Capacity of Multi-Channel Wireless Networks: Impact of Number of Channels and Interfaces

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ABSTRACT

This paper studies how the capacity of a static multi-channel network scales as the number of nodes, n, increases. Gupta and Kumar have determined the capacity of single-channel networks, and those bounds are applicable to multi-channel networks as well, provided each node in the network has a dedicated interface per channel.

In this work, we establish the capacity of general multi-channel networks wherein the number of interfaces, m, may be smaller than the number of channels, c. We show that the capacity of multi-channel networks exhibits different bounds that are dependent on the ratio between c and m. When the number of interfaces per node is smaller than the number of channels, there is a degradation in the network capacity in many scenarios. However, one important exception is a random network with up to $O(\log n)$ channels, wherein the network capacity remains at the Gupta and Kumar bound of $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec, independent of the number of interfaces available at each node. Since in many practical networks, number of channels available is small (e.g., IEEE 802.11 networks), this bound is of practical interest. This implies that it may be possible to build capacity-optimal multi-channel networks with as few as one interface per node. We also extend our model to consider the impact of interface switching delay, and show that in a random network with up to $O(\log n)$ channels, switching delay may not affect capacity if multiple interfaces are used.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*

General Terms

Theory, Performance

MobiCom'05, August 28–September 2, 2005, Cologne, Germany.

Keywords

Capacity, Multiple Channels, Multiple Interfaces, Ad hoc networks, Mesh networks

1. INTRODUCTION

Previous research (e.g., [9, 10]) has characterized the capacity of wireless networks. One approach for enhancing the network capacity is to use multiple channels. Past research on wireless network capacity has typically considered wireless networks with a single channel, although the results are applicable to a wireless network with multiple channels as well, provided that at each node there is a dedicated interface per channel. With a dedicated interface per channel, a node can use all the available channels simultaneously. However, the number of available channels in a wireless network can be fairly large (e.g., IEEE 802.11a [11] has provisioned for up to 12 non-overlapping channels), and it may not be feasible to have a dedicated interface per channel at each node. When nodes are not equipped with a dedicated interface per channel, then *capacity degradation* may occur, compared to using a dedicated interface per channel. In this paper, we characterize the impact of number of channels and interfaces per node on the network capacity, and show that in a random network with up to $O(\log n)$ channels, *even with a single interface per node, there is no capacity degradation.* This implies that it may be possible to build capacity-optimal multi-channel networks with as few as one interface per node.

When a dedicated interface per channel is not available, the available interfaces can potentially be switched among different channels to use any of the available channels. Such an *interface switching technique* is often used to improve channel utilization [15, 22, 23]. However, interface switching incurs a delay, which may reduce the achievable network capacity. In this paper, we include a preliminary study of the impact of interface switching delay on network capacity. We show that in a random network with up to $O(\log n)$ channels, interface switching delay has no impact on network capacity, even when there are end-to-end delay constraints, provided that a few additional interfaces are provisioned for at each node.

1.1 Modeling multi-channel multi-interface networks

We consider a static wireless network containing n nodes. We use the term "channel" to refer to a part of the frequency spectrum with some specified bandwidth. There are c channels, and we assume that every node is equipped with m interfaces, $1 \leq m \leq c$. We assume that an interface is capable of transmitting or receiving data on any one channel at a given time. We use the notation (m, c)-network to refer to a network with m interfaces per node, and c channels.

We define two channel models to represent the data rate supported by each channel:

Channel Model 1: In model 1, we assume that the total data rate possible by using all channels is W. The total data rate is divided equally among the channels, and therefore the data rate supported by any one of the c channels is W/c. This was the channel model used by Gupta and Kumar [10], and we primarily use this model in our analysis. In this model, as the number of channels increases, each channel supports a smaller data rate. This model is applicable to the scenario where the total available bandwidth is fixed, and new channels are created by partitioning existing channels.

Channel Model 2: In model 2, we assume that each channel can support a fixed data rate of W, independent of the number of channels. Therefore, the aggregate data rate possible by using all c channels is Wc. This model is applicable to the scenario where new channels are created by utilizing additional frequency spectrum.

The results presented in this paper are derived assuming channel model 1. However, all the derivations are applicable for channel model 2 as well, and the results for model 2 can be obtained by replacing W in the results of model 1 by Wc [14]. In the rest of this paper, we will only present the results for channel model 1, but discuss implications of the results with channel model 2 where appropriate.

1.2 Definitions

We study the capacity of static multi-channel wireless networks under the two settings introduced by Gupta and Kumar [10].

Arbitrary Networks: In the arbitrary network setting, the location of nodes, and traffic patterns can be controlled. Since any suitable traffic pattern and node placement can be used, the bounds for this scenario are applicable to any network. Therefore, the arbitrary network bounds may be viewed as the *best case* bounds on network capacity, as the bounds are applicable to all networks. The network capacity is measured in terms of "bit-meters/sec" (originally introduced by Gupta and Kumar [10]). The network is said to transport one "bit-meter/sec" when one bit has been transported across a distance of one meter in one second.

Random Networks: In the random network setting, node locations are randomly chosen, i.e. independently and uniformly chosen, on the surface of an unit torus. Each node sets up one flow to a randomly chosen destination. The network capacity is defined to be the aggregate throughput over all the flows in the network, and is measured in terms of bits/sec.

We use the following notation to represent asymptotic bounds:

1. $f(n) = O(g(n))$ implies there exists some constant d and integer N such that $f(n) \leq dg(n)$ for $n > N$.
2. $f(n) = o(g(n))$ implies that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.
3. $f(n) = \Omega(g(n))$ implies $g(n) = O(f(n))$.
4. $f(n) = \omega(g(n))$ implies $g(n) = o(f(n))$.
5. $f(n) = \Theta(g(n))$ implies $f(n) = O(g(n))$ and $g(n) = O(f(n))$.
6. $MIN_O\,(f(n), g(n))$ is equal to $f(n)$, if $f(n) = O(g(n))$, else, is equal to $g(n)$.

The bounds for random networks hold *with high probability (whp)*. In this paper, *whp* implies with "probability 1 when $n \to \infty$."

1.3 Main Results

Gupta and Kumar [10] have shown that in an arbitrary network, network capacity scales as $\Theta(W\sqrt{n})$ bit-meters/sec, and in a random network, the network capacity scales as $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec. Under the channel model 1, which was the model used by Gupta and Kumar [10], the capacity of a network with a single channel and one interface per node (that is, a $(1, 1)$-network in our notation) is equal to the capacity of a network with c channels and $m = c$ interfaces per node (that is, a (c, c)-network). This equivalence arises because the c interfaces can operate in parallel over channels of data rate $\frac{W}{c}$ to mimic the operation of one interface operating over a channel of data rate W (this is formally proved in Lemma 1). Furthermore, under both channel models, the capacity of a (c, c)-network is at least as large as the capacity of a (m, c)-network, when $m \leq c$ (this is trivially true, by not using $c - m$ interfaces in the (c, c)-network). In the results presented in this paper, we capture the impact of using fewer than c interfaces per node by establishing the *loss in capacity*, if any, of a (m, c)-network in comparison to a (c, c)-network.

The goal of this work is to study the impact of the number of channels c, and the number of interfaces per node m, on the capacity of arbitrary and random networks. Our results show that the capacity is dependent on the ratio $\frac{c}{m}$, and not on the exact values of either c or m (as proven in Lemma 2). We now state our main results under channel model 1.

1. Results for arbitrary network: The network capacity of a (m, c)-network has two regions (see Figure 1) as follows (from Theorem 1 and Theorem 2):

1. When $\frac{c}{m}$ is $O(n)$, the network capacity is $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec (segment A-B in Figure 1). Compared to a (c, c)-network, there is a capacity loss by a factor of $1 - \sqrt{\frac{m}{c}}$.
2. When $\frac{c}{m}$ is $\Omega(n)$, the network capacity is $\Theta\left(W\frac{nm}{c}\right)$ bit-meters/sec (line B-C in Figure 1). In this case, there is a larger capacity degradation than case 1, as $\frac{nm}{c} \leq \sqrt{\frac{nm}{c}}$ when $\frac{c}{m} \geq n$.

Therefore, there is always a capacity loss in arbitrary networks whenever the number of interfaces per node is fewer than the number of channels.

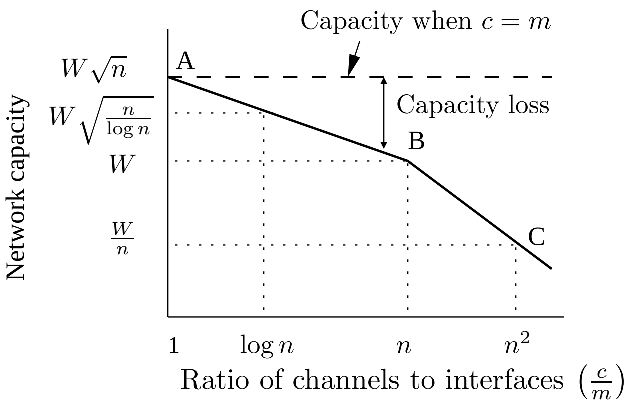

Figure 1: Impact of number of channels on capacity scaling in arbitrary networks (figure is not to scale). There is a degradation in capacity when the ratio of channels to interfaces is $\omega(1)$.

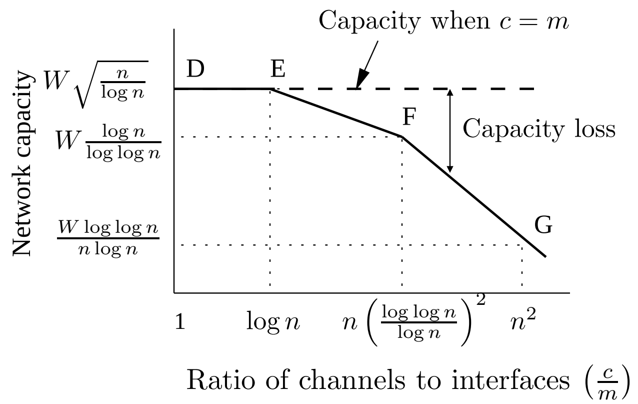

Figure 2: Impact of number of channels on capacity scaling in random networks (figure is not to scale). There is no degradation in capacity when the ratio of channels to interfaces is $O(\log n)$.

