Connectivity and Capacity of Multi-Channel Wireless Networks with Channel Switching Constraints

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Abstract

This paper argues for the need to address the issue of multi-channel network performance under constraints on channel switching. We present examples from emergent directions in wireless networking to motivate the need for such a study, and introduce some models to capture channel switching constraints. For some of these models, we study connectivity and capacity of a wireless network comprising n randomly deployed nodes, equipped with a single interface each, when there are $c = O(\log n)$ channels of equal bandwidth $\frac{W}{c}$ available. We consider an adjacent (c, f) channel assignment where a node may switch between f adjacent channels, but the adjacent channel block is randomly assigned. We show that the *per-flow* capacity for this channel assignment model is $\Theta(W\sqrt{\frac{f}{cn\log n}})$. We then show how the adjacent $(c,2)$ assignment maps to the case of untuned radios. We also consider a random (c, f) assignment where each node may switch between a pre-assigned random subset of f channels. For this model, we prove that *per-flow* capacity is $O(W\sqrt{\frac{p_{rnd}}{n\log n}})$ (where $p_{rnd} = 1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$) and $\Omega(W\sqrt{\frac{f}{cn\log n}})$.

I. Introduction

Earlier work on protocols for multi-channel wireless networks [1] has assumed that each node is capable of switching on all channels. This assumption may be challenged by emerging paradigms in wireless networking, such as envisioned large-scale deployment of extremely inexpensive wireless devices embedded in the environment, and dynamic spectrum access via cognitive radio. We briefly summarize some such scenarios:

- The need for low-cost, low-power radio transceivers to be used in inexpensive sensor nodes can give rise to many situations involving constrained switching. Hardware complexity (and hence cost),

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and/or power consumption may be significantly reduced if each node operates only in a small spectral range, and switches between a small subset of adjacent channels (e.g., if the transceiver uses an oscillator with limited tunability). However, if more spectrum is available than a single device can utilize, it may be possible at time of manufacture to lock different devices on to different frequency ranges. Also, potentially a transceiver may have an RF channel selector comprising *a bank of switchable µmechanical filters* [2], from which it may select one to use for transmission/reception.

- In the case of cognitive radio, given a multi-hop network of secondary users attempting to utilize unused spectrum, some channels may be locally unavailable to some nodes due to the presence of an active primary user in the vicinity.
- In wide-area mesh network deployments with a large number of available channels, different nodes may see different SINR conditions on different channels, and poor channel quality may render some channels unattractive to use, and *effectively unusable* over significant time-periods.

Thus, there is need to address the issue of multi-channel network performance in the presence of constraints on channel-switching, both in terms of determining how asymptotic transport capacity is affected by the constraints, and designing protocols for efficient channel-coordination, and data-transfer.

It has been proposed in [3] that extremely inexpensive wireless devices can be manufactured if it is possible to handle untuned radios whose operating frequency may lie randomly within some band. Also considered in [3] is the possibility that each device may have a small number of such untuned radios, and a random network coding based approach is proposed to relay information between a single source-destination pair. Some work on cognitive radio has addressed the issue of coordination in the face of restricted and variable channel availability at individual nodes due to active primary users [4], [5].

However, no formal theoretical models have been developed for the various types of switching constraints encountered in these previous works, and in other anticipated scenarios, and the impact of the constraints on network performance in a general multi-hop setting has not been quantified.

In this paper we present an initial foundation for this domain by introducing some models for constrained channel assignment, and exploring issues of connectivity and transport capacity for some of these models.

We consider an adjacent (c, f) channel assignment, and show that the *per-flow* capacity for this case is $\Theta(W\sqrt{\frac{f}{cn\log n}})$. We then establish a mapping between the adjacent $(2c+2,3)$ and adjacent $(4c+1,2)$ models and the case of untuned radios [3], and obtain asymptotic capacity results for untuned radios with random source-destination pairs. We also study a random (c, f) assignment, where each node is assigned a random subset of f channels at time of deployment, and can thereafter only switch between these channels. For this model, we prove that *per-flow* capacity is $O(W\sqrt{\frac{p_{rnd}}{n\log n}})$ (where $p_{rnd} = 1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$) and $\Omega(W\sqrt{\frac{f}{cn\log n}})$.

We also introduce and briefly discuss a spatially correlated channel assignment model.

II. Some Models for Constrained Channel Assignment

In this section we elaborate on some of the models for constrained channel assignment that we propose. These models assume that nodes possess only one interface each, there are c channels available, and all

channels are orthogonal. However, they may potentially be extended to the case where multiple interfaces are available at each node.

A. Adjacent (c, f) Assignment

We introduce an assignment model wherein a node can switch between a set of f contiguous channels $(2 \leq f \leq c)$. Thus, if the frequency band is divided into c channels numbered 1, 2, ..., c in order of increasing frequency, then, at deployment time, each node is assigned a block location i uniformly at random from $\{1,...,c-f+1\}$ and thereafter it can switch between the set $\{i,...,i+f-1\}$. This model is relevant when each individual node has a tranceiver with limited tunability, and thus may only switch between a small set of contiguous channels. It is also possible to establish a mapping between adjacent $(2c+1,2)$ assignment, and the case of untuned radios.

B. Random (c, f) Assignment

In this assignment model, a node is assigned a subset of f channels $(2 \leq f \leq c)$ uniformly at random from the set of all possible channel subsets of size f. This model can capture situations where tiny low-cost sensor nodes may be equipped with a transceiver having a bank of f filters (e.g., such a design has been proposed in [2]). It is possible to envisage scenarios where each filter operates on some random channel determined at time of manufacture.

C. Spatially Correlated Channel Assignment

In this model, a set of N pseudo-nodes is placed uninformly at random in the network, in addition to the regular network nodes. Each of these pseudo-nodes is assigned a randomly chosen channel. All network nodes within a distance R of a pseudonode with assigned channel i are blocked from using channel i. This model captures channel unavailability due to an active primary user in the vicinity in cognitive radio networks, as well as situations where an external source of noise leads to poor channel quality in a certain region.

While these models assume a single-interface at each node, they may be extended to cover the case where each node has m interfaces. It is also possible to envision situations involving a combination of these models, e.g., a node may have a bank of filters, each capable of tuning on a few channels in a contiguous sub-band, with each filter operating on some random sub-band.

