Comparison of Duplex and Triplex Memory Reliability

Nitin H. Vaidya[1]

Abstract *A large number of choices exist when designing a reliable memory system. The choices range from simple replication to complex error control codes (ECC). An intermediate solution is to use combination of replication and simple ECC. Such a system consists of multiple memory modules, data stored in each module being encoded using an ECC. This paper compares reliability of memory systems formed using simple triplication (without ECC) with memory systems formed by duplicating memory modules that use ECC. It is shown that reliability achieved by duplication of memory modules using codes capable of only error* detection *or only* single *error correction (SEC), is always worse than simple triplication. However, it is also shown that duplication of memory modules, with codes capable of single error correction* and *double error detection (SEC-DED), can achieve better reliability than simple triplication when bit error probability is small.*

Index Terms: Reliability, replication, coding, modular redundancy.

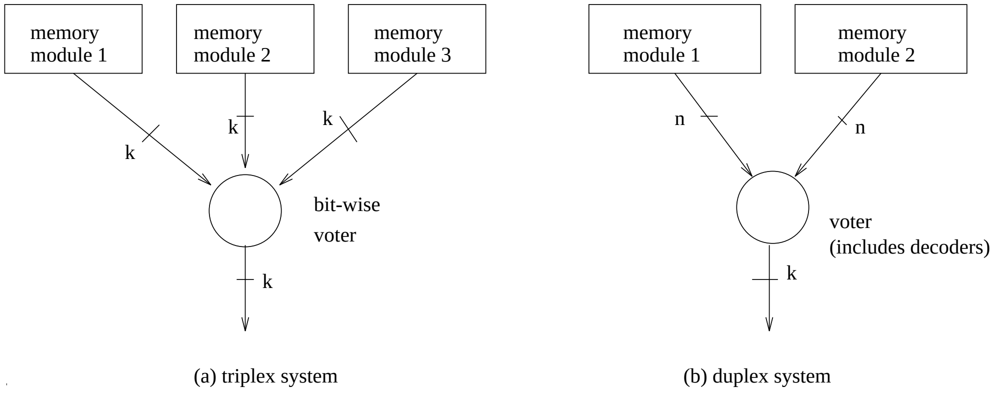
1. Introduction

A large number of choices exist when designing a reliable memory system. The choices range from simple replication to complex error control codes (ECC) [3]. Simple replication (without ECC) can achieve a better performance as compared to systems using ECC, because ECC necessitate decoding. An intermediate solution is to use combination of replication and simple ECC. Such a system consists of multiple memory modules, data stored in each module being encoded using an ECC. A similar approach has been used in commercial systems [1].

Reliability of memory systems formed by simple triplication (without ECC) is compared here with that of memory systems formed by duplicating memory modules that use ECC. These two systems, referred to as *triplex* and *duplex* systems, respectively, are illustrated in Figure 1. The triplex system in Figure 1(a) is formed by simple triplication of memory modules that do not use any error control coding. The memory system output is obtained by bit-wise voting on the output of the three modules in the triplex system. The duplex system in Figure 1(b) consists of two identical memory modules. Each memory module uses an (n, k) error control code. Encoded outputs of the two modules are available

[1] Department of Computer Science, Texas A&M University, College Station, TX 77843-3112.

to a voter that can decode the outputs of the two memory modules. The exact function of the voter in duplex systems will be defined more precisely in Section 3.



(a) triplex system

(b) duplex system

Figure 1: Triplex and duplex memory systems

The objective here is to determine the minimum *capability* required in the ECC such that the duplex system can achieve a higher reliability than the triplex system. To our knowledge, such an analysis has not been carried out before. The work presented here is motivated by our previous research on modular redundant system reliability and safety [7].

Capability of an ECC is characterized by the number of errors it can correct and detect. A t_1-error correcting-d_1 error detecting code is said to be *less* capable than a t_2-error correcting-d_2 error detecting code if (i) $t_1 < t_2$ or (ii) $t_1 = t_2$ and $d_1 < d_2$. Reliability of the triplex system is compared here with duplex systems that use error control codes (ECC) of three different capabilities:

- Error detection only.
- Single error correction (and no error detection, i.e., more than one error is assumed to result in erroneous decoding).
- Single error correction and double error detection (SEC-DED).

