

A Result on Hybrid Scheduling in Wireless Networks

Technical Report (March 2009)

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I. INTRODUCTION

Medium access control is an exceedingly important aspect of the operation of any network. This is particularly true in wireless settings where the broadcast nature of the medium accentuates the need for effective medium access algorithms which provide good throughput and fairness characteristics. By far the most popular MAC techniques for wireless networks are those based on Carrier Sense Multiple Access (CSMA) due to their simplicity and amenability to distributed implementation. The IEEE 802.11 MAC is an example of a widely used MAC protocol that uses CSMA. However, CSMA protocols are often inadequate when fairness is of importance. Moreover, the performance of CSMA protocols degrades when the number of active transmitters increases since the probability of collisions increases. In such scenarios, Time Division Multiple Access (TDMA) based approaches are much more suitable. In TDMA protocols, each user receives a pre-allocated time-slice during which it can access the channel. This provides for fair distribution of bandwidth across users. However, when only a few users are active, TDMA leads to waste of bandwidth, as a time-slice will go unutilized if the owner of that time-slice has no traffic to send. Evidently, it is desirable for a MAC procedure to have the good characteristics of both classes of MAC techniques, i.e., it should provide fair medium access when there are many active users, but should avoid wastage of bandwidth when only a few users are active. This has led to the design of hybrid TDMA-CSMA approaches, e.g. Z-MAC. However, there has been little effort toward developing formal theoretical results to examine and understand such hybrid strategies.

At the same time, there has been much recent work in the theoretical domain on designing distributed scheduling algorithms that can schedule traffic while maintaining finite (expected) queues. An important class of scheduling algorithms that has received much attention lately are the so-called *maximal* schedulers. The performance of a maximal scheduler with a local threshold rule is studied in [1], [2], while the rate-stability of certain prioritized maximal schedulers is considered in [3]. Approximations to maximal schedulers can potentially be implemented using backoff mechanisms in conjunction with CSMA [4], [5]. It is also to be noted that some characteristics of CSMA based medium access protocols, e.g., 802.11, are approximately similar to those of a maximal scheduler, since a node that has data to send will try to access the channel and unless it detects the channel to be occupied by another interfering transmission it will gain access to it after executing the specified medium access procedure.¹ Thus, maximal scheduling provides a tractable theoretical abstraction that can assist in building intuition.

Here, we present a performance bound for a hybrid TDMA-Maximal scheduler.

This research is supported in part by US Army Research Office grant W911NF-05-1-0246.

¹One must keep in mind that maximal schedulers do not capture the possibility for collisions that arises in CSMA based protocols. Thus, any results for the former do not provide intuition regarding that aspect.

II. PRELIMINARIES

We assume the availability of a single channel for communication. The wireless network is viewed as a directed graph, with each directed link in the graph representing an available communication link. We model interference using a *conflict* relation between links. Two links are said to conflict with each other if it is only feasible to schedule at most one of the links at any given time. The conflict relation is assumed to be symmetric. The conflict-based interference model provides a tractable approximation of reality – while it does not capture the wireless channel precisely, it is more amenable to analysis. Such conflict-based interference models have also been used in past related work (e.g., [6], [7]), etc.

We assume a single channel of operation. Time is assumed to be slotted, with the slot duration being 1 unit time (i.e., we use slot duration as the time unit). In each time slot, the scheduler used in the network determines which links should transmit in that time slot.

We now introduce some notation and terminology.

The network is viewed as a collection of directed links, where each link is a pair of nodes that are capable of direct communication with non-zero rate.

- \mathcal{L} denotes the set of directed links in the network.
- $\mathbf{I}(l)$ denotes the set of links that conflict with link l . As a matter of convention we assume that $l \in \mathbf{I}(l)$.
- K_l denotes the maximum number of links in $\mathbf{I}(l)$ that can be scheduled simultaneously if l is not scheduled.
- K is the largest value of K_l over all links l , i.e., $K = \max_l K_l$.
- $\tilde{K}_l = \max\{1, K_l\}$.
- $\tilde{K} = \max\{1, K\}$.
- $I_{max} = \max_{l \in \mathcal{L}} |\mathbf{I}(l)|$
- χ denotes the chromatic number of the link-interference graph.

We limit our focus to single-hop flows. Thus, all traffic over link l can be viewed as a single aggregated flow.