III. Network Model

In the assumed network model, n nodes are located uniformly at random in a unit area toroidal region. Nodes use a common transmission range $r(n)$. Interference is modeled using the Protocol Model [6]. There are c available channels of bandwidth $\frac{W}{c}$ each. We focus on the case where the total number of available channels $c = O(\log n)$. We believe this is justified because in large scale deployments, the number of nodes will typically be much larger than the number of available channels. Besides, when $c = \omega(\log n)$, there is a huge capacity degradation even with unconstrained channel switching (as shown

in [1]), thus making channelization an increasing liability, and constrained switching can only lead to additional degradation, and potentially unacceptable performance. [1]

As in [6], each node is source of exactly one flow. It chooses a point uniformly at random (we shall refer to these points as *pseudo-destinations* throughout this paper), and selects the node (other than itself) lying closest to that point as its destination.

IV. Notation and Terminology

We use the following asymptotic notation:

- $f(n) = O(g(n))$ means that $\exists c, N_o,$ such that $f(n) \leq cg(n)$ for $n > N_o$
- $f(n) = o(g(n))$ means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$
- $f(n) = \omega(g(n))$ means that $g(n) = o(f(n))$
- $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$
- $f(n) = \Theta(g(n))$means that $\exists c_1, c_2, N_o,$ such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for $n > N_o$

When $f(n) = O(g(n))$, any function $h(n) = O(f(n))$ is also $O(g(n))$. We often refer to such a situation as $h(n) = O(f(n)) \implies O(g(n))$.

As in [6], we say that the per flow network capacity is $\lambda(n)$ if each flow in the network can be guaranteed a throughput of at least λ. Whenever we use log without explicitly specifying the base, we imply the *natural* logarithm.

V. Some Useful Results

Theorem 1: (Vapnik-Chervonenkis Theorem) Let S be a set with finite VC dimension VCdim($\mathcal{S}$). Let $\{X_i\}$ be i.i.d. random variables with distribution P. Then for $\varepsilon, \delta > 0$:

$$Pr\left(\sup_{D\in\mathcal{S}} \left|\frac{1}{N}\sum_{i=1}^{N} I_{X_i\in D} - P(D)\right| \leq \varepsilon\right) > 1-\delta$$

$$\text{whenever } N > \max\left(\frac{8\text{VCdim}(\mathcal{S})}{\varepsilon}\log_2\frac{16e}{\varepsilon}, \frac{4}{\varepsilon}\log_2\frac{2}{\delta}\right)$$

Theorem 2: (Chernoff Bound[7]) Let $X_1, ..., X_n$ be independent Poisson trials, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^{n} X_i$. Then, for any $\beta > 0$:

$$Pr[X \geq (1+\beta)E[X]] < \left(\frac{e^{\beta}}{(1+\beta)^{(1+\beta)}}\right)^{E[X]} \tag{1}$$

Theorem 3: (Chernoff Upper Tail Bound[7]) Let $X_1, ..., X_n$ be independent Poisson trials, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^{n} X_i$. Then, for $0 < \beta \leq 1$:

$$Pr[X \geq (1+\beta)E[X]] \leq \exp(-\frac{\beta^2}{3}E[X]) \tag{2}$$

[1]We believe that the described capacity constructions in this paper can be easily extended upto larger values of c, and are in the process of doing so

Theorem 4: (Chernoff Lower Tail Bound[7]) Let $X_1, ..., X_n$ be independent Poisson trials, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^{n} X_i$. Then, for $0 < \beta < 1$:

$$Pr[X \leq (1-\beta)E[X]] \leq \exp(-\frac{\beta^2}{2}E[X]) \tag{3}$$

Lemma 1: [7] When n balls are thrown independently and uniformly at random into n bins, then $Pr[$ any bin has $> \frac{3\log n}{\log\log n}$ balls $] \leq \frac{1}{n}$ for sufficiently large n.

Lemma 2: If m balls are thrown into b bins independently and uniformly at random, then $Pr[$ any bin has $> \frac{2m}{b}$ balls $] \leq b \cdot exp(-\frac{m}{3b})$.

Proof: For bin i, let I_{i1}, I_{i2}, ..., I_{im} be indicator variables indicating whether ball j ($j = 1, 2, ..., m$) fell into bin i. Then, $Pr[I_{ik} = 1] = \frac{1}{b}$. Let $X_i = \sum I_{ik}$. Then $E[X_i] = \frac{m}{b}$. By application of the Chernoff bound from Theorem 3 (setting $\beta = 1$), we then obtain that $Pr[X > \frac{2m}{b}] \leq Pr[X \geq \frac{2m}{b}] \leq exp(-\frac{m}{3b})$. Thereafter, application of the union bound yields that $Pr[$ any bin has $> \frac{2m}{b}$ balls $] \leq b \cdot exp(-\frac{m}{3b})$. ■

Lemma 3: Suppose we are given a unit toroidal region with n points located uniformly at random, and the region is sub-divided into axis-parallel square cells of area $a(n)$ each. If $a(n) = \frac{100\alpha(n)\log n}{n}, 1 \leq \alpha(n) \leq \frac{n}{100\log n}$, then each cell has at least $100\alpha(n)\log n - 50\log n \geq 50\alpha(n)\log n$ points and at most $100\alpha(n)\log n + 50\log n \leq 150\alpha(n)\log n$ points, with probability at least $1 - \frac{50\log n}{n}$.