2. Results for random network: The network capacity of a (m, c)-network has three regions (see Figure 2) as follows (from Theorem 3 and Theorem 4):

1. When $\frac{c}{m}$ is $O(\log n)$, network capacity is $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec (segment D-E in Figure 2). In this case, *there is no loss* compared to a (c, c)-network. Hence, in many practical scenarios where c may be constant or small, *a single interface per node suffices.*

2. When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec (segment E-F in Figure 2). In this case, there is some capacity loss. Furthermore, in this region, the capacity of a (m, c)-random network *is the same* as that of a (m, c)-arbitrary network (segment E-F in Figure 2 overlaps part of segment A-B in Figure 1), implying "randomness" does not incur a capacity penalty.

3. When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $\Theta\left(\frac{Wnm\log\log n}{c\log n}\right)$ bits/sec (line F-G in Figure 2). In this case, there is a larger capacity degradation than case 2. Furthermore, in this region, the capacity of a (m, c)-random network *is smaller than* that of a (m, c)-arbitrary network, in contrast to case 2.

3. Other results: The results presented above are derived under the assumption that there is no delay in switching an interface from one channel to another. However, we show that in a random network with up to $O(\log n)$ channels, even if interface switching delay is considered, the network capacity is not reduced, provided a few additional interfaces are provisioned for at each node. This implies that it may be possible to hide the interface switching delay by using extra interfaces in conjunction with carefully designed routing and transmission scheduling protocols.

The rest of the paper is organized as follows. We present related work in Section 2. In Section 3, we establish the capacity of multi-channel networks under arbitrary network setting. Section 4 establishes the capacity of multi-channel networks under random network setting. Section 5 characterizes the impact of interface switching delay. Section 6 discusses the practical implications of the theoretical results. We conclude in Section 7.

2. RELATED WORK

In their seminal work, Gupta and Kumar [10] derived the capacity of ad hoc wireless networks. The results are applicable to single channel wireless networks, or multi-channel wireless networks where every node has a dedicated interface per channel. We extend the results of Gupta and Kumar to those multi-channel wireless networks where nodes may not have a dedicated interface per channel, and also consider the impact of interface switching delay on network capacity.

Grossglauser and Tse [9] showed that mobility can improve network capacity, though at the cost of increased end-to-end delay. Subsequently, other research [3, 20] has analyzed the trade-off between delay and capacity in mobile networks. Gamal et al. [7] characterize the optimal throughput-delay trade-off for both static and mobile networks. In this paper, we adapt some of the proof techniques presented by Gamal et al. [7] to the multi-channel capacity problem.

Recent results have shown that the capacity of wireless networks can be enhanced by introducing infrastructure support [1, 12, 17]. Other approaches for improving network capacity include the use of directional antennas [31], and the use of unlimited bandwidth resources (UWB) albeit with power constraints [19, 32].

Li et al. [16] have used simulations to evaluate the capacity of multi-channel networks based on IEEE 802.11. Other research on capacity is based on considerations of alternate communication models [8, 26, 27].

Several researchers have proposed wireless protocols for multi-channel networks (cf. [2, 6, 15, 18, 22, 23]). Some solutions are based on using a single interface at each node [2, 23–25], while other solutions require a dedicated interface for each channel [6, 18]. More recently, solutions have been proposed that require multiple interfaces, but fewer interfaces than the number of channels [13, 15, 22]. Although, there are several proposals for multi-channel networks, it is not apparent in those proposals how many interfaces are actually required to maximally utilize the available channels.

3. CAPACITY RESULTS FOR ARBITRARY NETWORKS

We model the impact of interference by using the protocol model proposed by Gupta and Kumar [10]. The transmission from a node i to a node j on some channel x is successful, if for every other node k simultaneously transmitting on channel x, the following condition holds

$$d(k,j) \geq (1+\Delta)d(i,j),\ \Delta > 0$$

where $d(i,j)$ is the distance between nodes i and j, and Δ is a "guard" parameter that ensures that concurrently transmitting nodes are sufficiently farther away from the receiver to prevent excessive interference.

It is shown in [10] that the protocol model is equivalent to an alternate physical model that is based on received Signal-to-Interference-Noise Ratio (SINR) (when path loss exponent is greater than 2). Therefore, the results in this paper are applicable under the physical model as well. We do not consider other physical layer characteristics such as channel fading in our analysis.

We derive the capacity results for arbitrary and random networks under the assumption that there is *no switching delay*. We extend our model to consider the impact of switching delay in Section 5.

In an arbitrary network, the location of nodes, and traffic patterns can be controlled. Recall that the network is said to transport one "bit-meter/sec" when one bit has been transported across a distance of one meter in a second. The network capacity of an arbitrary network is measured in terms of bit-meters per second, instead of bits per second. The bit-meters/sec metric is a measure of the "work" that is done by the network in transporting bits. In the case of random networks, the average distance traveled by any bit is $\Theta(1)$, and therefore the "bit-meters/sec" and "bits/sec" capacity is of the same order.

We assume that n nodes can be located anywhere on the surface of a torus of unit area, as in [7]. The assumption of a torus enables us to avoid technicalities arising out of edge effects, but the results are applicable for nodes located on an unit square as well. We first establish an upper bound on the network capacity of arbitrary networks, and then construct a network to prove that the bound is tight.

3.1 Upper bound on capacity

The capacity of multi-channel arbitrary networks is limited by two constraints (described below), and each of them is used to obtain a bound on the network capacity. The minimum of the two bounds (the bounds depend on ratio between the number of channels c and the number of interfaces m) is an upper bound on the network capacity. While there may be other constraints on capacity as well, the constraints we consider are sufficient to provide a tight bound. Later in this section, we will present a lower bound that matches the upper bound established by the two constraints, which validates our claim that the constraints are tight. We derive the bounds under channel model 1, although the derivation can be applied to channel model 2 as well[1].

Constraint 1 – Interference constraint: The capacity of any wireless network is constrained by interference. Since the wireless channel is a shared medium, under the assumed protocol model of interference, two nodes simultaneously receiving a packet from two different transmitters must have a minimum separation between them, which depends on Δ. This implies that there is a bound on the maximum number of simultaneous transmissions in the network. Based on this observation, using the proof techniques presented in [10] with some modifications to account for multiple interfaces and channels, one bound on the network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec. The detailed derivation is in Appendix A.

Constraint 2 – Interface bottleneck constraint: The capacity of a wireless network is also constrained by the maximum number of bits that can be transmitted simultaneously over all interfaces in the network. Since each node has m interfaces, there are a total of mn interfaces in the (m,c)-network. Each interface can transmit at a rate of $\frac{W}{c}$ bits/sec. Also, the maximum distance a bit can travel in the network is $O(1)$ meters. Hence, the total network capacity is at most $O\left(W\frac{nm}{c}\right)$ bit-meters/sec. This bound is tight when $\frac{c}{m}$ is $\Omega(n)$.

Combining the two constraints, the network capacity is $O\left(MIN_O\left(W\sqrt{\frac{nm}{c}}, W\frac{nm}{c}\right)\right)$ bit-meters/sec, under channel model 1. Therefore, we have the following theorem on the network capacity of arbitrary networks (Figure 1 has a pictorial representation).

THEOREM 1. *The upper bound on the capacity of a (m,c)-arbitrary network under channel model 1 is as follows:*

1. *When $\frac{c}{m}$ is $O(n)$, network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec.*
2. *When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $O\left(W\frac{nm}{c}\right)$ bit-meters/sec.*

The network capacity of a (c,c)-network is $O\left(W\sqrt{n}\right)$ bit-meters/sec under channel model 1, which was the result obtained by Gupta and Kumar [10]. When fewer interfaces are available, there is a capacity degradation by at least a factor of $1-\sqrt{\frac{m}{c}}$. Intuitively, the capacity degradation arises because the total bits that can be simultaneously transmitted decreases.

3.2 Constructive lower bound

In this section, we construct a network to establish a lower bound on the network capacity. The lower bound matches the upper bound, implying that the bounds are tight. We first establish two results that we use in the rest of the paper. The results are proved under channel model 1, but hold for channel model 2 as well.

LEMMA 1. *Suppose m, c, $\tilde{c}$ are positive integers such that $\tilde{c} = \frac{c}{m}$. Then, a (m,c)-network can support at least the capacity supported by a $(1,\tilde{c})$-network.*

[1]Recall that the results under channel model 2 can be obtained by replacing W with Wc in the results derived under channel model 1.

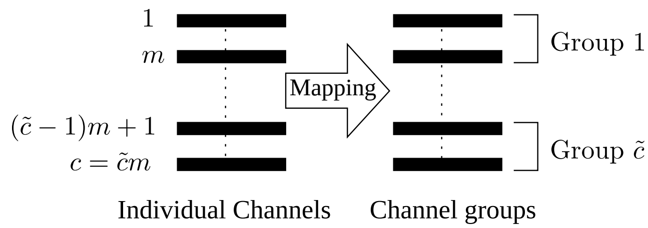

Figure 3: Lemma 1 construction: Forming $\tilde{c}$ channel groups, with m channels per group, in a (m,c)-network.

PROOF. Consider a (m,c)-network. We group the c channels into $\tilde{c}$ groups (numbered from 1 to $\tilde{c}$), with m channels per group as shown in Figure 3. Specifically, channel group i, $1 \le i \le \tilde{c}$, contains all channels j such that $(i-1)m+1 \le j \le im$.