In the following sections, an expression for the reliability of the triplex system is presented, followed by evaluation of reliability of the three duplex systems and comparison with the triplex system. Although the analysis is somewhat lengthy, it leads to the interesting conclusion that the duplex system must incorporate at least a SEC-DED code before it can achieve higher reliability than the triplex system.

2. Reliability of the Triplex System

For reliability analysis, we use the independent symmetric error model [4]. It is assumed that each bit in memory may become erroneous independently with probability p. In practice, *bit error probability* p is expected to be quite small. Each data word contains k bits. Let the number of words in the memory be W, i.e., let each memory module contain W words.

Reliability of the voters is not included in our analysis; we will discuss the effect of voter reliability in Section 4. Note that reliability is evaluated here using *error* probability, not *failure* probability. The two probabilities are different, as a failure does not necessarily result in an error.[2] Also, our analysis does not take into account *compensating* failures[3] [5].

Definition Reliability R_S *of a memory system* S *is defined as the probability that all words in the memory system can be accessed correctly.*

A data word contains k bits, therefore, the probability that a *given* word in the triplex system can be accessed correctly is [3]

$$R^*_{triplex} = \left[(1-p)^3 + 3p(1-p)^2\right]^k = (1-p)^{2k}(1+2p)^k \tag{1}$$

Therefore, reliability of the triplex system is given by $R_{triplex} = (R^*_{triplex})^W$.

3. Reliability of Duplex Systems

This section evaluates reliability of three different types of duplex systems and compares them with the triplex system.

3.1 Duplex system using error detecting codes

In this section we show that the reliability achieved by a duplex system, using an ECC for error detection *only*, is always less than the triplex system. Let the reliability of the duplex system under consideration in this section be denoted by $R_{duplex1}$. Each memory module in the duplex system uses an (n, k) error detecting code. Let P_u denote the probability that an

[2] For example, a stuck-at-0 failure will not cause an error if the memory location contained 0.

[3] Compensating failures occur in the triplex system, for example, when a bit in one module is stuck-at-0 and the corresponding bit in another module is stuck-at-1. The system will produce correct output in this situation, even though two modules are faulty.

undetected error occurs in a codeword of this code. The function of the voter in this system is as follows: When a word is to be read from the memory, the corresponding codewords from the two memory modules are provided as input to the voter. The voter decodes the two codewords to detect errors. If errors are detected in both codewords, then the voter does not produce any data. If exactly one codeword is detected to contain an error, then the other decoded codeword is produced as output. If neither codeword is detected to contain an error, then any one decoded codeword is produced as the output. This voter will maximize the reliability under the constraint that each codeword is to be used only for error detection. With such a voter, the reliability is given by $R_{duplex1} = (R^*_{duplex1})^W$, where

$$\begin{aligned} R^*_{duplex1} &= (1-p)^{2n} + 2\left(1 - P_u - (1-p)^n\right)(1-p)^n + \frac{1}{2}2(1-p)^n P_u && (2) \\ &= (2 - P_u)(1-p)^n - (1-p)^{2n} && (3) \end{aligned}$$

In Equation 2, $(1-p)^{2n}$ is the probability that both codewords are error-free. The term $2\left(1 - P_u - (1-p)^n\right)(1-p)^n$ is the probability that one of the codewords contains a detectable error and the other codeword is error-free. The term $\frac{1}{2}2(1-p)^n P_u$ corresponds to the probability that one of the codewords contains an undetectable error, the other codeword is error-free and the voter chooses the error-free codeword. Note that in this situation, the voter will choose the error-free codeword with probability $\frac{1}{2}$.

The theorem below states that triplex memory reliability is larger than that of a duplex system using ECC for error detection only.

Theorem 1 *When $0 < p < 1/2$, $R_{duplex1}$ is smaller than $R_{triplex}$ independent of the error detecting code used in the duplex system.*

To be able to prove the theorem, we first prove the following lemma.

Lemma 1 $(1-p)^{2k-n}(1+2p)^k + (1-p)^n > 2$ *provided* $0 < p < 1/2$, $n \geq 2$ *and* $k + 1 \leq n$.