Proof: We know that the set of axis-parallel squares has VC-dimension 3. In our construction, we have a set of axis-parallel square cells $\mathcal{S}$ such that the cells all have area $a(n) = \frac{100\alpha\log n}{n}$. Then considering the n random variables X_i denoting node positions, $Pr[X_i \in D (D \in \mathcal{S}] = \frac{100\alpha\log n}{n}$. Then, from the VC-theorem (Theorem 1):

$$Pr\left(\sup_{D\in\mathcal{S}} |\frac{\text{No. of nodes in} D}{n} - \frac{100\alpha(n)\log n}{n}| \leq \varepsilon(n)\right) > 1 - \delta(n)$$
$$\text{whenever } n > \max\left(\frac{24}{\varepsilon}\log_2\frac{16e}{\varepsilon}, \frac{4}{\varepsilon}\log_2\frac{2}{\delta}\right)$$

This is satisfied when $\varepsilon(n) = \delta(n) = \frac{50\log n}{n}$. Thus, with probability at least $1 - \frac{50\alpha(n)\log n}{n}$, the population $Pop(D)$ of cell D satisfies:

$$\frac{50\alpha(n)\log n}{n} \leq 100\alpha(n)\log n - 50\log n \leq Pop(D) \leq 100\alpha(n)\log n + 50\log n \leq \frac{150\alpha(n)\log n}{n} \tag{4}$$

■

Lemma 4: Suppose n points are thrown uniformly at random on the unit torus. Let us consider the set of all circles of radius R and area $A(n) = \pi R^2$ on the unit toroid. If $A(n) = \frac{100\alpha(n)\log n}{n}, 1 \leq \alpha(n) \leq \frac{n}{100\log n}$, then each circle has at most $150\alpha(n)\log n$ points, with probability at least $1 - \frac{50\log n}{n}$.

Proof: The set of all circles of radius R in the plane has VC-dimension 3. Thereafter by the same argument as in the proof of Lemma 3, the result proceeds. ■

Lemma 5: If n pairs of points (P_i, Q_i) are chosen uniformly at random in the unit area network, the resultant set of straight-line formed by each pair $L_i = P_iQ_i$ satisfies the condition that no cell has more than $n\sqrt{a(n)}$ lines passing through it.

Proof: Given the lines L_i are i.i.d., the proof of Lemma 3 in [8] can be applied to prove this result. ■

VI. Related Work

It was shown by Gupta and Kumar [6] that for a single-channel single-interface scenario, in an arbitrary network, the per flow capacity scales as $\Theta(\frac{W}{\sqrt{n}})$ bit-m/s per flow, while in a random network, it scales as $\Theta(\frac{W}{\sqrt{n\log n}})$ bits/s. It was also shown in [6] that if the available bandwidth W is split into c channels, with each node having a dedicated interface per channel, the results remain the same.

It was shown in [9] that mobility can increase the capacity, and in fact $\Theta(1)$ throughput per flow is attainable when each node is source and destination for exactly one flow each. The capacity of hybrid networks (those having some infrastructure support in the form of access points) was studied in [10] and [11].

The throughput-delay trade-off was studied in [8], and it was shown that the optimal trade-off is given by $D(n) = \Theta(nT(n))$ where $D(n)$ is delay, and $T(n)$ is throughput. The capacity of ultra-wideband (UWB) networks was studied in [12], and [13].

In the multi-channel context, an interesting scenario arises when the number of interfaces m at each node may be smaller than the number of available channels c. This issue was analyzed in [1] and it was shown that the capacity results are a function of the channel-to-interface ratio $\frac{c}{m}$. It was also shown that in the random network case, there are three distinct capacity regions: when $\frac{c}{m} = O(\log n)$, the per-flow capacity is $\frac{W}{\sqrt{n\log n}}$, when $\frac{c}{m} = \Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the per flow capacity is $\Theta(W\sqrt{\frac{m}{nc}})$, and when $\frac{c}{m} = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, the per-flow capacity is $\Theta(\frac{Wm\log\log n}{\log n})$.

Another relevant body of work is that on bond percolation in wireless networks, e.g. [14]. The constrained assignments considered by us also lead to nodes within range being able to communicate only with a certain probability. However, unlike percolation, in our case the probabilities are not independent for all pairs of nodes. Besides, percolation results deal with existence of an unbounded cluster, whereas our goal is to have full connectivity.

A multi-channel multi-hop network architecture has been considered in [15] in which each node has a single transceiver, and nodes have a *quiescent* channel to which they tune when not transmitting. A node wishing to communicate with a destination tunes to its quiescent channel, and transmits the packet to a neighbor whose quiescent channel is the same as that of the destination. Thereafter, the packet proceeds towards the destination on the quiescent channel. This has some similarity to our model and

constructions in that a flow seeks to transition to a target destination channel (see Sections X and XIV for our constructions). However, in their case, the transition can happen trivially at the very first hop, since the source node is always capable of tuning to the destination's quiescent channel, while in our models nodes can only switch on some channels, and this needs to be taken into account.

Situations in which some channels may be unavailable to some nodes has been considered in some work on cognitive radio. An area-blocking model (with a notion of a *protected radius* around a primary user) similar to that in proposed spatially correlated assignment is considered in [16]. However, the goal of that work is not to determine multi-hop capacity. In [5], a model is considered where channel-sets of neighboring nodes may differ by at most k channels. This is also similar in spirit to the spatially correlated assignment, in that nearby nodes tend to have limited variability in their channel-sets. As mentioned earlier, none of these works has focued on obtaining a formal model of such anticipated spatially correlated constraints for connectivity and capacity analysis.

VII. Upper Bounds on Capacity

Some general constraints on the capacity of the network (for any channel assignment model) are as follows:

a) Source-Destination Constraint for $f = 1$: If $f = 1$, but $c > 1$, then a source and its destination should have the same channel for communication between them to be possible. This may not always happen if the channels are assigned randomly. To illustrate, consider the class of assignment models where the assignment to individual nodes is i.i.d. Suppose, $Pr[\text{i and } dst(i) \text{ share a channel }] \leq p$. If the traffic model is such that any single node can be the destination of only upto $D(n)$ flows, then we argue thus:

We can obtain at least $\lfloor \frac{n}{2D(n)} \rfloor$ pairs with distinct nodes (thus leading to independent probabilities). The probability that at least one of the n source-destination pairs have different channels can be lower bounded by the probability that at least one of these distinct pairs do not share a common channel, and this is at least $1 - p^{\lfloor \frac{n}{2D(n)} \rfloor}$. When $\log\left(\frac{1}{p}\right) = \omega(\frac{2D(n)}{n})$, it grows to 1, as $n \to \infty$. Thus, the network capacity would be 0. For the adjacent (c, f) and random (c, f) assignments studied in this paper, when $c > 1$, this condition holds, and so $f = 1$ when $c > 1$ yields zero capacity.