Assume that time on the channels is divided into slots of duration τ. Consider any slot s. Suppose a node X in the $(1,\tilde{c})$-network has its interface on some channel i, $1 \le i \le \tilde{c}$, in slot s. We simulate this behavior in the (m,c)-network by assigning the m interfaces of X in the slot s to the m channels in the channel group i. In this fashion, in any slot, the m interfaces of any node in the (m,c)-network are mapped to a channel group. The aggregate data rate of each channel group is $Wm/c = W/\tilde{c}$ (since $c = m\tilde{c}$). Therefore, a channel group in the (m,c)-network can support the same data rate as a channel in the $(1,\tilde{c})$-network. This mapping allows the (m,c)-network to mimic the behavior of $(1,\tilde{c})$-network; the $W\tau/\tilde{c}$ bits sent on some channel in any time slot s in the $(1,\tilde{c})$-network can be simulated by sending $W\tau/c$ bits (in the same slot s) on each of the m channels in the corresponding channel group of the (m,c)-network. Hence, a (m,c)-network can support the capacity of a $(1,\tilde{c})$ network, when $c = m\tilde{c}$. □

LEMMA 2. *Suppose m and c are positive integers. Then, a (m,c)-network can support at least $\frac{1}{2}$ the capacity supported by a $\left(1, \left\lfloor \frac{c}{m} \right\rfloor\right)$-network.*

PROOF. Suppose $\left\lfloor \frac{c}{m} \right\rfloor = \frac{c}{m}$. Then the result directly follows from the previous lemma. Otherwise, $m < c$, and we use $c' = m \left\lfloor \frac{c}{m} \right\rfloor$ of the channels in the (m,c)-network, and ignore the rest of the channels. This can be viewed as a (m,c')-network, with a total data rate of $W' = W\frac{m}{c}\left\lfloor \frac{c}{m} \right\rfloor$ (as each channel supports $\frac{W}{c}$ bits/sec). Using Lemma 1, a (m,c')-network with total data rate of W' can support at least the capacity of a $\left(1, \left\lfloor \frac{c}{m} \right\rfloor\right)$-network with total data rate of W'. However, when $W' < W$, the (m,c')-network with total data rate W' can achieve only a fraction $\frac{W'}{W}$ of the capacity of a $\left(1, \left\lfloor \frac{c}{m} \right\rfloor\right)$-network with total data rate W (instead of W'). Now,

$$\begin{aligned}
\frac{W'}{W} &= \frac{m}{c}\left\lfloor \frac{c}{m} \right\rfloor \\
&= \frac{\left\lfloor \frac{c}{m} \right\rfloor}{\frac{c}{m}} \\
&\ge \frac{\left\lfloor \frac{c}{m} \right\rfloor}{\left\lfloor \frac{c}{m} \right\rfloor + 1}, \text{ since } \frac{c}{m} \le \left\lfloor \frac{c}{m} \right\rfloor + 1 \\
&\ge \frac{1}{2}, \text{ since } \left\lfloor \frac{c}{m} \right\rfloor \ge 1
\end{aligned}$$

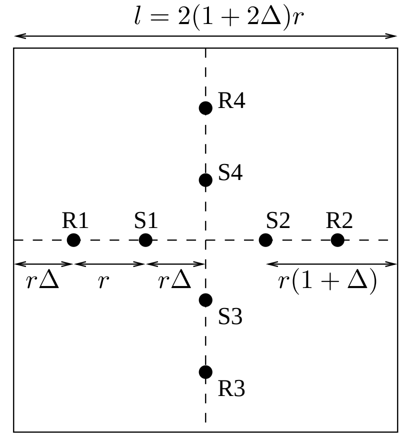

Figure 4: The placement of nodes within a cell. There are k nodes at each of the labeled positions.

Hence, a (m,c)-network can support at least $\frac{1}{2}$ the capacity supported by a $\left(1, \left\lfloor \frac{c}{m} \right\rfloor\right)$ network. This implies that asymptotically, a (m,c)-network has the same order of capacity as a $\left(1, \left\lfloor \frac{c}{m} \right\rfloor\right)$-network.
□

We now provide the following construction to establish that a capacity of $\Omega\left(MIN_O\left(W\sqrt{\frac{nm}{c}}, W\frac{nm}{c}\right)\right)$ bit-meters/sec is achievable in a $(1,c)$-network under the channel model 1. The result is then extended to a (m,c)-network by using Lemma 2.

Step 1: We consider a torus of unit area. Let $k = \min\left(c, \frac{n}{8}\right)$. This implies that $k \le c$. Partition the square area into $\frac{n}{8k}$ equal-sized square cells, and place $8k$ nodes in each cell. Since the total area is 1, each cell has an area of $\frac{8k}{n}$, and sides of length $l = \sqrt{\frac{8k}{n}}$.

Step 2: The $8k$ nodes within each cell are distributed by placing k nodes at each of the eight positions shown in Figure 4. Nodes placed at locations S1, S2, S3, S4 act as senders, and nodes placed at remaining locations act as receivers. The sender locations S1 through S4 are at a distance of $r\Delta$ from the center of the cell (recall that Δ is the "guard" parameter from the protocol model of interference), where $r = \frac{l}{2(1+2\Delta)} = \frac{1}{(1+2\Delta)}\sqrt{\frac{2k}{n}}$. The receiver locations R1 through R4 are at a distance of $r(1+\Delta)$ from the center of the cell. Therefore, the distance between S1-R1, S2-R2, S3-R3, and S4-R4 is equal to r. Each receiver location is at a distance of $r\Delta$ from nearest edge of the cell, and each sender location is at a distance of $r(1+\Delta)$ from the nearest edge of the cell.

Step 3: Label the k nodes in any location (S1 through S4, R1 through R4) as 1 through k. The j^{th} node in each sender location, $1 \le j \le k$, communicates with the j^{th} node in the nearest receiver location (at a distance of r) on channel j. Consider any pair of communicating nodes A and B that are located at, say, S1 and R1 respectively. Then, the nearest senders within the cell, other than A (located at S1), which are sending on the same channel as A are located at one of S2, S3, S4, and are at least a distance of $r(1+\Delta)$ away from B (located at R1). Similarly, in every cell, senders are at least $r(1+\Delta)$ distance from the cell boundary. Therefore, senders in adjacent cells of B are at least a distance of $r(1+\Delta)$ away from B as well. Hence, under the protocol model of interference, the transmission between A and B is not interfered with by any other transmission in the network, and this property holds for all communicating pairs.

From the above construction, there are $\frac{n}{2}$ pairs of nodes in the $(1,c)$-network, each transmitting at a rate of $\frac{W}{c}$ over a distance $r = \frac{1}{(1+2\Delta)}\sqrt{\frac{2k}{n}}$. Hence, the total capacity of the network (summing over all n nodes) is $\frac{n}{2}\frac{W}{c}r = \frac{W}{c}\frac{1}{(1+2\Delta)}\sqrt{\frac{nk}{2}}$ bit-meters/sec. Recall that $k = \min\left(c, \frac{n}{8}\right)$. Substituting for k in the above derivation, we obtain the capacity of a $(1,c)$-network to be $\Omega\left(MIN_O\left(W\sqrt{\frac{n}{c}}, W\frac{n}{c}\right)\right)$ bit-meters/sec under channel model 1, since Δ is a constant.

Using Lemma 2, the capacity of a (m,c)-network under channel model 1 is $\Omega\left(MIN_O\left(W\sqrt{\frac{n}{\lfloor\frac{c}{m}\rfloor}}, W\frac{n}{\lfloor\frac{c}{m}\rfloor}\right)\right)$ bit-meters/sec. Since $\frac{1}{\lfloor\frac{c}{m}\rfloor} \geq \frac{1}{\frac{c}{m}}$, we have the capacity of arbitrary networks to be $\Omega\left(MIN_O\left(W\sqrt{\frac{mn}{c}}, W\frac{mn}{c}\right)\right)$ bit-meters/sec, which leads to the following theorem:

THEOREM 2. *The achievable network capacity of a (m,c)-arbitrary network under channel model 1 is as follows:*

1. *When $\frac{c}{m}$ is $O(n)$, network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec.*

2. *When $\frac{c}{m}$ is $\Omega(n)$, network capacity is $\Omega\left(W\frac{nm}{c}\right)$ bit-meters/sec.*

The upper bound (Theorem 1) and lower bound (Theorem 2) on the order of the capacity of arbitrary networks match, indicating the bounds are tight.

3.3 Implications

A common scenario of operation is when the number of channels is not too large ($\frac{c}{m} = O(n)$). Under this scenario, the capacity of a (m,c)-network in the arbitrary setting scales as $\Theta\left(W\sqrt{\frac{nm}{c}}\right)$ under channel model 1. Similarly, under channel model 2, the capacity of the network scales as $\Theta\left(W\sqrt{nmc}\right)$. Under either model, the capacity of a (m,c)-network goes down by a factor of $1-\sqrt{\frac{m}{c}}$, when compared with a (c,c)-network. Therefore, doubling the number of interfaces at each node (as long as number of interfaces is smaller than the number of channels) increases the channel capacity by a factor of only $\sqrt{2}$.

Furthermore, the ratio between m and c decides the capacity, rather than the individual values of m and c. Increasing the number of interfaces may result in a linear increase in the cost but only a sub-linear (proportional to square-root of number of interfaces) increase in the capacity. Therefore, the *optimal number of interfaces to use may be smaller than the number of channels* depending on the relationship between cost of interfaces and utility obtained by higher capacity.

Different network architectures have been proposed for utilizing multiple channels when the number of available interfaces is smaller than the number of available channels [6, 15, 22]. The construction used in proving lower bound implies that maximal capacity is achieved when *all channels are utilized*. One architecture used in the past [6] is to use only m channels when m interfaces are available, leading to wastage of the remaining $c-m$ channels. That architecture results in a factor of $1-\frac{m}{c}$ loss in capacity which can be significantly higher than the optimal $1-\sqrt{\frac{m}{c}}$ loss (when $\frac{c}{m} = O(n)$). Hence, in general, *higher capacity may be achievable by architectures that use all channels*, possibly by dynamically switching channels.

4. CAPACITY RESULTS FOR RANDOM NETWORKS

We assume that n nodes are randomly located on the surface of a torus of unit area. Each node selects a destination randomly to which it sends $\lambda(n)$ bits/sec. The highest value of $\lambda(n)$ which can be supported by *every* source-destination pair with high probability is defined as the *per-node throughput* of the network. The traffic between a source-destination pair is referred to as a "flow". Since there are a total of n flows, the network capacity is defined to be $n\lambda(n)$.

