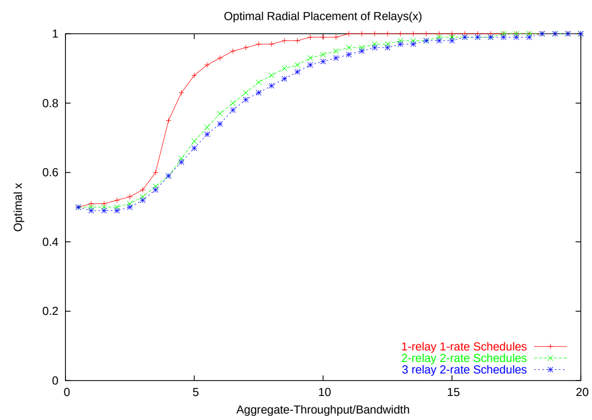

Fig. 8. Optimal Radial Position of Relays for Various Schedules

Discussion of Provable Trends vis-a-vis Numerical Plots

While the equations presented earlier do not lend themselves to simple closed-form solutions, we have obtained some numerical plots to determine trends in power consumption and optimal relay placement. When we use the term *optimal* in this context, we mean the optimal as determined by a search over the (x, θ) spatial configuration space for a given class of schedules. The value of the path-loss exponent α has been set to 4 in these plots. Lower values of α typically yield lower improvement in power consumption due to reduction in possible spatial re-use. Fig. 7 depicts the optimal power consumption for various aggregate rates obtained using the various schedule classes under consideration. We observe that as W increases, the minimum required power tends towards P_{direct}. This is to be expected, since at higher W, the feasible relay placement region shrinks towards the periphery and in the limit the relays and perimeter nodes are co-located, corresponding to direct communication. The noteworthy fact is that there is significant improvement to be achieved using 2-hop communication for a certain range of W. It is to be noted that the improvement obtained by increasing the number n of simultaneous transmissions in the outer ring diminishes rapidly. In fact most of the improvement is obtained within $n = 2, 3$ as compared to the lower bound on minimum power when n increases to infinity. This seems to indicate that it might be useful to design protocols that perform limited rate control (i.e. use a small value of n) to get the major part of the advantage in terms of energy-efficiency. Fig. 8 depicts the variation in optimal radial placement of relays for various aggregate rates. There is a clear trend indicating that optimal radial placement is a monotonically increasing function of W. In the absence of an analytical proof, we currently state this as a conjecture:

Conjecture 1: Let $x_{opt}(W)$ = minimum value of x at which an optimal power consumption for aggregate rate of W is achieved. Then:

$$W_1 < W_2 \longrightarrow x_{opt}(W_1) \leq x_{opt}(W_2)$$

We also plot the special case of midway relays i.e. $x = 0.5$ in Fig. 9. As may be seen, the power consumption with midway relays increases steeply with increase in aggregate rate and even exceeds that of direct communication at some point. In fact, very high rates are not feasible/achievable with this placement.

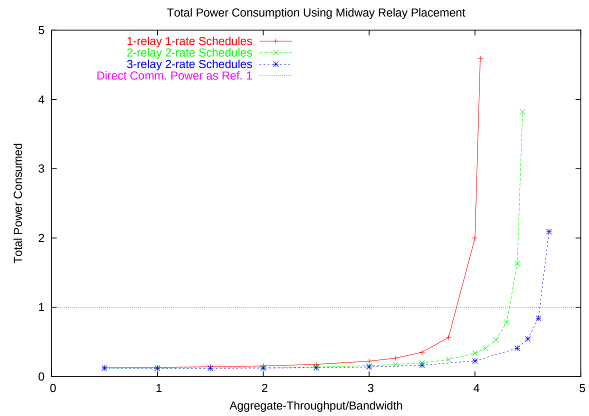

Fig. 9. Minimum Power Consumption with Midway Relay Placement (x = 0.5)