When $f > 1$, as in the rest of this paper, this constraint does not apply.

b) Connectivity Constraint: This was first formulated in [6]. Suppose the necessary condition for connectivity is that $r(n) = \Omega(g(n))$. Thus, the spatial re-use in the network is limited to $O(\frac{1}{(g(n))^2})$ concurrent transmissions on any single channel. Besides, each source-destination is separated by average $\Theta(1)$ distance (see [6] for details) and hence average $\Theta(\frac{1}{r(n)})$ hops. Thus per flow throughput is limited to $O(\frac{W}{nr(n)})$.

c) Interference Constraint: In [1], it was established that the per flow capacity is constrained to $O(W\sqrt{\frac{1}{cn}})$, when single-interface nodes can switch to any channel. This upper bound still applies to the adjacent (c, f)-assignment case, since whatever is achievable with adjacent (c, f) assignment, is also achievable when nodes can switch to any channel.

d) Destination Bottleneck Constraint: This was first formulated in [1]. If the traffic model is such that some node can be the destination of upto $D(n)$ flows, the per-flow throughput is constrained to be $O(\frac{W}{D(n)})$.

Note that since we are only interested in the region $c = O(\log n)$, the connectivity constraint is asymptotically dominant, and shall drive the upper bounds, as well as the lower bound constructions presented for the various channel assignment models.

VIII. Adjacent (c, f) Channel Assignment

Recall that in this model, the frequency band is divided into c channels numbered 1, 2, ..., c in order of increasing frequency, but an individual node can only use f channels ($2 \leq f \leq c$). At deployment time, each node is assigned a block location i uniformly at random from $1, ..., c-f+1$ and thereafter it can switch between the set $i, ..., i+f-1$. Thus, the probability that a node is capable of switching to channel i is given by $p_s^{adj}(i) = \frac{\min\{i, c-i+1, f, c-f+1\}}{c-f+1}$, since channel i occurs in $\min\{i, c-i+1, f, c-f+1\}$ blocks, and each block is randomly chosen with probability $\frac{1}{c-f+1}$. When the minimum is f, $p_s^{adj}(i) = \frac{f}{c-f+1} > \frac{f}{c}$. When the minimum is $c-f+1$, $p_s^{adj}(i) = 1$. When the minimum is i, then for $i \geq \lceil \frac{f}{2} \rceil$, $p_s^{adj}(i) \geq \frac{f}{2c}$. When the minimum is $c-i+1$, then for $i \leq c - \lfloor \frac{f}{2} \rfloor$, $p_s^{adj}(i) \geq \frac{f}{2c}$.

Let us call channels with $p_s^{adj}(i) \geq \frac{f}{2c}$ the *preferred* channels. Then, it can be seem that for any set of f contiguous channels, at least $\lceil \frac{f}{2} \rceil$ of the channels have $p_s^{adj}(i) \geq \frac{f}{2c}$. Hence, each node can switch on $x \geq \lceil \frac{f}{2} \rceil \geq \frac{f}{2}$ *preferred* channels. Also note that non-preferred channels only occur at the fringes of the frequency band.

The probability that a node with block location i shares a channel with another randomly chosen node is given by $p_{adj}(i) = \frac{(1+\min\{i-1, f-1\}+\min\{c-f+1-i, f-1\})}{c-f+1}$. Since block locations are chosen uniformly at random from $1, ..., c-f+1$, the probability that two randomly chosen nodes share at least one channel is given by:

$$p_{adj} = \frac{1}{c-f+1} \sum_{i=1}^{c-f+1} p_{adj}(i) \tag{5}$$

It can be seen that $\frac{\min\{f, c-f+1\}}{c-f+1} \leq p_{adj}(i) \leq \frac{\min\{2f-1, c-f+1\}}{c-f+1}$. Thus, $\frac{\min\{f, c-f+1\}}{c-f+1} \leq p_{adj} \leq \frac{\min\{2f-1, c-f+1\}}{c-f+1}$.

A. Necessary Condition for Connectivity

An adaptation of the proof techniques used to obtain the necessary condition for connectivity in [17], enables one to handle connectivity with adjacent (c, f) assignment. The following lemma was stated and proved in [17].

Lemma 6: (i) For any $p \in [0, 1]$

$$(1-p) \leq e^{-p}$$

(ii) For any given $\theta \geq 1$, there exists $p_0 \in [0, 1]$, such that

$$e^{-\theta p} \leq (1-p), \forall 0 \leq p \leq p_0$$

If $\theta > 1$, then $p_0 > 0$.

Proof: See Lemma 2.1 in [17]. ■

Lemma 7: If $\pi r^2(n) = \frac{(\log n + b)}{pn}$, then, for any fixed $\theta < 1$:

$$n(1 - p\pi r^2(n))^{(n-1)} \geq \theta e^{-b} \tag{6}$$

for sufficiently large n.

Proof: The proof proceeds from the proof of Lemma 2.2 in [17] by replacing $\pi r^2(n)$ with $p\pi r^2(n)$ everywhere. For the sake of clarity, the proof is elaborated in the appendix. ■

Theorem 5: With an adjacent (c, f) channel assignment (when $c = O(\log n)$), if $p = \min\{\frac{2f-1}{c-f+1}, 1\}$, and $\pi r^2(n) = \frac{(\log n + b(n))}{pn}$, where $b = \lim_{n\to\infty} b(n) < +\infty$ then:

$$\liminf_{n\to\infty} Pr[\text{ disconnection }] \geq e^{-b}(1 - e^{-b}) > 0$$

where by *disconnection* we imply the event that there is a partition of the network.

Proof:

The probability that two nodes in range of each other share a channel is $\min\{\frac{f}{c-f+1}, 1\} \leq p_{adj} \leq \min\{\frac{2f-1}{c-f+1}, 1\}$ Note that $p_{adj}(i)$ is different for different block locations i primarily because nodes with locks at the fringes of the band are less likely to share channels with each other. Since, we are deriving a *necessary* condition for connectivity, it is possible to make the following assumption for the purpose of this proof: channel pairs $(i, c-f+i+1), 1 \leq i \leq f-1$ possess magical capabilities, such that communication on channel i ends up being visible on channel $c-f+i+1$,and vice-versa. Thus, if a node has channel i, then it can also communicate with a node that does not share any channel with it, but has channel $c-f+i+1$. Another way to view this situation is that although nodes are assigned channels as per the adjacent (c, f) model, at time of network operation, a node having channel $c-f+i+1, 1 \leq i \leq f-1$ uses channel i instead (i.e., $c-f+i+1$ serves as an alias for i). Under this assumption, $p_{adj}(i) = \min\{\frac{2f-1}{c-f+1}, 1\}, \forall i$. If the network is disconnected under this assumption, then it must necessarily be so otherwise. This can be seen thus: suppose we are given a network instance with nodes assigned adjacent channels as per the adjacent (c, f) model, and we then impose the assumption stated above. Suppose this network is disconnected. Now the imposed assumption is removed, but the channel block assigned to each node remains unchanged. Then, in the new scenario, some nodes that were earlier able to communicate, will not be able to do so anymore; however those nodes that were incapable of communicating will preserve their status quo. Thus, the necessary condition would remain valid.