Note that each node picks a destination node randomly, and so a node may be the destination of multiple flows. Let $D(n)$ be the maximum number of flows for which a node in the network is a destination. We use the following result to bound $D(n)$.

LEMMA 3. *The maximum number of flows for which a node in the network is a destination, $D(n)$, is $\Theta\left(\frac{\log n}{\log\log n}\right)$, with high probability.*

PROOF. The process of nodes selecting a random destination may be mapped to the well-known "Balls into Bins" problem [21]. Each source node may be viewed as a "ball", and each destination node may be viewed as a "bin". The process of selecting a destination node may be viewed as randomly dropping a "ball" into a "bin". Based on this mapping, the proof of the lemma follows from well-known results (cf. [21], Section 4). □

4.1 Upper bound

The capacity of multi-channel random networks is limited by three constraints, and each of them is used to obtain a bound on the network capacity. The minimum of the three bounds (the bounds depend on ratio between the number of channels c and the number of interfaces m) is an upper bound on the network capacity. While there may be other constraints on capacity as well, the constraints we consider are sufficient to provide a tight bound. We derive the bounds under channel model 1, but the results are applicable under channel model 2 as well.

Constraint 1 – Connectivity constraint: The capacity of random networks is constrained by the need to ensure the network is connected, so that every source-destination pair can successfully communicate. Since node locations are randomly chosen, there is some minimum transmission range each node should use to ensure the network is connected. Since all transmissions cover at least an area proportional to the square of the minimum transmission range, there is a bound on the number of simultaneous transmissions that can occur in the network. Based on this observation, Gupta and Kumar [10] have presented one bound on the network capacity to be $O\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec. This bound is applicable to multi-channel networks as well.

Constraint 2 – Interference constraint: A random network is a special case of an arbitrary network, and therefore the arbitrary network constraints are applicable to random networks as well. Therefore, the capacity of multi-channel random networks is also constrained by interference (this is same as constraint 1 listed for arbitrary networks in Section 3.1). This constraint was already captured in the upper bound for arbitrary networks, and we had obtained a bound of $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec. In a random network, each of the n source-destination pairs are separated by an average distance of $\Theta(1)$ meter. Consequently, the network capacity of random networks is at most $O\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec. We do not explicitly use the second arbitrary network constraint ("Interface bottleneck constraint" from Section 3.1) in the random network proof as the bounds established by that constraint are not tight, and that bound is subsumed by the bound for "destination bottleneck constraint".

Constraint 3 – Destination bottleneck constraint: The capacity of a multi-channel network is constrained by the data that can be received by a destination node. Consider a node X which is the destination of the maximum number (that is, $D(n)$) of flows. Recall that in a (m, c)-network, each channel supports a data rate of $\frac{W}{c}$ bits/sec. Therefore, the total data rate at which X can receive data over m interfaces is $\frac{Wm}{c}$ bits/sec. Since X has $D(n)$ incoming flows, the data rate of the minimum rate flow is at most $\frac{Wm}{cD(n)}$ bits/sec. Therefore, by definition of $\lambda(n)$, $\lambda(n) \leq \frac{Wm}{cD(n)}$, implying that network capacity (which by definition is $n\lambda(n)$) is at most $O\left(\frac{Wmn}{cD(n)}\right)$ bits/sec. Substituting for $D(n)$ from Lemma 3, the network capacity is at most $O\left(\frac{Wmn \log\log n}{c \log n}\right)$ bits/sec.

The bound obtained from constraint 3 is applicable to *any network, including mobile networks,* as long as the destination of every flow is randomly chosen among the nodes in the network. Even when $m = c$, this bound implies that the per-flow throughput, $\lambda(n)$, is at most $O\left(\frac{W \log\log n}{\log n}\right)$ bits/sec. Previous results on capacity of mobile networks [4,7,9] have stated a per-flow throughput of $O(W)$ bits/sec is possible, as in their models, each node does not randomly select a destination node. In our work, we choose the destination of a flow randomly from among $n-1$ possible destinations, similar to Gupta and Kumar [10]. Considering our discussion above, the $O(W)$ bits/sec bound with mobility cannot apply when destination nodes are randomly chosen. The previous results for mobile networks hold under other models of selecting destination nodes, wherein each node is the destination of at most $O(1)$ flows (for example, such a constraint is satisfied when permutation routing is used).

Combining the three bounds, the network capacity is at most $O\left(MIN_O\left(W\sqrt{\frac{n}{\log n}}, W\sqrt{\frac{nm}{c}}, \frac{Wmn \log\log n}{c \log n}\right)\right)$ bits/sec under channel model 1. From this, we have the following theorem on the upper bound on capacity of random networks (Figure 2 has a pictorial representation).

THEOREM 3. *The upper bound on the capacity of a (m, c)-random network under channel model 1 is as follows:*

1. *When $\frac{c}{m}$ is $O(\log n)$, network capacity is $O\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.*
2. *When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, network capacity is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.*
3. *When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the network capacity is $O\left(\frac{Wmn \log\log n}{c \log n}\right)$ bits/sec.*

An interesting observation from this theorem is that as long as $\frac{c}{m}$ is $O(\log n)$, the number of interfaces has no impact on channel capacity. This implies that when the number of channels is $O(\log n)$ (which is the common case today), *there is no loss in network capacity even if each node has a single interface.*

4.2 Lower bound

The lower bound is established by constructing a routing scheme and a transmission schedule for any random network. The lower bound matches the upper bound implying that the bounds are tight. We will provide a construction for a $(1, c)$-network (a network wherein each node has a single interface) under channel model 1, and then invoke Lemma 2 to extend the result to a (m, c)-network. The steps involved in the construction are described next.

4.2.1 Cell construction

The surface of the unit torus is divided using a square grid into square cells (see Figure 5), each of area $a(n)$, similar to the approach used in [7]. The key difference in our work from [7] is that the size of the cell, $a(n)$, varies with the number of channels, and has to be carefully chosen to meet multiple constraints (which are described later in the text). In particular, we set $a(n) = \min\left(\max\left(\frac{100 \log n}{n}, \frac{c}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$ as described before. Intuitively, the three values that influence $a(n)$ are based on the three constraints that were described in the upper bound proof: cell size needed to ensure connectivity, cell size needed when capacity is constrained by interference, and cell size needed when capacity is constrained by the maximum number of flows to any destination node, respectively.

We need to bound the number of nodes that are present in each cell. We state the bound here, and present a proof of the bound in Appendix B.

LEMMA 4. *If $a(n) > \frac{50 \log n}{n}$, then each cell has $\Theta(na(n))$ nodes per cell, with high probability.*

PROOF. See Appendix B. ☐

By construction, we ensure that $a(n) \geq \frac{100 \log n}{n}$ for large n (as $\max\left(\frac{100 \log n}{n}, \frac{c}{n}\right)$ is at least $\frac{100 \log n}{n}$, and $\left(\frac{1}{D(n)}\right)^2$ is asymptotically larger than $\frac{100 \log n}{n}$). Thus, with our choice of $a(n)$, Lemma 4 holds for suitably large n, and each cell has $\Theta(na(n))$ nodes per cell, *whp*.

The transmission range[2] of each node, $r(n)$, is set to be $\sqrt{8a(n)}$. With this transmission range, a node in one cell

[2]Transmission range is defined to be the maximum distance over which any node can communicate.

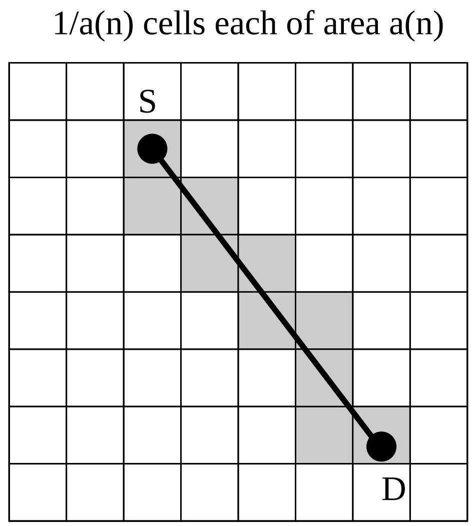

Figure 5: Routing through cells: Packets are routed through the cells intersected by the line joining the source and the destination. Within each cell, a specific node is chosen for forwarding all packets of a flow.

can communicate with any node in its eight neighboring cells. Note that when the cell size $a(n)$ increases, larger transmission range is required, as $r(n)$ is dependent on $a(n)$.

A transmission originating from a node S interferes with another transmission from A to B, only if S is within a distance of $(1+\Delta)r(n)$ of receiver B (using the interference definition of protocol model). Since the distance between A and B is at most $r(n)$, the distance between the two transmitters, S and A, must be less than $(2+\Delta)r(n)$ if the transmissions were to interfere. Thus, any transmission can possibly interfere with only those transmissions from transmitters within a distance of $(2+\Delta)r(n)$. Therefore, nodes in a cell can be interfered with by only nodes in cells within a distance of $(2+\Delta)r(n)$, and this interfering area can be completely enclosed in a larger square of side $3(2+\Delta)r(n)$ (this is a loose bound). Consequently, there are at most $\frac{(3(2+\Delta)r(n))^2}{a(n)} = 72(2+\Delta)^2$ interfering cells (recall $r(n) = \sqrt{8a(n)}$). Hence, the number of interfering cells, $k_{inter} \leq 72(2+\Delta)^2$, is a constant that only depends on Δ (and is independent of $a(n)$ and n).

4.2.2 Routing Scheme

Packets are routed through the cells that lie along the straight line joining the source and the destination node. A node in each cell through which the line passes is used to relay traffic along that flow (we will describe the choice of the node later). Figure 5 shows an example of the cells used to route data for a flow between source S and destination D.