Proof: Assume that $0 < p < 1/2$, $n \geq 2$ and $k + 1 \leq n$. Let function $g(k, n, p) = (1-p)^{2k-n}(1+2p)^k + (1-p)^n$. Then,

$$\frac{\partial g}{\partial k} = \frac{(1-p)^{2k}(1+2p)^k}{(1-p)^n} \ln[(1-p)^2(1+2p)].$$

As $0 < p < 1/2$, $0 < (1-p)^2(1+2p) < 1$, and it follows that $\frac{\partial g}{\partial k} < 0$. Thus, g is a monotonically decreasing function of k. Therefore, we will choose the largest possible value of k, i.e., $k = n-1$, and show that g is larger than 2 for this value of k. When $k = n-1$, we have $g(n-1,n,p) = (1-p)^{n-2}(1+2p)^{n-1} + (1-p)^n = [(1-p)(1+2p)]^{n-2}(1+2p) + (1-p)^n$. Let function $f(n,p) = g(n-1,n,p)$. Thus, our goal now is to prove that $f(n,p) > 2$. Now,

$$\frac{\partial f}{\partial n} = (1-p)^{n-2}(1+2p)^{n-1}\ln[(1-p)(1+2p)] + (1-p)^n \ln(1-p).$$

To find the extrema of f, we set $\frac{\partial f}{\partial n} = 0$. This implies that, at the extrema (i.e. minima or maxima),

$$(1+2p)^{n-1} = -(1-p)^2 \, \frac{\ln(1-p)}{\ln[(1-p)(1+2p)]}.$$

This equation can hold for only one real value of n. Thus, there exists only one extrema of f with respect to n. Looking at $f(n,p)$ it is clear that by increasing n, $f(n,p)$ can be made arbitrarily large.[4] Therefore, the above extrema must be a minimum. Let the minimum occur at $n = n^*$. Two cases can occur: (a) $n^* > 2$ and (b) $n^* \leq 2$. We consider the two cases separately.

Case (a) $n^* > 2$ **:** In this case, we have

$$(1+2p)^{n^*-1} = -(1-p)^2 \, \frac{\ln(1-p)}{\ln[(1-p)(1+2p)]} \tag{4}$$

and $f(n,p) \geq f(n^*,p)$. Our goal now is to prove that $f(n^*,p) > 2$. From Equation 4, we get

$$(1-p)^{n^*} = -(1-p)^{n^*-2}(1+2p)^{n^*-1} \, \frac{\ln[(1-p)(1+2p)]}{\ln(1-p)}.$$

Substituting this expression into $f(n^*,p)$, we get

$$\begin{aligned} f(n^*,p) &= (1-p)^{n^*-2}(1+2p)^{n^*-1} - (1-p)^{n^*-2}(1+2p)^{n^*-1} \, \frac{\ln[(1-p)(1+2p)]}{\ln(1-p)} \\ &= (1-p)^{n^*-2}(1+2p)^{n^*-1} \, \frac{\ln(1+2p)}{-\ln(1-p)} \;=\; [(1-p)(1+2p)]^{n^*-2}(1+2p)\frac{\ln(1+2p)}{-\ln(1-p)} \\ &> (1+2p)\frac{\ln(1+2p)}{-\ln(1-p)}, \qquad \text{because } n^* > 2 \text{ and } (1-p)(1+2p) > 1 \text{ for } 0 < p < 1/2 \end{aligned}$$

[4] Observe that, for $0 < p < 1/2$, $(1-p)(1+2p) = 1 + p(1-2p) > 1$.

Define function $h(p) = (1+2p)\ln(1+2p)+2\ln(1-p)$. $h(p) > 0$ implies that $(1+2p)\frac{\ln(1+2p)}{-\ln(1-p)} > 2$ which in turn (by the above inequality) implies that $f(n^*, p) > 2$. Therefore, our goal now is to prove that $h(p) > 0$.