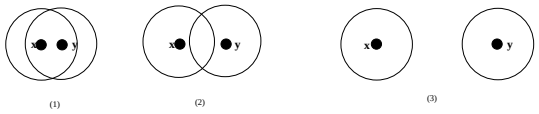

Fig. 1. Three Cases: Necessary Condition for Connectivity

This proof is an adaptation of a similar proof in [17]. We focus on the disconnection event where singleton sets are partitioned from the rest of network. Recall that $p = \min\{\frac{2f-1}{c-f+1}, 1\}$. When $f \geq \frac{c+2}{3}$, $p = 1$, i.e. any pair of nodes that are within range can communicate with each other, and the necessary condition result from [17] applies directly. Thus we need to consider only $f < \frac{c+2}{3}$ for which $p = \frac{2f-1}{c-f+1}$. Also note that:

$$\begin{aligned} \pi r^2(n) \leq \frac{2\log n}{pn} \leq \frac{2\alpha \log^2(n)}{n} \\ (\because p \geq \frac{1}{c-f+1} > \frac{1}{c} \text{ and} \\ c \leq \alpha \log n \text{ for some constant } \alpha \text{ and} \\ b(n) < \log n \text{ for large } n \cdot\because \limsup b(n) < +\infty) \end{aligned} \tag{7}$$

The probability that a node x is isolated, i.e., cannot communicate with any neighbor is give by $p_1 = (1 - p\pi r^2(n))^{(n-1)}$. Consider the event that nodes x and y are both isolated. There are three different cases for this (also see Fig. 1): (1) x and y lie within distance $r(n)$ of each other, but do not share a common channel (2) x and y do not lie within distance $r(n)$ of each other, but have overlapping neighborhood regions, i.e. lie within distance $2r(n)$ of each other (3) their neighborhood regions are disjoint, i.e., the distance between them is greater than $2r(n)$.

Then, the probability that both x and y are isolated is given by the probability that they cannot communicate with each other, and none of the other $n-2$ nodes can communicate with either of them.

Let us first consider case (1), i.e., the distance between x and y is $d(n) \leq r(n)$. We view it as two sub-cases, viz., (i) y is at distance $d(n) \leq r'(n) = \left(\frac{16\log\log n}{\log n}\right) r(n)$ of x, and (ii) y is at distance $d(n) > r'(n) = \left(\frac{16\log\log n}{\log n}\right) r(n)$ of x. Also, note that the probability a node $z \neq x,y$ within range of both x and y is capable of communicating with either one of x and y, given that they cannot communicate with each other is $q \geq \frac{\min\{3f-1, c-f+1\}}{c-f+1}$ (note that when $f > \frac{c}{3}$, $q = 1$, as it is impossible to have three disjoint blocks of f channels each). Also, when $f \leq \frac{c+2}{4}, 3f-1 \leq c-f+1$, and $q \geq \frac{3p}{2}$.

For sub-case (i) of case (1), the overlap area between the neighborhoods of x and y is at least $(1-\delta)\pi r^2(n)$ for any $\delta > 0$ and large enough n, since the separation $r'(n) \leq \left(\frac{16\log\log n}{\log n}\right) r(n)$. For our purpose, it suffices to take $\delta = \frac{1}{5}$, yielding an overlap area of at least $\frac{4\pi r^2(n)}{5}$. Then the probability that a node can communicate with either x or y or both is at least q times the probability of lying in the overlap area.

Thus, the contribution of subcase (i) of case (1) to the probability that both x and y are isolated is upper-bounded as follows:

When $f \leq \frac{c+2}{4} \implies q \geq \frac{3p}{2}$:

$$\begin{aligned}
p_{21(i)} &\leq \pi r'^2(n)(1-p)\left(1-q\frac{4\pi r^2(n)}{5}\right)^{n-2} \\
&< \pi r^2(n)\left(1-\frac{4q\pi r^2(n)}{5}\right)^{n-2} \\
&\leq \pi r^2(n)\left(1-\frac{6p\pi r^2(n)}{5}\right)^{n-2} \\
&\leq \pi r^2(n)e^{-(n-2)\frac{6}{5}p\pi r^2(n)} \text{ from Lemma 6} \\
&\leq \frac{2\alpha\log^2 n}{n}e^{-(n-2)\frac{6(\log n+b(n))}{5n}} \\
&\leq e^{-\frac{6(\log n+b(n))}{5}+\frac{12(\log n+b(n))}{5n}+\log 2\alpha+2\log\log n-\log n} \\
&\leq e^{-\frac{11\log n}{5}-\frac{6b(n)}{5}+\frac{12(\log n+b(n))}{5n}+\log 2\alpha+2\log\log n} \\
&\leq e^{-\frac{21\log n}{10}-b(n)} \text{ for large } n
\end{aligned} \tag{8}$$

When $f > \frac{c+2}{4}$, $p = \min\{\frac{2f-1}{c-f+1}, 1\} \geq \frac{1}{2}, \forall c \geq 2$. For this situation, we merely consider the probability that one of the remaining $n-2$ nodes can communicate with one of x and y (say x) to obtain the upper bound on both x and y being isolated:

$$\begin{aligned}
p_{21(i)} &\leq \pi r'^2(n)(1-p)(1-p\pi r^2(n))^{n-2} \\
&\leq \frac{256(\log\log n)^2}{\log^2 n}\pi r^2(n)(1-p\pi r^2(n))^{n-2} \\
&\leq \frac{256(\log\log n)^2}{\log^2 n}\pi r^2(n)e^{-(n-2)p\pi r^2(n)} \text{ from Lemma 6} \\
&\leq \frac{256(\log\log n)^2}{\log^2 n}\frac{(\log n+b(n))}{pn}e^{-(n-2)\frac{(\log n+b(n))}{n}} \\
&\leq \frac{256(\log\log n)^2(2(2\log n))}{n\log^2(n)}e^{-(n-2)\frac{(\log n+b(n))}{n}} \\
&\leq e^{-\log n-b(n)+\frac{2(\log n+b(n))}{n}+\log 256+\log 4-\log n-\log\log n+2\log\log\log n} \\
&\leq e^{-2\log n-b(n)-\frac{1}{2}\log\log n} \text{ for large } n
\end{aligned} \tag{9}$$