In previously proposed constructions for proving lower bound on capacity [7,10], it was immaterial which node in a chosen cell forwarded packets for some flow. However, such an approach may "*overload*" certain nodes, leading to capacity degradation, when the number of interfaces per node is smaller than the number of channels. Consequently, it is important to ensure that the routing load is distributed among the nodes in a cell. This is a key extension to the routing procedure used in earlier capacity results [10], and the extension is described next.

For each flow passing through a cell, one node in the cell is "assigned" to the flow. The assigned node of a flow in a cell is the only node in that cell which may receive/transmit data along that flow. The assignment is done using a *flow distribution procedure* as below:

Step 1 – Assign source and destination nodes: For any flow that originates in a cell, the source node S is assigned to the flow (S is necessarily in the originating cell). Similarly, for any flow that terminates in a cell, the destination node D is assigned to the flow. Since a single node in each cell is allowed to receive or transmit data for a flow, it is required that the source and destination nodes be assigned to flows originating or terminating from them.

Step 2 – Balance distribution of remaining flows: After step 1 is complete, we are left with only those flows that pass through a cell. Each such remaining flow passing through a cell is assigned to the node in the cell that has the least number of flows assigned to it so far. This step balances the assignment of flows to ensure that all nodes are assigned (nearly) the same number of flows. The node assigned to a flow will receive packets from some node in the previous cell and send the packet to a node in the next cell.

Each node is the originator of one flow. Each node is the destination of at most $D(n)$ flows, which by Lemma 3 is $\Theta\left(\frac{\log n}{\log\log n}\right)$. Therefore, step 1 of the flow distribution procedure assigns to each node at most $1 + D(n)$ flows.

We use the following result to bound the number of source-destination lines that pass through any cell (and are assigned in step 2); we omit the proof as it has already been presented earlier in [7].

LEMMA 5. *The maximum number of source-destination lines that intersect any cell (including lines originating and terminating in the cell) is* $O\left(n\sqrt{a(n)}\right)$, *with high probability.*

Step 2 of the flow distribution procedure carefully assigns the remaining flows among the nodes in the cell to ensure that all nodes end up with nearly same number of flows. By Lemma 4, each cell has $\Theta(na(n))$ nodes, and by Lemma 5 at most $O\left(n\sqrt{a(n)}\right)$ flows pass through a cell. Therefore, step 2 will assign to any node in the network at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows. Therefore the total flows assigned to any node is at most $O\left(1 + D(n) + \frac{1}{\sqrt{a(n)}}\right)$. When choosing the size of $a(n)$ earlier, the maximum value of $a(n)$ was at most $\left(\frac{1}{D(n)}\right)^2$, which implies $\frac{1}{\sqrt{a(n)}}$ is at least $D(n)$. Hence, the total flows assigned to any node is always asymptotically dominated by $\frac{1}{\sqrt{a(n)}}$, and is therefore equal to $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows.

4.2.3 Scheduling transmissions

The transmission scheduling scheme is responsible for generating a transmission schedule for each node in the $(1, c)$-network that satisfies the following constraints:

Constraint 1: When a node X transmits a packet to a node Y over a channel j for some flow, X and Y should not be scheduled to transmit/receive at the same time for any other flow (since each node is assumed to have a single interface in the construction).

Constraint 2: Any two simultaneous transmissions on any channel should not interfere.

The multi-channel construction differs from the mechanisms used in earlier constructions [7,10] in two ways. First, the scheduling is on a per-node basis since flows are distributed among nodes, whereas in the past work it was sufficient to schedule on a per-cell basis. Second, since there is a single interface, but c channels are available (recall that we are assuming a $(1, c)$-network for now), the schedule has to additionally ensure that at most a single transmission/reception is scheduled for a node at any time (constraint 1 above).

We build a suitable schedule using a two-step process. In the first step, we satisfy constraint 1 by scheduling transmissions in "edge-color" slots so that at every node during any edge-color slot, at most one transmission or reception is scheduled. In the second step, we satisfy constraint 2 by dividing each edge-color slot into "mini-slots", and assigning mini-slots to channels such that any scheduled transmission is interference-free. By using the two-step process, each transmission in a mini-slot satisfies both constraint 1 and constraint 2.

Step 1 – Build a routing graph: We build a graph, called the "routing graph", whose vertices are the nodes in the network. One edge is inserted between all node pairs, say A and B, for every flow on which A and B are consecutive nodes (the routing scheme for selecting nodes along a flow was described earlier). Therefore, by this construction, every hop[3] in the network along any flow is associated with one edge in the routing graph. The resulting routing graph is a multi-graph[4] in which each node has at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ edges, since each flow through a node can result in at most two edges, one incoming and one outgoing, and we have already shown that each node is assigned to at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows. It is a well-known result [30] that a multi-graph with at most e edges per vertex can be edge-colored[5] with at most $\frac{3e}{2}$ colors. Therefore, the routing graph can be edge colored with at most some $f = O\left(\frac{1}{\sqrt{a(n)}}\right)$ colors.

We use edge coloring to ensure that when a transmission is scheduled along an edge, the interfaces on the nodes at either end of the edge are free, thereby satisfying constraint 1. We divide every 1 second period into f $(O\left(\frac{1}{\sqrt{a(n)}}\right))$ "edge-color" slots, each of length $\frac{1}{f}$ $(\Omega\left(\sqrt{a(n)}\right))$ seconds. Each of these edge-color slots is associated with an unique edge color. An edge is scheduled for transmission in the slot associated with its edge color. Since edge coloring ensures that at a vertex, all edges connected to the vertex use different colors, each node will have at most one transmission/reception scheduled in any edge-color slot. By construction, each edge corresponds to a hop in the network. Therefore this scheme ensures that during every 1 second interval, along any flow in the network, one transmission is scheduled on each hop of a flow.

Step 2 – Build an interference graph: In step 2, each edge-color slot is further sub-divided into "mini-slots" as explained below, and every node has an opportunity to transmit in some mini-slot. We develop a schedule for using mini-slots, which satisfies constraint 2. The schedule decides on which mini-slot within an edge-color slot and on what channel a node may transmit, and the same schedule is used in every edge-color slot.

We build another graph, called the "interference graph", wherein, vertices are nodes in the network, and there is an edge between two nodes if they may interfere with each other. Since every cell has at most some constant k_{inter} number of cells that may interfere with each other, and each cell has $\Theta\left(na(n)\right)$ nodes, each node has at most $g = O\left(na(n)\right)$ edges in the interference graph. It is well-known that a graph with maximum degree e can be vertex-colored[6] with at most $e + 1$ colors [30]. Therefore, the graph can be vertex-colored with some $O\left(na(n)\right)$ colors, i.e., at most $k_1na(n)$ colors for some constant k_1. Transmissions of two nodes assigned the same vertex-color do not interfere with each other. Hence, they can be scheduled to transmit on the same channel at the same time. On the other hand, nodes colored with different colors may interfere with each other, and need to be scheduled either on different channels, or at different time slots on the same channel.

We divide each edge-color slot into $\left\lceil\frac{k_1na(n)}{c}\right\rceil$ mini-slots on every channel, and number the slots on each channel from 1 to $\left\lceil\frac{k_1na(n)}{c}\right\rceil$. There is a total of $c\left\lceil\frac{k_1na(n)}{c}\right\rceil$ mini-slots across the c channels. Channels are numbered from 1 to c. A node which is allocated a color $p, 1 \leq p \leq k_1na(n)$ is allowed to transmit in mini-slot $\left\lceil\frac{p}{c}\right\rceil$ on channel $(p \bmod c) + 1$. The node may actually transmit if the edge-coloring has allocated an outgoing edge from the node to the corresponding edge-color slot.

Figure 6 depicts a schedule of transmissions on the network developed after the two-step scheduling process. The first step allocates one edge-color slot for each hop of every flow. The second step decides within each edge-color slot when the transmitter node on a hop may actually transmit a packet.

As seen in step 1, each edge-color slot is of length $\Omega\left(\sqrt{a(n)}\right)$ seconds. As seen in step 2, each edge-color slot is sub-

[3] A hop is a pair of consecutive nodes on a flow.

[4] A graph with possibly multiple edges between a pair of nodes.

[5] Edge-coloring requires any two edges incident on a common vertex to use different colors.

[6] Vertex-coloring requires any two vertices sharing a common edge to use different colors.

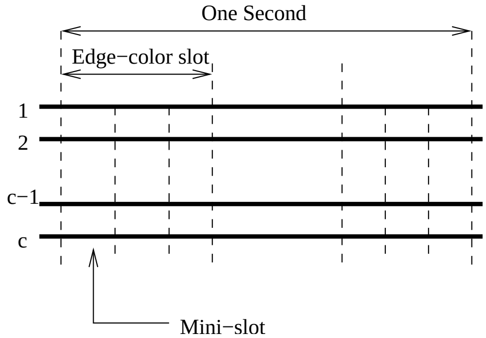

Figure 6: Transmission schedule: Every hop along every flow is assigned to exactly one edge-color slot in each one second interval. Within the edge-color slot assigned to a hop, a specific mini-slot is chosen during which the transmitter node on that hop may transmit.

divided into $\left\lceil\frac{k_1 n a(n)}{c}\right\rceil$ mini-slots. Therefore, each mini-slot is of length $\Omega\left(\frac{\sqrt{a(n)}}{\left\lceil\frac{k_1 n a(n)}{c}\right\rceil}\right)$ seconds. Each channel can transmit at the rate of $\frac{W}{c}$ bits/second. Hence, in each mini-slot, $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{c\left\lceil\frac{k_1 n a(n)}{c}\right\rceil}\right)$ bits can be transported. Since $\left\lceil\frac{k_1 n a(n)}{c}\right\rceil \leq \frac{k_1 n a(n)}{c} + 1$, we have, $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{k_1 n a(n)+c}\right)$ bits/sec. Depending on the asymptotic order of c, either $na(n)$ or c will dominate the denominator of $\lambda(n)$. Hence, $\lambda(n) = \Omega\left(MIN_O\left(\frac{W}{n\sqrt{a(n)}}, \frac{W\sqrt{a(n)}}{c}\right)\right)$ bits/sec. Since each flow is scheduled to receive one mini-slot on each hop during every 1 second interval, every source-destination flow can support a per-node throughput of $\lambda(n)$ bits/sec. Therefore, the total network capacity is equal to $n\lambda(n)$ which is equal to $\Omega\left(MIN_O\left(\frac{W}{\sqrt{a(n)}}, \frac{Wn\sqrt{a(n)}}{c}\right)\right)$ bits/sec.