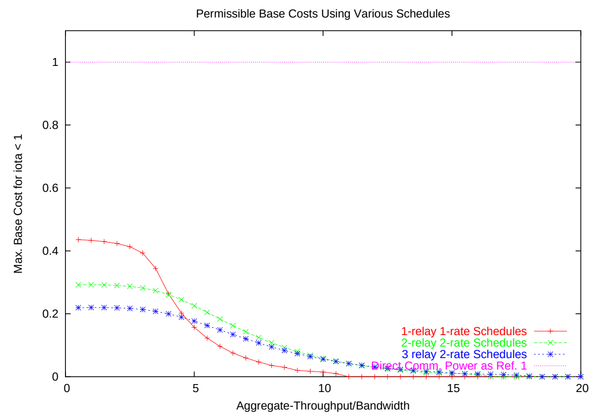

Fig. 10. Max. Permissible Base Cost ($P_{send} + P_{rcv}$) for Various Schedules

A Note on the Impact of Base Communication Costs

The immediate applicability of this analysis to the design of a protocol is dependent on how the base (rate-independent) costs of sending and receiving data compare with the power of the transmitted signal (rate-dependent). We present a simple formulation.

We assume that while transmitting, a certain fixed power P_{send} is always consumed. Similarly, while receiving a fixed power P_{rcv} is consumed. Then for an n-relay 2-rate schedule:

$$P_{actual} = P_{total} + (1+n)(P_{send} + P_{rcv}) \text{ where } P_{total} \text{ is as used elsewhere} \tag{51}$$

Therefore, for a schedule to yield improvement over direct communication:

$$P_{send} + P_{rcv} < P_{direct}\left(\frac{1-\iota}{1+n}\right) \tag{52}$$

Since the R.H.S. is a decreasing function of n, this indicates that increasing n would make the constraint on the base costs increasingly stringent, and accentuates the earlier observation that a very large n may not be of practical value.

Fig. 10 is a numerical plot that indicates the maximum value $\left(\frac{1-\iota}{1+n}\right)$ for $n = 1, 2$ and 3 over all spatial configurations belonging to these respective classes. As is to be expected, the practical utility of any protocol based on the proposed mechanism is dependent on the magnitude of P_{direct} which depends on the size of the network. For large values of R (network radius), P_{direct} is expected to be large, and hence the condition in Eqn. 52 is more likely to be satisfied. For small values of R, direct communication might often prove to be the better option, as the benefits of reduced propagation distance are far outweighed by the increased number of senders and receivers who also consume a a certain base power (apart from the actual transmission power).

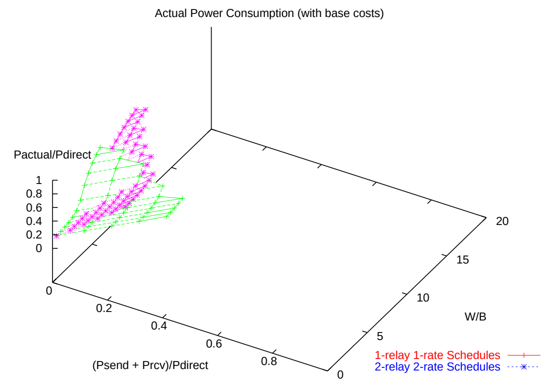

Fig. 11. Actual Power Consumption (with base costs) for Various Schedules

We also note that for low rates, the constraints on the base costs seem to be significantly less stringent for the 1-relay 1-rate schedules. However at somewhat higher rates, the maximum permissible base cost value dips sharply for this schedule class, and $n = 2$ seems to be a good practical choice. To further investigate this, we plot P_{actual} as in Eqn. 51 for the 1-relay 1-rate and 2-relay 2-rate schedules, while varying the base costs and W (Fig. 11). As may be seen, for very low rates the 1-relay 1-rate schedules perform better, but at somewhat higher rates, 2-relay 2-rate schedules can possibly yield better power consumption.

Future Work

As part of future work we intend to attempt obtaining analytical proofs of trends that are evident from the numerical plots but have not been proved as yet, and also possibly obtain some interesting bounds for the general k-hop case. As a culmination of the analysis, we intend to utilize the insights obtained from it to design a suitable rate-and-power control protocol for energy-efficient many-to-one communication.

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