Then, from Eqns. 8 and 9, for any f:

$$p_{21(i)} \leq e^{-2\log n-b(n)-\frac{1}{2}\log\log n} \text{ for large } n \tag{10}$$

For sub-case (ii), the situation is depicted in Fig. 2. The probability that some node can talk to either x or y is lower bounded by the probability that it lies in range of x ($\pi r^2(n)$) and shares a channel with it (p), or it lies out of range of x but within range of y (at least $\frac{\sqrt{3}r(n)r'(n)}{2}$ for large enough n [2]), and

[2]The area within range of y but out of range of x is given by $\pi r^2(n) -$ overlap area ; where overlap area = 2 (area of quadrant subtending angle $2\theta -$ area of $\triangle ABC$) $\leq \pi r^2(n) - r^2(n)\sin(2\theta)$. Note that $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$. Thus non-overlap area $\geq r^2(n)\sin(2\theta) = r^2(n)(2\sin\theta\cos\theta) = r^2(n)2\sin\theta\frac{d(n)}{2r(n)} \geq 2r^2(n)\sin\frac{\pi}{3}\frac{r'(n)}{2r(n)} \geq \frac{\sqrt{3}r(n)r'(n)}{2}$

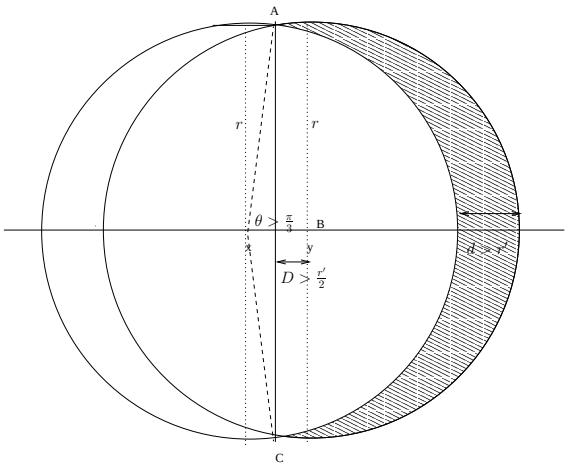

Fig. 2. First Case: Necessary Condition for Connectivity

shares a channel with y (p). The contribution to the probability that both x and y are isolated is thus at most:

$$\begin{aligned}
p_{21(ii)} &\leq (\pi r^2(n) - \pi r'^2(n))(1-p)(1-p(\pi r^2(n) \\
&\quad + \frac{\sqrt{3}r(n)r'(n)}{2}))^{n-2} \\
&\leq \pi r^2(n)\left(1-p\left(\pi r^2(n)+\frac{\sqrt{3}r(n)r'(n)}{2}\right)\right)^{n-2} \\
&\leq \pi r^2(n)\left(1-p\pi r^2(n)\left(1+\frac{\sqrt{3}r'(n)}{2\pi r(n)}\right)\right)^{n-2} \\
&\leq \pi r^2(n)\left(1-p\pi r^2(n)\left(1+\frac{8\sqrt{3}\log\log n}{\pi\log n}\right)\right)^{n-2} \qquad (11) \\
&\leq \pi r^2(n) e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})} \text{ from Lemma 6}(\because \pi < 2\sqrt{3})) \\
&\leq \frac{2\alpha\log^2 n}{n} e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})} \text{ from Eqn. 7} \\
&\leq e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})+\log 2\alpha+2\log\log n-\log n} \\
&\leq e^{-\log n-b(n)-4\log\log n+\frac{2(\log n+b(n))(1+\frac{4\log\log n}{\log n})}{n}+\log 2\alpha+2\log\log n-\log n} \\
&\leq e^{-2\log n-b(n)-\log\log n} \text{ for large } n
\end{aligned}$$

For case 2, the probability that some node can communicate with either x or y is lower bounded by the probability that it lies in range of x ($\pi r^2(n)$) and shares a channel with it (p), or it lies out of range of x but within range of y (disjunction of the two circles in Fig. 1 (2) is at least $\frac{1}{2}\pi r^2(n)$), and shares a channel with it. Thus the contribution of this case to the probability that both x and y are isolated is

upper bounded by:

$$\begin{aligned} p_{22} &= (4\pi r^2(n) - \pi r^2(n))(1 - \frac{3}{2}p\pi r^2(n))^{n-2} \\ &\leq 3\pi r^2(n) e^{-\frac{3(n-2)p\pi r^2(n)}{2}} \text{ from Lemma 6} \\ &\leq \frac{3(\log n + b(n)}{pn} e^{-(n-2)\frac{3(\log n + b(n))}{2n}} \\ &\leq \frac{6\alpha \log^2 n}{n} e^{-(n-2)\frac{3(\log n + b(n))}{2n}} \text{ from Eqn. 7} \\ &\leq e^{-\frac{3}{2}\log n - \frac{3}{2}b(n) + \frac{3(\log n + b(n))}{n} + \log 6\alpha + 2\log\log n - \log n} \\ &\leq e^{-\frac{9}{4}\log n - b(n)} \text{ for large } n \end{aligned} \tag{12}$$

For case 3, the probability that both x and y are isolated is upper bounded by:

$$\begin{aligned} p_{23} &= (1 - 4\pi r^2)(1 - p(2\pi r^2(n)))^{n-2} \\ &\leq (1 - 2p\pi r^2(n))^{n-2} \\ &\leq e^{-2(n-2)p\pi r^2(n)} \text{ from Lemma 6} \\ &\leq e^{-2\log(n) - 2b(n) + \frac{4(\log n + b(n))}{n}} \end{aligned} \tag{13}$$