III. HYBRID TDMA-MAXIMAL SCHEDULING

The following assumptions are made about the arrival and channel rate processes:

The arrival process for link l is i.i.d. over all time-slots t , and is denoted by $\{\lambda_l(t)\}$, with $E[\lambda_l(t)] = \lambda_l$. We make no assumption about independence of arrival processes for two links l, k . However, we consider only the class of arrival processes for which $E[\lambda_l(t)\lambda_k(t)]$ is bounded, i.e., $E[\lambda_l(t)\lambda_k(t)] \leq \eta$ for all $l \in \mathcal{L}, k \in \mathcal{L}$, where η is a suitable constant. The rate r_l achievable on a link l is assumed to be time-invariant. \vec{r} is the link-rate vector, i.e., a vector of dimension $|\mathcal{L}|$ with component l being r_l .

Each time slot is sub-divided into m subslots labeled $1, \dots, m$, where $m \geq \chi$. Consider a valid coloring of the graph that uses m colors labeled $1, \dots, m$.

At the beginning of slot t , the schedule for each of the m sub-slots is computed as follows:

Only links l with $q_l(t) \geq r_l$ are eligible to participate. Amongst participating links, the schedule for each sub-slot i is computed as follows: links with color i have priority over links with other colors in being scheduled, i.e., all links with color i that participate are guaranteed to be scheduled in sub-slot i (note that no two such links conflict with each other); thereafter a maximal schedule is computed over participating links of other colors which have not been blocked by the scheduled color i links.

Theorem 1: A hybrid scheduler with $m \geq \chi$ sub-slots in each slot can stabilize any load-vector $\vec{\lambda}$ such that $\left(\frac{m\tilde{K}}{m+\tilde{K}-1}\right) \left(\vec{\lambda} + \epsilon_o \vec{r}\right)$ lies in the stability region for some $\epsilon_o > 0$.

Proof:

The queue dynamics is as follows:

$$q_l(t+1) = q_l(t) + \lambda_l(t) - x_l(t) \tag{1}$$

where $x_l(t)$ is the amount of service link l receives during slot t .

Consider a partition of the set of links \mathcal{L} into two subsets:

- 1) \mathcal{L}_1 is the set of links for which $\frac{\lambda_l}{r_l} \leq \frac{1}{m} - \epsilon_o$
- 2) \mathcal{L}_2 is the set of links for which $\frac{\lambda_l}{r_l} > \frac{1}{m} - \epsilon_o$

We use the following Lyapunov function:

$$V_q(\vec{q}(t)) = \left(\frac{I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)}{r_l} \right)^2 + \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \left(\sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} \right) \quad (2)$$

It can be seen that:

$$\begin{aligned} V_q(\vec{q}(t+1)) - V_q(\vec{q}(t)) &= \left(\frac{I_{max}}{\epsilon_o} \right) \left[\sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t+1)}{r_l} \right)^2 - \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)}{r_l} \right)^2 \right] + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t+1)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1)}{r_k} \right] - \left[\sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} \right] \\ &= \left(\frac{I_{max}}{\epsilon_o} \right) \left[\sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t) + q_l(t+1) - q_l(t)}{r_l} \right)^2 - \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)}{r_l} \right)^2 \right] + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t) + q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t) + q_k(t+1) - q_k(t)}{r_k} \right. \\ &\quad \left. - \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} \right] \\ &= \left(\frac{I_{max}}{\epsilon_o} \right) \left[\sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)}{r_l} \right)^2 + \sum_{l \in \mathcal{L}_1} \frac{2q_l(t)(q_l(t+1) - q_l(t))}{r_l^2} + \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t+1) - q_l(t)}{r_l} \right)^2 - \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)}{r_l} \right)^2 \right] \\ &\quad + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} + \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \right] \\ &+ \left[\sum_{l \in \mathcal{L}_2} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} + \sum_{l \in \mathcal{L}} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} - \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} \right] \\ &= \left(\frac{I_{max}}{\epsilon_o} \right) \left[\sum_{l \in \mathcal{L}_1} \frac{2q_l(t)(q_l(t+1) - q_l(t))}{r_l^2} + \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t+1) - q_l(t)}{r_l} \right)^2 \right] + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \right] \\ &\quad + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t)}{r_k} + \sum_{l \in \mathcal{L}} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \right] \\ &= \left(\frac{2I_{max}}{\epsilon_o} \right) \left[\sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t)(q_l(t+1) - q_l(t))}{r_l^2} \right) \right] + \left[2 \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \right] \\ &\quad + \left(\frac{I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t+1) - q_l(t)}{r_l} \right)^2 + \left[\sum_{l \in \mathcal{L}} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \right] \end{aligned}$$

since $k \in \mathbf{I}(l) \implies l \in \mathbf{I}(k)$ from the symmetric conflicts assumption

(3)

Let $\mathcal{L}'(t)$ denote the set of links for which $q_l(t) \geq r_l$. Thus, $\mathcal{L}'(t)$ constitutes the set of links that participate in the scheduling process during slot t .