Recall that $a(n)$ is set to $\min\left(\max\left(\frac{100\log n}{n}, \frac{c}{n}\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$. Substituting for the three values, and then applying Lemma 2 to extend the results to an (m, c)-network, we have the following theorem.

THEOREM 4. *The achievable capacity of a (m, c)-random network under channel model 1 is as follows:*

1. *When $\frac{c}{m}$ is $O(\log n)$, $a(n) = \Theta\left(\frac{\log n}{n}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/sec.*
2. *When $\frac{c}{m}$ is $\Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\frac{c}{mn}\right)$, and the network capacity is $\Omega\left(W\sqrt{\frac{nm}{c}}\right)$ bits/sec.*
3. *When $\frac{c}{m}$ is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $a(n) = \Theta\left(\left(\frac{\log\log n}{\log n}\right)^2\right)$, and the network capacity is $\Omega\left(\frac{Wmn\log\log n}{c\log n}\right)$ bits/sec.*

The lower bound matches the upper bound (Theorem 3) implying that the bounds are tight. Recall that the transmission range $r(n)$ has been set to $\sqrt{8a(n)}$. Hence, the *transmission range is larger* in case 2 and case 3 of Theorem 4 as compared to case 1 (since $a(n)$ increases). This implies that in multi-channel networks with large number of channels, higher transmission power is necessary for meeting capacity bounds than is required in a single channel network.

4.3 Implications

The above result implies that the capacity of multi-channel random networks with total channel data rate of W is the same as that of a single channel network with data rate W as long as the ratio $\frac{c}{m}$ is $O(\log n)$. When the number of nodes n in the network increases, we can also scale the number of channels (for example, by using additional bandwidth, or by dividing available bandwidth into multiple sub-channels). Even then, as long as the channels are *scaled at a rate not more than* $\log n$, *there is no loss in capacity even if a single interface is available at each node.* In particular, if the number of channels c is a fixed constant, independent of the node density, then as the node density increases beyond some threshold density (at which point $c \leq \log n$), there is no loss in capacity even if just a single interface is available per node. Thus, this result may be used to roughly estimate the number of interfaces each node has to be equipped with for a given node density and a given number of channels.

In a single channel random network, i.e., a $(1, 1)$-network, the capacity bottleneck arises out of the channel becoming fully utilized, and not because interface at any node is fully utilized. On an average, the interface of a node in a single channel network is busy only for $\frac{1}{X}$ fraction of the time, where X is the average number of nodes that interfere with a given node. In a $(1, 1)$-random network with n nodes, each node on an average has $\Theta(\log n)$ neighbors to maintain connectivity [10]. This implies that in a single channel network, each interface is busy for only $\Theta\left(\frac{1}{\log n}\right)$ time. Intuitively, our construction above utilizes this slack time of interfaces to support up to $O(\log n)$ channels without loss in capacity. In general, there is no loss in capacity in a random network as long as *the number of channels is smaller than the average number of nodes in any neighborhood*[7] *of a node.*

In earlier capacity results [7, 10], the transmission range, and therefore the neighborhood size, is a function of only the node density. However, for multi-channel networks, the transmission range has to be chosen based on ratio of channels to interfaces, in addition to the node density. For example, with a given node density, when the number of channels to interfaces is large (specifically, $\omega(\log n)$), the number of interfaces in a neighborhood will be smaller than the total number of channels. Therefore, even if all the interfaces are being used continuously, it is not possible to fully saturate the available channels. This can result in significant capacity degradation.

The capacity degradation can be reduced by increasing the size of a neighborhood, thereby ensuring the number of interfaces in a neighborhood is equal to the number of channels. Therefore, the lower bound construction requires the cell size to be chosen such that the number of interfaces (or nodes, when each node has a single interface) in each neighborhood is greater than or equal to the number

[7]The neighborhood of a node consists of all other nodes that may interfere with it.

of channels. Thus, it turns out that the optimal strategy for maximizing capacity when number of channels is large is to sufficiently increase the cell size $a(n)$, which implies a *larger transmission range* $r(n)$ *is needed* to allow communication with neighboring cells. However, there is still some capacity loss because larger transmission range (than that is needed for connectivity alone) lowers capacity by "consuming" more area. In summary, in a single channel random network, the transmission range is chosen to be large enough to ensure connectivity. However, in the case of multi-channel networks, the transmission range has to be chosen such that it is sufficiently large to ensure that all channels are utilized, in addition to guaranteeing connectivity.

4.4 Optimal routing and transmission scheduling approaches

The construction used in demonstrating that the lower bound is achievable can be used to develop optimal routing and transmission scheduling approaches. The lower bound construction suggests that *load balancing* (i.e., distributing flows) among nodes in a given neighborhood is essential for full utilization of multiple channels. In a single channel network, load balancing is sometimes used to balance energy consumption across nodes, or to improve resilience of the network. However, load balancing in the same neighborhood is not always required in single channel networks for maximizing capacity.

For example, consider a simple scenario with two flows from A to B and C to D as shown in Figure 7. The flows pass through a cell with two nodes E and F. Assume that node E is being used to forward data for flow A-B. In a single channel network with channel rate W, the per-flow throughput is the same whether node E or node F is chosen to forward data along flow C-D. In particular, the per-flow throughput is $\frac{W}{4}$ as the channel rate is split between receiving and sending data at the intermediate nodes. This is because E and F interfere with each other, and therefore cannot simultaneously transmit. Now, consider the scenario wherein two channels of data rate $\frac{W}{2}$ are available, but each node has a single interface. In this scenario, if node E is chosen to forward data along both flows A-B and C-D, the interface on node E can transmit/receive at most $\frac{W}{2}$ bits/sec, leading to a lower per-flow throughput of $\frac{W}{8}$. Instead, if node F is chosen to forward data along C-D, and links A-E and E-B use one channel, while C-F and F-D use the other channel, a higher per-flow throughput of $\frac{W}{4}$ can be achieved. This example highlights the need for distributing flows ("load"). The routing protocol should therefore *explicitly try to balance load among nodes in every neighborhood*, and select routes with lower load.

In the transmission scheduling scheme used for lower bound construction, *it suffices for a node to always transmit on a specific channel without requiring to switch channels* for different packets (recall that the same mini-slot on a specific channel is used by a node in all "edge-color" slots). However, a node may have to switch channels for receiving data. An alternate construction is to use a scheduling scheme which ensures that a node receives all data on a specific channel, but may have to switch channels when sending data. It can be shown that the alternate construction is equivalent to the lower bound construction by modifying the mini-slot assignment to be done on a per-receiver basis instead of a per-sender basis. This intuition can be used to develop a practical scheme that uses two interfaces per node. One interface can be used for receiving data and is always fixed to a single channel. The second interface can be used for sending data and is switched between channels, as necessary. Existing multi-channel protocols have often required tight synchronization among nodes. The use of two interfaces, with a dedicated interface on a fixed channel obviates the need for tight synchronization as a node receives data on a well-known channel. Furthermore, using a fixed channel for reception does not degrade capacity since it is based on the (optimal) alternate construction.

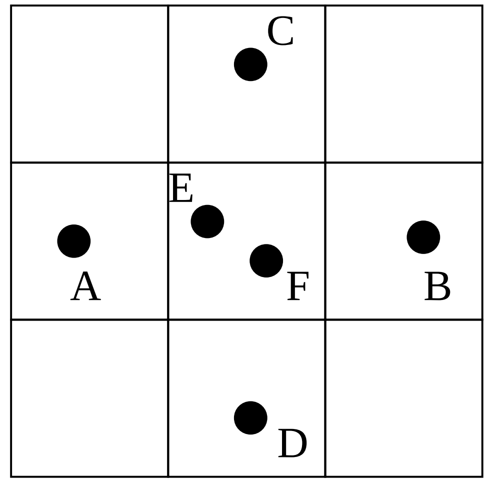

Figure 7: Need for balancing load among nodes in a neighborhood. If A is transmitting to B through E, it is better for C to transmit to D through F.

We have already used some of the insights gained from this work to develop routing and channel assignment protocols [13, 15] that are well-suited for multi-channel networks.

5. IMPACT OF SWITCHING DELAY

The previous discussion on multi-channel capacity has not considered the impact of interface switching delay. When the number of interfaces at each node is smaller than the number of channels, interfaces may have to be switched between channels. Switching an interface from one channel to another may incur a switching delay, say S. For example, existing IEEE 802.11-based wireless interfaces require [22] between few tens to hundreds of microseconds to switch from one channel to another. Switching delay is, however, independent of the number of nodes in the network.

We will show that if there are no end-to-end delay constraints, switching delay will not affect network capacity. For this, we will use the end-to-end delay constraint definition from [7]. Each packet is assumed to have a size L, and L is scaled with respect to the throughput obtained for each end-to-end flow. If each flow can transport λ bits/sec, then each flow is assumed to send packets of size $L = \lambda$. In the lower bound construction provided before, if packet sizes are set to λ bits, each packet traverses at least one hop in one second. Therefore, the end-to-end delay of a flow will be bounded by the number of hops on the flow, when there is no interface switching latency. Let us assume that the minimum end-to-end delay in the absence of interface switching latency is D_{opt}. A reasonable delay constraint in the presence of switching latency is to require that the end-to-end delay is at most a small constant multiple of D_{opt}; otherwise applications may see a large increase in the end-to-end delay. This requirement may be equivalently translated to allow a maximum packet size of L.

5.1 Capacity in the absence of end-to-end delay constraints

In the case of arbitrary networks, capacity bounds are met without requiring interface switching at all (as was shown in the construction used for lower bound). Hence, switching delay will not impact the capacity of arbitrary networks, even if there is an end-to-end delay constraint. In the absence of any end-to-end delay constraints, we show next that the capacity of random networks is independent of switching delay (the construction is described next).