Then, the probability p_2 that nodes x and y are both isolated is given by:

$$p_2 \leq p_{21(i)} + p_{21(ii)} + p_{22} + p_{23} \tag{14}$$

Let us first consider the case where $b(n) = b$ is a constant.

$$\begin{aligned} Pr[\text{ disconnection }] &\geq \sum_x Pr[x \text{ is only isolated node}] \\ &\geq \sum_x Pr[x \text{ isolated }] - \sum_{x,y} Pr[x \text{ and } y \text{ both isolated }] \\ &= np_1 - n(n-1)p_2 \\ &\geq n(1 - p\pi r^2(n))^{(n-1)} \\ &\quad - n(n-1)\left(p_{21(i)} + p_{21(ii)} + p_{22} + p_{23}\right) \\ &\geq \theta e^{-b} - n(n-1)(e^{-2\log n - b(n) - \frac{1}{2}\log\log n} \\ &\quad + e^{-2\log n - b - \log\log n} \\ &\quad + e^{-\frac{9}{4}\log n - b} + e^{-2\log n - 2b + \frac{4(\log n + b)}{n}}) \\ &\geq \theta e^{-b} - (1+\varepsilon)e^{-2b} \end{aligned} \tag{15}$$

for any $\theta < 1, \varepsilon > 0,$ and large n by Lemmas 6 and 7

Now, let us consider the case where $b(n)$ is not constant, and $\lim_{n\to\infty} b(n) = b$. Then, for any $\varepsilon > 0$, $b(n) - b \leq \varepsilon$ for large n. Since the probability of disconnection monotonically decreases in $b(n)$, we can take the following bound:

$$Pr[disconnection] \geq \theta e^{-(b+\varepsilon)} - (1+\varepsilon)e^{-2(b+\varepsilon)} \qquad (16)$$
$$(\text{ for large enough } n)$$

Thus, if $\limsup_{n\to\infty} b(n) < +\infty$, the network is asymptotically disconnected with some positive probability. ■

Corollary 1: With an adjacent (c, f) assignment, the necessary condition for connectivity is that $r(n) = \Omega(\sqrt{\frac{c\log n}{fn}})$, else the network is disconnected with some positive probability.

Proof: Whenever $f \geq \frac{c+2}{3}$, $p = 1 < \frac{3f}{c}$ in Theorem 5, and the necessary condition require $\pi r^2(n) > \frac{\log n}{n} > \frac{c\log n}{3fn}$. Whenever, $f < \frac{c+2}{3}$, $p = \frac{2f-1}{c-f+1} \leq \frac{3f}{c}$, and the necessary condition again requires that $\pi r^2(n) > \frac{c\log n}{3fn}$. Hence with adjacent (c, f) assignment, connectivity requires that $r(n) = \Omega(\sqrt{\frac{c\log n}{fn}})$. Note that this can be viewed as $r(n) = \Omega(\sqrt{\frac{\log n}{p_{adj}n}})$. ■

B. Sufficient Condition for Connectivity

It can be shown that having $r(n) = a_1\sqrt{\frac{\log n}{p_{adj}n}} = a_1\sqrt{\frac{c\log n}{fn}}$, for some suitable constant a_1, suffices for connectivity. This will be evident from our lower bound construction for capacity, and the proof is hence not presented separately.

IX. Adjacent (c, f) Assignment: Capacity Upper Bound

A. Upper Bound on Capacity

We proved that the necessary condition for connectivity implies $r(n) = \Omega(\sqrt{\frac{c\log n}{fn}})$. Then by the connectivity constraint mentioned in Section VII, the per flow throughput is limited to $O(W\sqrt{\frac{f}{cn\log n}})$ (recall that, as in [17], the disconnection events we considered involved individual nodes getting isolated, and thus some source node would be unable to communicate with its destination).

X. Adjacent (c, f) Assignment: Capacity Lower Bound

We present a constructive proof that achieves $\Omega(W\sqrt{\frac{f}{cn\log n}})$. This construction has similarity to the constructions in [6], [8], and [1], but must now handle the constraint that a node may not switch on all channels. The surface of the unit torus is divided into square cells of area $a(n)$ each. The transmission range $r(n)$ is set to $\sqrt{8a(n)}$, thereby ensuring that any node in a given cell is within range of any other node in any adjoining cell. Since we utilize the *Protocol Model* [6], a node C can potentially interfere with an ongoing transmission from node A to node B, only if $BC \leq (1+\Delta)r(n)$. Thus, a transmission by A in a given cell can only be affected by transmissions in cells with some point within a distance $(2+\Delta)r(n)$ from it, and all such cells must lie within a circle of radius $O((1+\Delta)r(n))$. Since Δ is independent of n, the number of cells that interfere with a given cell is only some constant (say β).

We choose $a(n) = \frac{100c\log n}{fn}$ (i.e. $r(n) = \sqrt{\frac{800c\log n}{fn}}$). Then by Lemma 3, the number of nodes in any cell lies between $\frac{50c\log n}{f}$ and $\frac{150c\log n}{f}$ with probability at least $1 - \frac{50\log n}{n}$.

Lemma 8: If there are at least $\frac{50c\log n}{f}$ nodes in every cell D, then there are at least $12\log n$ nodes in each cell on each of the *preferred* channels, with probability at least $1 - q_1$, where $q_1 = O(\frac{1}{n^2})$.

Proof: Let us consider one particular cell D, with $x_D \geq \frac{50c\log n}{f}$ nodes. Let $X_{ij} = 1$ if node j is on channel i, and 0 else. $Pr[X_{ij} = 1] = p_s^{adj} > \frac{f}{2c}$ if i is a *preferred* channel. Let $X_i = \sum_{i \in D} X_{ij}$. Then $E[X_i] \geq 25\log n$. By application of the Chernoff bound in Theorem 4 (with $\beta = \frac{1}{2}$) , we obtain:

$$\begin{aligned} Pr[X_i \leq 12\log n] &\leq Pr[X_j \leq \frac{25}{2}\log n] \\ &\leq \exp(-\frac{25\log n}{8}) \leq \frac{1}{n^{\frac{25}{8}}} \end{aligned} \tag{17}$$

Since there are at most $c = O(\log n)$ *preferred* channels, the union bound yields that $Pr[X_j \leq 25\log n$ for any preferred $j] \leq \frac{c}{n^{\frac{25}{8}}} = O(\frac{\log n}{n^{\frac{25}{8}}})$. Also, since there are $\frac{1}{a(n)} = \frac{fn}{100c\log n} \leq n$ cells, another application of the union bound yields that:

$$Pr[X_i < 12\log n \text{ in any cell }] = O(\frac{1}{n^2}) \tag{18}$$

■

From Lemma 3, each cell has at least $\frac{50c\log n}{f}$ nodes w.h.p. Thus, a union bound argument (further elaborated in Section XV) can be invoked to show that each cell has at least $12\log n$ nodes on every *preferred* channel w.h.p.