From the scheduler definition, it follows that:

$$\frac{x_l(t)}{r_l} \geq \frac{1}{m} \text{ for all } l \in \mathcal{L}'(t) \quad (4)$$

Furthermore, for any link $l \in \mathcal{L}$, $q_l(t) \geq r_l$ implies that in each of the m sub-slots of slot t , either l is scheduled, or some other link $k \in \mathbf{I}(l)$ is scheduled. Then:

$$\sum_{k \in \mathbf{I}(l)} \frac{x_k(t)}{r_k} \geq 1 \text{ for all } l \in \mathcal{L}'(t) \quad (5)$$

As observed earlier, since $\left(\frac{m\tilde{K}}{m+\tilde{K}-1}\right)(\vec{\lambda} + \varepsilon_o \vec{r})$ can be stabilized by some algorithm, it follows that $\frac{m\tilde{K}}{m+\tilde{K}-1}(\vec{\lambda} + \varepsilon_o \vec{r})$ lies within the feasible rate region.

Since, all links $k \in \mathbf{I}(l) \setminus \{l\}$ conflict with l , therefore any such link can only be scheduled when l is not scheduled. This implies that:

$$\frac{m\tilde{K}}{m+\tilde{K}-1} \sum_{k \in \mathbf{I}(l)} \left(\frac{\lambda_k}{r_k} + \varepsilon_o \right) \leq K \left(1 - \frac{m\tilde{K}}{(m+\tilde{K}-1)} \left(\frac{\lambda_l}{r_l} + \varepsilon_o \right) \right) + \frac{m\tilde{K}}{m+\tilde{K}-1} \left(\frac{\lambda_l}{r_l} + \varepsilon_o \right) \text{ for all } l \in \mathcal{L} \quad (6)$$

In the special case $K = 0$ (which occur when $\mathbf{I}(l) = \{l\}$), it is trivially true that $\tilde{K} = 1$, and that:

$$\sum_{k \in \mathbf{I}(l)} \frac{\lambda_k}{r_k} \leq 1 - \varepsilon_o \text{ for all } l \in \mathcal{L} \quad (7)$$

Let us now consider that $K \geq 1$, for which $\tilde{K} = K$. Since $\frac{\lambda_l}{r_l} > \frac{1}{m} - \varepsilon_o$ for all $l \in \mathcal{L}_2$, it follows that:

$$\begin{aligned} \frac{m\tilde{K}}{m+\tilde{K}-1} \sum_{k \in \mathbf{I}(l)} \left(\frac{\lambda_k}{r_k} + \varepsilon_o \right) &\leq K \left(1 - \frac{m\tilde{K}}{(m+\tilde{K}-1)} \left(\frac{\lambda_l}{r_l} + \varepsilon_o \right) \right) + \frac{m\tilde{K}}{(m+\tilde{K}-1)} \left(\frac{\lambda_l}{r_l} + \varepsilon_o \right) \\ &= K - (K-1) \frac{m\tilde{K}}{m+\tilde{K}-1} \left(\frac{\lambda_l}{r_l} + \varepsilon_o \right) \leq K - (K-1) \frac{\tilde{K}}{m+\tilde{K}-1} \\ &\text{for all } l \in \mathcal{L}_2 \end{aligned} \quad (8)$$

Therefore:

$$\begin{aligned} \sum_{k \in \mathbf{I}(l)} \frac{\lambda_k}{r_k} &\leq \frac{K(m+\tilde{K}-1)}{m\tilde{K}} - \frac{(K-1)}{m} - \varepsilon_o \quad (\because |\mathbf{I}(l)| \geq 1) \\ &= \frac{m+\tilde{K}-1}{m} - \frac{\tilde{K}-1}{m} - \varepsilon_o \text{ for all } l \in \mathcal{L}_2 \\ \therefore \sum_{k \in \mathbf{I}(l)} \frac{\lambda_k}{r_k} &\leq 1 - \varepsilon_o \text{ for all } l \in \mathcal{L}_2 \end{aligned} \quad (9)$$

Therefore, for all values of K , it is true that:

$$\sum_{k \in \mathbf{I}(l)} \frac{\lambda_k}{r_k} \leq 1 - \varepsilon_o \text{ for all } l \in \mathcal{L}_2 \quad (10)$$