Note that $h(0) = 0$ and $h(1/2) = 0$. Also function h is differentiable in $[0, 1/2]$. Therefore, by Rolle's theorem [2], at least one extrema (maxima or minima) exists between $p = 0$ and $p = 1/2$. Now, $\frac{dh}{dp} = 2 + 2\ln(1 + 2p) - 2/(1 - p)$ and $\frac{d^2h}{dp^2} = \frac{4}{1+2p} - \frac{2}{(1-p)^2}$. Note that for $p = 0$, $\frac{dh}{dp} = 0$ and $\frac{d^2h}{dp^2} > 0$. Thus, h has a minimum at $p = 0$ and at least one maximum in [0,1/2] (by Rolle's theorem). Let the maximum closest to 0 occur at p_{max}. As $p = p_{max}$ is a maximum, $\frac{d^2h}{dp^2}$ must be negative at p_{max}. $\frac{d^2h}{dp^2}$ is a decreasing function of p, therefore, it will remain negative for $p > p_{max}$. This implies that in the interval $(p_{max}, 1/2)$, no minimum exists. This in turn implies that between 0 and 1/2, there exists only one maximum and no minimum. As $h(0) = h(1/2) = 0$, it implies that $h(p) > 0$ for $0 < p < 1/2$. This implies that $f(n^*, p) > 2$. Now, $2 < f(n^*, p) \leq f(n, p) \leq g(k, n, p)$. Therefore, $g(k, n, p) = (1 - p)^{2k-n}(1 + 2p)^k + (1 - p)^n > 2$.

Case (b) $n^* \leq 2$ **:** Observe that the range of interest for parameter n is $n \geq 2$. If n^* is no larger than 2, then in the range of interest, function $f(n, p)$ will be minimized at $n = 2$, i.e. $f(n, p) \geq f(2, p)$. Therefore, our goal in this case is to prove that $f(2, p) > 2$. Now,

$$\begin{aligned} f(2,p) &= (1 + 2p) + (1 - p)^2 = 2 + p^2 \\ &> 2 \quad \text{because } 0 < p < 1/2. \end{aligned}$$

This implies that $f(n, p) > 2$. As $f(n, p) \leq g(k, n, p)$, we have $g(k, n, p) = (1 - p)^{2k-n}(1 + 2p)^k + (1 - p)^n > 2$. □

Proof of Theorem 1

If $n = k$, then it is clear that all errors in a codeword will be undetected. In other words, P_u in Equation 3 is $1-(1-p)^n$. It can easily seen that, in this case, $R_{triplex} > R_{duplex1}$. Now we assume that $n \geq k + 1$. As $k \geq 1$, this implies that $n \geq 2$. To summarize, we have $n \geq 2$, $k + 1 \leq n$ and $0 < p < 1/2$. Under these conditions, the result proved in Lemma 1 is applicable. Therefore, we have

$$(1 - p)^{2k-n}(1 + 2p)^k + (1 - p)^n \quad > \quad 2$$

$$
\begin{aligned}
\Longrightarrow (1-p)^{2k}(1+2p)^k + (1-p)^{2n} &> 2(1-p)^n \\
\Longrightarrow (1-p)^{2k}(1+2p)^k + (1-p)^{2n} &> (2-P_u)(1-p)^n, \quad \text{because } P_u \geq 0 \\
\Longrightarrow (1-p)^{2k}(1+2p)^k &> (2-P_u)(1-p)^n - (1-p)^{2n} \\
\Longrightarrow R^*_{triplex} &> R^*_{duplex1} \quad \text{by Equations 1 and 3} \\
\Longrightarrow R_{triplex} &> R_{duplex1}
\end{aligned}
$$

3.2 Duplex systems using single error correcting (SEC) codes

In this section, we assume that the error control code used in the duplex system can correct a single error and not detect any other errors. In other words, it is assumed that more than one error will result in incorrect decoding of this code. In the next section, we will consider a single error correcting and double error detecting code.

For the duplex system considered here, the voter function is as follows: The voter decodes the two codewords and corrects any error that may be detected. Then, it outputs any one of the decoded codewords. This voter will maximize the reliability under the constraint that each codeword can be used only to correct a single error and that more than one error in a codeword causes erroneous decoding.