In the construction we use to establish lower bound for random networks, interfaces may have to be switched between channels (when receiving data). In the worst case, an interface may have to be switched between channels for every packet transmission. If there is no end-to-end delay constraint, then we propose a simple "guard slot" approach which ensures that capacity loss can be made arbitrarily small even in the presence of switching delay.

The "guard slot" approach is as follows. Suppose that each packet is L bits long. This implies that the length of each edge color slot is $T = \frac{Lc}{W}$ seconds (since each channel supports a data rate of $\frac{W}{c}$ bits/sec under channel model 1). One simple way of hiding the interface switching delay S is to insert a "guard" slot of duration S between two "edge-color" slots during which all channels are idle, to ensure there is sufficient time for interface switching. With this approach, the network capacity will be only $\frac{T}{T+S}$ fraction of the capacity when there is no switching delay. However, the capacity reduction can be made arbitrarily small by sending extremely large packets ($L \gg \lambda$) resulting in $T \gg S$, leading to large end-to-end delay. Therefore, in the absence of end-to-end delay constraints, by using large data packets, the capacity degradation in random networks can be made arbitrarily small.

5.2 Capacity in the presence of end-to-end delay constraints

As we discussed above, even in the presence of delay constraints, the capacity of arbitrary networks is not affected by switching delay, since switching is not required to meet the capacity bounds. In the case of random networks as well, the upper bound proofs do not mandate interfaces to be switched, and therefore, even with switching delay, there may be no change in the capacity. However, it is still an open question *if the capacity of random networks is independent of the switching delay when there are end-to-end delay constraints.*

In the presence of end-to-end delay constraints, switching delay *does reduce* the achievable network capacity in the lower bound constructions proposed earlier. For example, considering the guard-slot approach described above, when there is a restriction on the maximum packet size, each edge-color slot is bounded by some length T, and the network capacity will be only $\frac{T}{T+S}$ of the capacity without switching delay, We next describe an approach that shows using additional interfaces at each node is *sufficient* in many scenarios to hide the switching delay, even with end-to-end delay constraints.

The new approach simulates a *virtual interface* having 0 switching delay using multiple physical interfaces that each have a switching delay S. By this construction, the use of v additional interfaces per node can *hide the switching delay*, i.e., a (v, c)-network using interfaces with switching delay S can achieve the same capacity and end-to-end delay bounds as a $(1, c)$-network using one interface with 0 switching delay. This construction suggests that multiple interfaces are *sufficient* to overcome the impact of switching delay, though multiple interfaces may not be *necessary.*

LEMMA 6. *Suppose that the time required for packet transmission in a $(1, c)$-network is $T = \frac{Lc}{W}$, and suppose $v = \lceil \frac{S}{T} \rceil + 1$. Then a (v, c)-network built with interfaces having switching delay S, can achieve the same capacity and end-to-end delay as a $(1, c)$-network built with interfaces having 0 switching delay.*

PROOF. Let us assume that each node has $v = \lceil \frac{S}{T} \rceil + 1$ interfaces, each having a switching delay S. We build a *virtual interface* with zero switching delay by using the v physical interfaces, as shown in Figure 8. We consider any time interval of length vT. We divide this time into v slots of length T, and only allow the i^{th} interface, $1 \leq i \leq v$, to transmit/receive in slot i. Thus, each physical interface is used for transmission/reception in one slot, and is idle for the next $(v - 1)$ slots of total duration $(v - 1)T$ seconds. Since $v = \lceil \frac{S}{T} \rceil + 1$, we have:

$$\begin{aligned} (v-1)T &= \left\lceil \frac{S}{T} \right\rceil T \\ &\geq S \end{aligned}$$

Hence, between two successive operations of a physical interface there is at least a gap of S, which ensures that switching delay is provisioned for. By this construction, the simulated *virtual interface* can continuously transmit/receive, with 0 switching delay. Therefore, a network using v interfaces having switching delay S, can mimic the behavior of a $(1, c)$-network built with interfaces having switching delay 0. □

From the previous lemma, *by increasing the number of interfaces at each node by a factor of v, switching delay is completely hidden.* We next discuss the capacity implications of using v physical interfaces at each node to construct a virtual interface, instead of directly using the v interfaces to send data in parallel.

From Theorem 4, we note that when the number of channels is $O(\log n)$ and there is no switching delay, the capacity of a (v, c)-network is the same as that of a $(1, c)$-network. Using this observation along with Lemma 6, we can conclude that by using the virtual interface technique, the capacity of a (v, c)-network with each interface having switching delay S is the same as the capacity of a (v, c)-network with each interface having switching delay 0. Hence, when the number of channels is $O(\log n)$, which is a scenario of significant

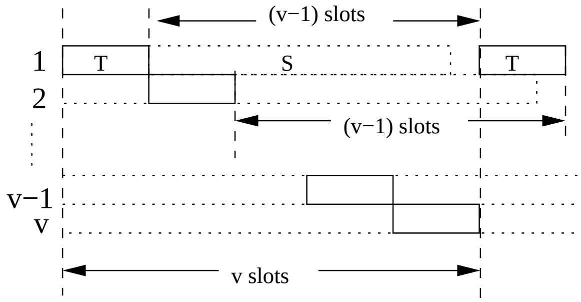

Figure 8: Constructing one virtual interface with zero switching delay by using v physical interfaces with switching delay S. Each packet transmission requires T seconds.

practical interest, *there is no capacity loss even with switching delay, provided multiple interfaces are used.*

Again, from Theorem 4, we note that when the number of channels is larger ($\Omega(\log n)$) and there is no switching delay, the capacity of a $(1, c)$-network is the *lower* than that of a (v, c)-network. Hence, using this observation along with Lemma 6, we can conclude that using the virtual interface technique when the number of channels is larger ($\Omega(\log n)$), a (v, c)-network with each interface having switching delay S *will have lower capacity* than a (v, c)-network with each interface having switching delay 0.

Using Theorem 4, we can show that in this case, the capacity will be lower by a factor of $\frac{1}{\sqrt{v}} \approx \sqrt{\frac{T}{T+S}}$ (since $v \approx \frac{T+S}{T}$) when number of channels is between $\Omega(\log n)$ and $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, and by a factor of $\frac{1}{v} \approx \frac{T}{T+S}$ when number of channels is $\Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$. In contrast, if the guard slot approach is used, the capacity is lower by a factor $\frac{T}{T+S}$ in all cases, independent of the number of channels. Therefore, although there is a capacity loss with switching delay for certain scenarios using the virtual interface technique, it is still significantly better than the guard slot approach when the number of channels is small. It is part of our future work to study if alternate constructions are possible that will not have any capacity loss at all.

6. PRACTICAL IMPLICATIONS

The theoretical analysis has studied the capacity of wireless networks with the number of channels varying across a wide range. The region where the number of channels is scaled as $O(\log n)$ seems to be of immediate practical interest, since the number of channels provisioned for in current wireless technologies is not too large. However, there are many recent efforts aimed at utilizing frequency spectrum in higher frequency bands, where significantly larger bandwidth is available for use. For example, there is around 7 GHz of spectrum available for unlicensed use in the 60 GHz band [5], whereas the total bandwidth used in current wireless technologies, such as IEEE 802.11, is less than 500 MHz. The bandwidth that may become available in higher frequency bands can be split up into a large number of channels, and therefore the region with number of channels greater than $\Omega(\log n)$ may be of practical interest in the near future.

The capacity analysis has shown that a single interface may suffice for random networks with up to $O(\log n)$ channels. The capacity-optimal lower bound construction used to support the above claim is based on certain assumptions, all of which may not be satisfied in practice. For example, we assume that interface switching delay is zero, transmission range of interfaces can be carefully controlled, and there is a centralized mechanism for co-ordinating route assignment and scheduling. In addition, the theoretical analysis derives asymptotic results, and capacity can be improved by constant factors in the lower bound constructions by using multiple interfaces. From Section 5, we note that when interface switching delay is not zero, having more than one interface may be beneficial. Furthermore, our research on protocol design [15] has identified many benefits of using at least two interfaces at each node, such as allowing full-duplex transfer, and simplifying the development of distributed protocols for utilizing multiple channels.

Simulation and testbed experiments [13, 22] have shown that having more than one interface may be beneficial in practice. However, these experiments do not prove multiple interfaces are necessary for obtaining all the observed performance improvement. In addition, some results also show that [13] it is not necessary to have one interface per channel to utilize all the channels, and in fact even many (e.g., 12) channels can be fully utilized by using only two interfaces, which partly validates the theoretical claim.

Furthermore, there are other proposals [2,25], which show that a single interface solution can also effectively utilize multiple channels, though at the cost of increased protocol complexity. Therefore, in practice, the theoretical claim that a single interface suffices with $O(\log n)$ channels is reasonably accurate, with the caveat that additional interfaces may be useful in simplifying protocol design and hiding switching delay.

7. CONCLUSIONS

In this paper, we have derived the lower and upper bounds on the capacity of static multi-channel wireless networks. We have considered wireless networks having c channels, and $m \leq c$ interfaces per node. Each interface is capable of selecting appropriate transmission power, and lower bound constructions require global knowledge. Under this model, we have shown that in an arbitrary network, there is a loss in the network capacity when the number of interfaces per node is smaller than the number of channels. However, we have shown that surprisingly, in a random network, a single interface may suffice for utilizing multiple channels, as long as the number of channels is scaled as $O(\log n)$. We have then considered the impact of non-zero interface switching delay on capacity, and shown that in a random network with up to $O(\log n)$ channels, interface switching delay has no impact on capacity, provided each node is provisioned with a few extra interfaces. As part of our future work, we intend to apply the insights gained from this work to build practical routing and MAC algorithms that approach the capacity limit.

8. ACKNOWLEDGMENTS

We would like to thank the anonymous referees for their valuable comments, and our shepherd Thyaga Nandagopal for guiding us in improving the paper. This research was supported in part by NSF grant ANI-0125859 and a Vodafone Graduate Fellowship.