In light of this, it follows that:

$$\begin{aligned}
E[V_q(\vec{q}(t+1)) - V_q(\vec{q}(t)) | \vec{q}(t)] &= E \left[\left(\frac{2I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)(q_l(t+1) - q_l(t))}{r_l^2} \middle| \vec{q}(t) \right] + E \left[2 \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \middle| \vec{q}(t) \right] \\
&\quad + E \left[\left(\frac{I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \left(\frac{q_l(t+1) - q_l(t)}{r_l} \right)^2 + \sum_{l \in \mathcal{L}} \frac{q_l(t+1) - q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \middle| \vec{q}(t) \right] \\
&\leq E \left[\left(\frac{2I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)(q_l(t+1) - q_l(t))}{r_l} \middle| \vec{q}(t) \right] + E \left[2 \sum_{l \in \mathcal{L}} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{q_k(t+1) - q_k(t)}{r_k} \middle| \vec{q}(t) \right] + C_1 \\
&= E \left[\left(\frac{2I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)(\lambda_l(t) - x_l(t))}{r_l} \middle| \vec{q}(t) \right] + \left[2 \sum_{l \in \mathcal{L}_1} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{\lambda_k(t) - x_k(t)}{r_k} \middle| \vec{q}(t) \right] + E \left[2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \frac{\lambda_k(t) - x_k(t)}{r_k} \middle| \vec{q}(t) \right] + C_1 \\
&= \left[\left(\frac{2I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)}{r_l} \left(\frac{\lambda_l}{r_l} - E \left[\frac{x_l(t)}{r_l} \middle| \vec{q}(t) \right] \right) \right] + \left[2 \sum_{l \in \mathcal{L}_1} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \left(\frac{\lambda_k}{r_k} - E \left[\frac{x_k(t)}{r_k} \middle| \vec{q}(t) \right] \right) \right] \\
&\quad + \left[2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} \sum_{k \in \mathbf{I}(l)} \left(\frac{\lambda_k}{r_k} - E \left[\frac{x_k(t)}{r_k} \middle| \vec{q}(t) \right] \right) \right] + C_1 \\
&\leq \left[\left(\frac{2I_{max}}{\epsilon_o} \right) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)}{r_l} \left(\frac{1}{m} - \epsilon_o - \frac{1}{m} \right) \right] + \left[2(I_{max} - 1 + \frac{1}{m} - \epsilon_o) \sum_{l \in \mathcal{L}_1} \frac{q_l(t)}{r_l} \right] + \left[2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} (1 - \epsilon_o - 1) \right] \\
&\quad + \left(\frac{2I_{max}}{\epsilon_o} \right) \left(\frac{1}{m} \right) |\mathcal{L}_1| + 2I_{max} |\mathcal{L}_1| + 2I_{max} |\mathcal{L}_2| + C_1
\end{aligned}$$

using (4), (5) and (10), and compensating for links $l \notin \mathcal{L}'(t)$ by adding the last three terms

$$\begin{aligned}
&\leq \sum_{l \in \mathcal{L}_1} \left(\frac{2I_{max}}{\epsilon_o} \right) \frac{q_l(t)}{r_l} (-\epsilon_o) + \sum_{l \in \mathcal{L}_1} \left(2(I_{max} - 1 + \frac{1}{m} - \epsilon_o) \right) \frac{q_l(t)}{r_l} + 2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} (-\epsilon_o) + C_2 \\
&\leq \sum_{l \in \mathcal{L}_1} \left(-2(1 - \frac{1}{m} + \epsilon_o) \right) \left(\frac{q_l(t)}{r_l} \right) + 2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} (-\epsilon_o) + C_2 \\
&\leq \sum_{l \in \mathcal{L}_1} (-2\epsilon_o) \left(\frac{q_l(t)}{r_l} \right) + 2 \sum_{l \in \mathcal{L}_2} \frac{q_l(t)}{r_l} (-\epsilon_o) + C_2
\end{aligned}$$

(11)

where $C_1 = \frac{I_{max} |\mathcal{L}| \eta}{\epsilon_o \min_{l \in \mathcal{L}} r_l^2} + \frac{|\mathcal{L}| I_{max} \eta}{\min_{l \in \mathcal{L}} r_l^2}$ and $C_2 = C_1 + \left(\frac{2I_{max}}{\epsilon_o} \right) \left(\frac{1}{m} \right) |\mathcal{L}_1| + 2I_{max} |\mathcal{L}_1| + 2I_{max} |\mathcal{L}_2|$.

Using Lemma 2 from [8], this proves stability. ■

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