Let the reliability of the duplex system being considered in this section be denoted by $R_{duplex2}$. A given word can be accessed correctly when the two codewords contain at most one error each. In the case where one of the codewords has at most one error and the other codeword contains more than one error, there is a 50% chance that the correct information will be obtained (recollect that multiple errors in a codeword are not detected). When both codewords contain more than one error, correct information cannot be obtained. Therefore, $R_{duplex2} = (R^*_{duplex2})^W$ where

$$
\begin{aligned}
R^*_{duplex2} &= ((1-p)^n + np(1-p)^{n-1})^2 \\
&\quad + \frac{1}{2}2((1-p)^n + np(1-p)^{n-1})\;(1-(1-p)^n - np(1-p)^{n-1}) \\
&= (1-p)^n + np(1-p)^{n-1}
\end{aligned}
$$

The above expression is identical to the reliability that would be obtained if just one memory module with a single error correcting code were used (instead of two). This implies that when the error control code is *only* capable of correcting a single error, it does not help to use two memory modules.

Theorem 2 *When $0 < p < 1/3$, $R_{duplex2}$ is smaller than $R_{triplex}$ independent of the single error correcting code used in the duplex system, provided $k > 1$. When $k = 1$, $R_{duplex2}$ can equal $R_{triplex}$.*

Proof: The number of checkbits in the (n, k) code is $r = n - k$. When $k = 1$, the triplex system essentially implements a single error correcting code using a total of 3 bits. Therefore $R_{duplex2}$ with $k = 1$ and $r = 2$ is identical to $R_{triplex}$ with $k = 1$.

It is not possible to design a single error correcting code with just one checkbit. Therefore, $r \geq 2$. Also, it is not possible to design a single error correcting code for $k > 1$ with $r = 2$. For $k \leq 4$, r may be equal to 3. For $k > 4$, r must be at least 4 for any single error correcting code. We consider the case of $r \geq 4$ first followed by $r = 3$.

Case 1: $r \geq 4,\ 0 < p < 1/3$ **:** To prove the theorem, we first derive three inequalities.

$$\begin{aligned}(1-p)^k(1+2p)^k &= [1+p(1-2p)]^k = \sum_{i=0}^{k}\binom{k}{i}[p(1-2p)]^i \\ \Longrightarrow (1-p)^k(1+2p)^k &\geq 1+kp(1-2p) \quad \text{as } k > 0 \qquad (5)\end{aligned}$$

$$\begin{aligned}1 &= (p+(1-p))^r = \sum_{i=0}^{r}\binom{r}{i}p^i(1-p)^{r-i} \\ \Longrightarrow 1 &> (1-p)^r + rp(1-p)^{r-1} \quad \text{as } r \geq 4 \qquad (6)\end{aligned}$$

When $0 < p < 1/3$, $1 - 2p > (1-p)^3$. Also, $(1-p)^3 > (1-p)^i$ for $i \geq 4$. Therefore, for $r \geq 4$, $(1-2p) > (1-p)^{r-1}$. By multiplying both sides of the inequality by $k\,p$, we get

$$k\,p(1-2p) > k\,p(1-p)^{r-1} \qquad (7)$$

By replacing the two terms on right hand side of inequality 5 by right hand sides of inequalities 6 and 7, respectively, we get

$$(1-p)^k(1+2p)^k > (1-p)^r + rp(1-p)^{r-1} + kp(1-p)^{r-1}$$

$$\Longrightarrow (1-p)^k(1+2p)^k > (1-p)^r + np(1-p)^{r-1} \quad \text{as } n = k+r$$

Multiplying both sides by $(1-p)^k$ and replacing $n = k+r$, we get

$$(1-p)^{2k}(1+2p)^k \;>\; (1-p)^n + np(1-p)^{n-1} \;\Longrightarrow\; R^*_{triplex} > R^*_{duplex2}$$

$$\Longrightarrow R_{triplex} \;>\; R_{duplex2}$$

Case 2: $r = 3$, $0 < p < 1/3$ **:** As discussed earlier, $r = 3$ implies that k can at most be 4. We consider each value of k separately. Note that

$$\frac{R^*_{duplex2}}{R^*_{triplex}} = \frac{(1-p)^n + np(1-p)^{n-1}}{(1-p)^{2k}(1+2p)^k} = \frac{(1-p)^r + np(1-p)^{r-1}}{(1-p)^k(1+2p)^k}$$

(i) $r = 3$, $k = 4$ **:** In this case, $n = 7$ and

$$\begin{aligned}\frac{R^*_{duplex2}}{R^*_{triplex}} &= \frac{(1-p)^3 + 7p(1-p)^2}{(1-p)^4(1+2p)^4} = \frac{1+6p}{(1-p)^2(1+2p)^4}\\ &= \frac{1+6p}{1+6p+9p^2-8p^3-24p^4+16p^6} = \frac{1+6p}{1+6p+p^2(9-8p-24p^2)+16p^6}\\ &< 1, \quad \text{because } 9-8p-24p^2 > 0 \text{ when } 0 < p < 1/3.\end{aligned}$$

Therefore, $R^*_{duplex2} < R^*_{triplex}$.