9. REFERENCES

[1] A. Agarwal and P. R. Kumar. Capacity bounds for ad-hoc and hybrid wireless networks. *ACM SIGCOMM Computer Communication Review*, 34(3):71–81, July 2004.

[2] P. Bahl, R. Chandra, and J. Dunagan. SSCH: Slotted seeded channel hopping for capacity improvement in IEEE 802.11 ad-hoc wireless networks. In *ACM Mobicom*, 2004.

[3] N. Bansal and Z. Liu. Capacity, delay and mobility in wireless ad-hoc networks. In *Infocom*, pages 1553–1563, 2003.

[4] S. Diggavi, M. Grossglauser, and D. Tse. Even one-dimensional mobility increases adhoc wireless capacity. Technical report, UC Berkeley, 2003.

[5] C. H. Doan, S. Emami, D. A. Sobel, A. M. Niknejad, and R. W. Brodersen. Design considerations for 60 GHz CMOS radios. *IEEE Communications Magazine*, 42(132-140), Dec 2004.

[6] R. Draves, J. Padhye, and B. Zill. Routing in multi-radio, multi-hop wireless mesh networks. In *ACM Mobicom*, 2004.

[7] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah. Throughput-delay trade-off in wireless networks. In *Infocom*, 2004.

[8] M. Gastpar and M. Vetterli. On the capacity of wireless networks: The relay case. In *Infocom*, pages 1577–1586, New York, USA, June 2002.

[9] M. Grossglauser and D. Tse. Mobility increases the capacity of ad-hoc wireless networks. In *Infocom*, 2001.

[10] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.

[11] *IEEE Standard for Wireless LAN-Medium Access Control and Physical Layer Specification, P802.11*, 1999.

[12] U. C. Kozat and L. Tassiulas. Throughput capacity of random ad hoc networks with infrastructure support. In *Mobicom*, 2003.

[13] P. Kyasanur and N. H. Vaidya. Routing in multi-channel multi-interface ad hoc wireless networks. Technical report, University of Illinois at Urbana-Champaign, December 2004.

[14] P. Kyasanur and N. H. Vaidya. Capacity of multi-channel wireless networks: Impact of number of channels and interfaces. Technical report, University of Illinois at Urbana-Champaign, March 2005.

[15] P. Kyasanur and N. H. Vaidya. Routing and interface assignment in multi-channel multi-interface wireless networks. In *WCNC*, 2005.

[16] J. Li, Z. J. Haas, , and M. Sheng. Capacity evaluation of multi-channel multi-hop ad hoc networks. In *International Conference on Personal Communications*, 2002.

[17] B. Liu, Z. Liu, and D. Towsley. On the capacity of hybrid wireless networks. In *Infocom*, 2003.

[18] A. Nasipuri, J. Zhuang, and S. Das. A multichannel CSMA MAC protocol for multihop wireless networks. In *WCNC*, September 1999.

[19] R. Negi and A. Rajeswaran. Capacity of power constrained ad-hoc networks. In *Infocom*, 2004.

[20] E. Perevalov and R. Blum. Delay limited capacity of ad hoc networks: Asymptotically optimal transmission and relaying strategy. In *Infocom*, 2003.

[21] M. Raab and A. Steger. “Balls into Bins” - A simple and tight analysis. In *2nd Workshop on Randomization and Approximation Techniques in Computer Science*, 1998.

[22] A. Raniwala and T. Chiueh. Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network. In *Infocom*, 2005.

[23] J. So and N. H. Vaidya. Multi-channel MAC for ad hoc networks: Handling multi-channel hidden terminals using a single transceiver. In *Mobihoc*, 2004.

[24] J. So and N. H. Vaidya. Routing and channel assignment in multi-channel multi-hop wireless networks with single-nic devices. Technical report, University of Illinois at Urbana-Champaign, December 2004.

[25] J. So and N. H. Vaidya. A routing protocol for utilizing multiple channels in multi-hop wireless networks with a single transceiver. Technical report, University of Illinois at Urbana-Champaign, October 2004.

[26] S. Toumpis. Capacity bounds for three classes of wireless networks: Asymmetric, cluster, and hybrid. In *Mobihoc*, 2004.

[27] S. Toumpis and A. J. Goldsmith. Large wireless networks under fading, mobility, and delay constraints. In *Infocom*, 2004.

[28] V. N. Vapnik. *Estimation of Dependences based on Empirical Data*. Springer-Verlag, New York, 1982.

[29] V. N. Vapnik and A. Chervonenkis. On the uniform convergence of relative frequencies of events to their probabilities. *Theory of Probability and its Applications*, 16(2):264–280, 1971. 1971.

[30] D. B. West. *Introduction to Graph Theory*. Prentice Hall, 2 edition, 2001.

[31] S. Yi, Y. Pei, and S. Kalyanaraman. On the capacity improvement of ad hoc wireless networks using directional antennas. In *Mobihoc*, pages 108–116, USA, 2003.

[32] H. Zhang and J. C. Hou. Capacity of wireless ad-hoc networks under ultra wide band with power constraint. In *Infocom*, 2005.

APPENDIX

A. UPPER BOUND IN ARBITRARY NETWORKS

THEOREM 5. *The arbitrary network capacity of a (m, c)-network is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec under channel model 1.*

PROOF. We prove the result under channel model 1. The proof is based on a proof in [10]. We assume that nodes are synchronized, and slotted transmissions of duration τ are used. We assume that each source node originates λ bits/sec. Let the average distance between source and destination pairs be $\bar{L}$. Therefore, the capacity of the network is $\lambda n \bar{L}$ bit-meters/sec.

We consider any time period of length one second. In this time interval, consider a bit b, $1 \leq b \leq \lambda n$. We assume that bit b traverses $h(b)$ hops on the path from its source to its destination, where the h-th hop traverses a distance of r_b^h. Since the distance traversed by a bit from its source to its destination is at least equal to the length of the line joining the source and the destination, by summing over all bits we obtain,

$$\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} r_b^h \geq \lambda n \bar{L} \tag{1}$$

Let us define H to be the total number of hops traversed by all bits in a second, i.e. $H = \sum_{b=1}^{\lambda n} h(b)$. Therefore, the number of bits transmitted by all nodes in a second (including bits relayed) is equal to H. Since each node has m interfaces, and each interface transmits over a channel with rate W/c (assuming channel model 1), the total bits that can be transmitted by all nodes over all interfaces is at most $\frac{Wmn}{2c}$ (Transporting a bit across one hop requires *two* interfaces, one each at the transmitting and the receiving nodes). Hence, we have,

$$H \leq \frac{Wmn}{2c} \tag{2}$$

Under the protocol model, a transmission over a hop of length r is successful only if there is no transmitter within a distance of $(1+\Delta)r$. Suppose node A is transmitting a bit to node B, while node C is simultaneously transmitting a bit to node D, and both the transmissions are over a common channel. Then, using the interference model, we have

$$d(C,B) \geq (1+\Delta) d(A,B)$$

$$d(A,D) \geq (1+\Delta) d(C,D)$$

Adding the above two expressions together, and applying triangle inequality, we obtain,

$$d(B,D) \geq \frac{\Delta}{2}(d(A,B) + d(C,D))$$

This implies that the receivers of two simultaneous transmissions are separated by a distance proportional to the distance from their senders. This may be viewed as each hop consuming a disk of radius $\frac{\Delta}{2}$ times the length of the hop around each receiver. Since the area "consumed" on each channel is bounded above by the area of the domain (1 sq meter), summing over all channels (which can in total potentially transport W bits) we have the constraint,

$$\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2}{4} (r_b^h)^2 \leq W \tag{3}$$

which can be rewritten as,

$$\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r_b^h)^2 \leq \frac{4W}{\pi \Delta^2 H} \tag{4}$$

Since the expression on the left hand side is convex, we have,

$$\left(\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} r_b^h\right)^2 \leq \sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} \frac{1}{H} (r_b^h)^2 \tag{5}$$

Therefore, from (4) and (5),

$$\sum_{b=1}^{\lambda n} \sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4WH}{\pi \Delta^2}} \tag{6}$$

Substituting for H from (2), and using (1) we have,

$$\lambda n \bar{L} \leq W \sqrt{\frac{2mn}{\pi \Delta^2 c}} \tag{7}$$

This proves that the network capacity of an arbitrary network is $O\left(W\sqrt{\frac{nm}{c}}\right)$ bit-meters/sec under channel model 1. □

B. RESULTS FOR ESTABLISHING LOWER BOUND IN RANDOM NETWORKS

Lemma 4: If $a(n) > \frac{50 \log n}{n}$, then each cell has $\Theta(na(n))$ nodes per cell, with high probability.

PROOF. A similar result was stated in [7] without proof. Here we provide a proof based on VC-theory (see [28] for details on VC-theory), similar to the approach used by Gupta and Kumar [10]. The total number of square cells is $\frac{1}{a(n)}$. Since nodes are randomly located on the torus, the probability that any given node will lie in a specific cell is $a(n)$. We want to derive bounds on number of nodes in *every* cell in the square grid, which requires a proof of uniform convergence. The set of axis-parallel squares $\mathcal{C}$ are known to have VC-dimension 3. By applying the Vapnik-Chervonekis theorem [29], similar to the approach used in [10], we have the following bound on the number of nodes N_C in any cell C:

$$\text{Prob}\left(\sup_{C \in \mathcal{C}} \left| \frac{N_C}{n} - a(n) \right| \leq \frac{50 \log n}{n}\right) > 1 - \frac{50 \log n}{n} \tag{8}$$

where the constants in the above expression have been carefully chosen to satisfy the Vapnik-Chervonekis theorem. The above result implies that with high probability, we have

$$na(n) - 50 \log n \leq N_C \leq na(n) + 50 \log n$$

provided $a(n) > \frac{50 \log n}{n}$.

Hence, we can conclude that the number of nodes in any cell is $\Theta(na(n))$ with high probability, as long as $a(n) > \frac{50 \log n}{n}$. □