(ii) $r = 3$, $k = 3$ **:** In this case, $n = 6$. By following similar steps as above, we get

$$\begin{aligned}\frac{R^*_{duplex2}}{R^*_{triplex}} &= \frac{1+5p}{(1-p)(1+2p)^3} = \frac{1+5p}{1+5p+2p^2(3-2p-4p^2)}\\ &< 1, \quad \text{because } 3-2p-4p^2 > 0 \text{ when } 0 < p < 1/3.\end{aligned}$$

Therefore, $R^*_{duplex2} < R^*_{triplex}$.

Similar to above, for $k = 1$ and 2 also, it can be shown that $R^*_{duplex2} < R^*_{triplex}$ [6].

This implies that $R_{duplex2} < R_{triplex}$. □

Although the result stated above is proved for $0 < p < 1/3$, we *conjecture* that it holds true when $0 < p < 1/2$. In practice, p is expected to be much smaller that $1/3$, therefore, the above result is adequate for real applications.

3.3 Duplex systems using SEC-DED codes

This section shows that a duplex system using a single error correcting and double error detecting (SEC-DED) code can achieve reliability better than a triplex system. This is demonstrated with the help of an example. Assume that the voter for the duplex system using SEC-DED code functions as follows: It decodes the codeword from one of the memory modules and if zero or one error is detected in this codeword, the decoded codeword is produced as the output. If two errors are detected in this codeword, then the second codeword is decoded. In this case, the second decoded codeword is produced as output if it is detected to contain at most one error.

Let the reliability of the duplex system being considered here be denoted by $R_{duplex3}$. Then, $R_{duplex3} = (R^*_{duplex3})^W$ where,

$$R^*_{duplex3} = (1-p)^n + np(1-p)^{n-1} + \binom{n}{2} p^2 (1-p)^{n-2} \left((1-p)^n + np(1-p)^{n-1} \right)$$

Unlike the results presented in Theorems 1 and 2, in this case, the duplex system can achieve a better reliability than the triplex system. We illustrate this with an example. Assume that the ECC used in the duplex system is a (n, k) SEC-DED code obtained by (possibly) shortening the distance-4 extended Hamming code [4]. For the SEC-DED code, $n = 39$ when $k = 32$. For $k = 32$, Figure 2 plots the *un*reliability (i.e., 1−reliability) for duplex and triplex systems as a function of p. Similar plots are obtained for other values of k.

From the unreliability plots, it can be seen that for sufficiently small values of p, $(1 - R^*_{duplex3})$ is smaller than $(1 - R^*_{triplex})$, implying that reliability $R_{duplex3} = (R^*_{duplex3})^W$ is larger than $R_{triplex} = (R^*_{triplex})^W$ when p is small. Specifically, for $k = 32$, when p is smaller than 0.009, $R^*_{duplex3}$ is larger than $R^*_{triplex}$. There are two aspects to this issue: (a) In practice, given realistic failure rates, the value of p is likely to be small enough to meet this bound. (b) Secondly, the duplex system with SEC-DED code uses much fewer bits than the triplex system (i.e., $3k > 2n$ or $n < 3k/2$). It should be possible to construct a single error correcting-triple error detecting code with n less than $3k/2$. The duplex system using this code would achieve reliability higher than the triplex system for larger values of p. In Figure 2, observe that, for $p = 0.001$, $(1 - R^*_{duplex3})$ is an order of magnitude smaller than $(1 - R^*_{triplex})$. Thus, when p is small, the duplex system can achieve significantly better reliability than the triplex system.

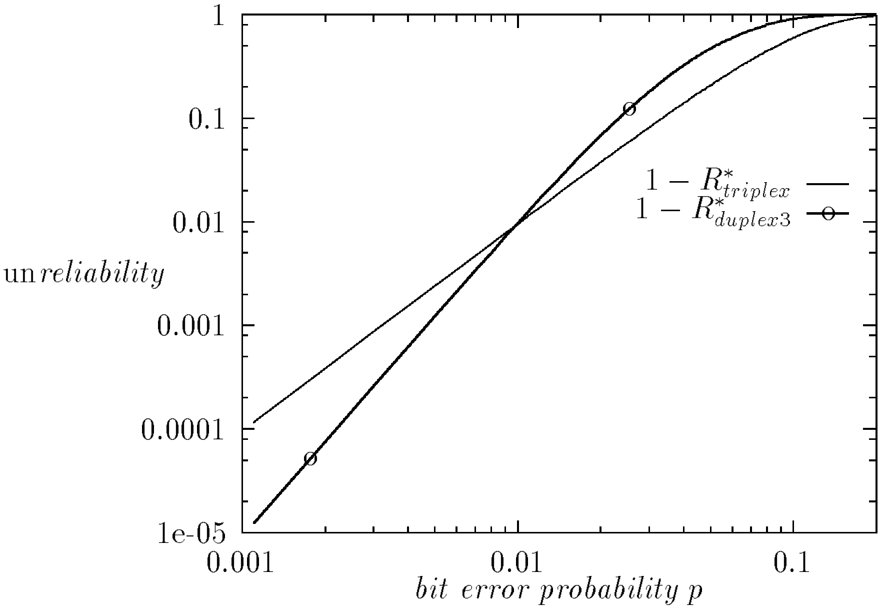


Figure 2: Comparison of $R^*_{duplex3}$ and $R^*_{triplex}$ for $k = 32$ and $n = 39$

The objective here was to demonstrate that a duplex system with a SEC-DED code can achieve reliability higher than the triplex system. We have shown this to be true provided the error probability p is small enough.

4. Discussion

This paper compares reliability of *triplex* memory systems formed using simple triplication (without ECC) with *duplex* memory systems formed by duplicating memory modules that use ECC. Reliability of a duplex system is shown to be worse than the triplex system if the ECC used in the duplex system is capable of only error detection or only single error correction. It is also shown that if the ECC is capable of single error correction as well as double error detection, for small bit error probability p, the duplex system achieves higher reliability than the triplex system. From these results the following conclusion can be drawn:

A necessary *condition, for the duplex system to be able to achieve higher reliability than the triplex system, is that the error control code must at least be capable of single error correction and double error detection (SEC-DED).*

The duplex system (depending on the complexity of the voter circuits and the size of the memory) is also expected to be more efficient in terms of chip area requirement.

Voter reliability was not taken into account in our analysis. In reality, the voter for the duplex system (including decoders) can be expected to be less reliable than that for the triplex system due to greater hardware complexity. Therefore, in practice, a SEC-DED code may not be *sufficient* for the duplex system to achieve a higher reliability than the triplex system (even when bit error probability is small); a code with greater capability may be required. Further work is necessary to evaluate the impact of voter reliability.

Acknowledgements

We thank Prasad Padmanabhan for his help in proving the correctness of Lemma 1. Thanks are also due to the referees for their helpful comments.

References

[1] P. A. Bernstein, "Sequoia: A fault-tolerant tightly coupled multiprocessor for transaction processing," *Computer*, pp. 37–45, February 1988.

[2] W. E. Boyce and R. C. DiPrima, *Calculus.* John Wiley & Sons, Inc., 1988.

[3] B. W. Johnson, *Design and Analysis of Fault Tolerant Digital Systems.* Addison-Wesley, 1989.

[4] T. R. N. Rao and E. Fujiwara, *Error-Control Coding for Computer Systems.* Prentice-Hall, 1989.

[5] D. P. Siewiorek, "Reliability modeling of compensating module failures in majority voted redundancy," *IEEE Trans. Computers*, vol. C-24, pp. 525–533, May 1975.

[6] N. H. Vaidya, "Duplex and triplex memory: Which is more reliable?," Tech. Rep. 94-025, Computer Science Department, Texas A&M University, College Station, February 1994.

[7] N. H. Vaidya and D. K. Pradhan, "Fault-tolerant design strategies for high reliability and safety," *IEEE Trans. Computers*, vol. 42, pp. 1195–1206, October 1993.