Lemma 9: If there are at least $\frac{50c\log n}{f}$ nodes in every cell D, then, for all adjacent *preferred* channels i and $i+1$, there are at least $12\log n$ nodes in the cell having both channels i and $i+1$, with probability at least $1-q_2$, where $q_2 = O(\frac{1}{n^2})$.

Proof: Let us consider one particular cell D with number of nodes $x_D \geq \frac{50c\log n}{f}$. Let $X_{ij} = 1$ if node j can switch on both channel i and $i+1$ (where both i and $i+1$ are *preferred*), and 0 else. Then $Pr[X_{ij} = 1] \geq \frac{\lceil \frac{f}{2} \rceil}{c-f+1} \geq \frac{f}{2c}$. Let $X_i = \sum_{j \in D} X_{ij}$. Then $E[X_i] \geq 25\log n$. By application of the Chernoff bound from Theorem 4 (with $\beta = \frac{1}{2}$) , we obtain:

$$\begin{aligned} Pr[X_i \leq 12\log n] &\leq Pr[X_i \leq \frac{25}{2}\log n] \\ &\leq \exp(-\frac{25\log n}{8}) \leq \frac{1}{n^{\frac{25}{8}}} \end{aligned} \tag{19}$$

Applying the union bound over the maximum possible $c-1 = O(\log n)$ possibilities for i, we obtain that $Pr[X_i \leq 12\log n$ for any preferred $i, i+1] \leq \frac{(c-1)}{n^{\frac{25}{8}}} = O(\frac{\log n}{n^{\frac{25}{8}}})$. Also, there are $\frac{1}{a(n)} = \frac{fn}{100c\log n} < n$ cells, and thus another application of the union bound yields that:

$$Pr[X_i \leq 12\log n \text{ in any cell}] = O(\frac{1}{n^2}) \tag{20}$$

■

From Lemma 3, each cell has at least $\frac{50c\log n}{f}$ nodes w.h.p. Thus, each cell has at least $12\log n$ nodes on every pair of adjacent *preferred* channels $(i, i+1)$ w.h.p.

Lemma 10: If there are at least $\frac{50c\log n}{f}$ nodes in every cell, and if i and $i+x$ are both *preferred* channels, where $x \leq \lfloor \frac{f}{2} \rfloor$, then there are at least $12\log n$ nodes in the cell having both channels i and $i+x$, with probability at least $1-q_3$, where $q_3 = O(\frac{1}{n^2})$.

Proof: A node can switch on both i and $i+x$, if its block location lies between $i+x-f+1$ and i. This probability is $\frac{f-x}{c-f+1}$. Since $x \leq \lfloor \frac{f}{2} \rfloor$, this probability is at least $\frac{\lceil \frac{f}{2} \rceil}{c-f+1} \geq \frac{f}{2c}$. Thereafter the proof argument is the same as that of Lemma 9. ■

A. Routing

Let us denote the source of a flow as S, the pseudo-destination as D', and the actual destination as D. If there were no constraints on switching, we could have used a routing strategy similar to that in [6], in which a flow traverses the cells intersected by the straight line SD', and thereafter needs to take at most one extra-hop to reach the actual destination D, which must necessarily lie either in the same cell as D' or in one of the 8 adjacent cells. If that were the case, it can be claimed that:

Lemma 11: The number of SD'D routes that traverse any cell is $O(n\sqrt{a(n)})$.

Proof: From Lemma 5 we know that the number of SD' straight-lines traversing a single cell are $O(n\sqrt{a(n)})$ We must now consider the number of routes whose last D'D hop may enter this cell. If D is in the same cell as D', there is no extra hop. Otherwise, the number of flows for which D' lies in one of the 8 adjacent cells is $O(na(n))$ w.h.p. from Lemma 3. Thus the total number of traversing routes is $O(n\sqrt{a(n)})$. ■

We shall hereafter refer to this routing as straight-line routing, as it basically comprises a straight-line except for the last hop.

Lemma 12: No node is the destination of more than $O(\log n) \implies O(na(n))$ flows.

Proof: Consider that a flow's pseudo-destination falls in a certain cell D. Consider a circle of radius $\sqrt{\frac{100\log n}{\pi n}}$, and hence area $\frac{100\log n}{n}$ centered around this pseudo-destination. Then, this circle falls entirely within cell D and the 8 cells adjacent to cell D, and from Lemma 4, all such circles contain $\Theta(\log n)$ nodes w.h.p. In the worst-case, one of these nodes could potentially be the source node for that flow. However, the circle still has more than one node other than the flow's source. Thus, the flow will select as its destination, some node within this circle. Hence a flow can only be assigned a destination within distance $\sqrt{\frac{100\log n}{\pi n}}$ from its pseudo-destination. Thus it proceeds that a node can only be the destination for flows whose pseudo-destination lies within a distance $\sqrt{\frac{100\log n}{\pi n}}$ from it. From Lemma 4, each circle of this size contains $O(\log n)$ pseudo-destinations w.h.p. Thus no node is the destination of more than $O(\log n$ flows. Given our choice of $a(n)$, it proceeds that $O(\log n) \implies O(na(n))$. This yields the result. It is also easy to see that a circle of radius $\sqrt{\frac{100\log n}{\pi n}}$ centered at a node will fall completely within the 8 cells adjacent to the node's cell. Hence if a node is destination of some flow, that flow's pseudo-destination must lie within either the same cell, or an adjacent cell.

■

In case of adjacent (c, f) assignment, we cannot stipulate that *all* flows be routed along the (almost) straight-line path SD'D. This is because the flow is required to traverse a minimum number of hops to

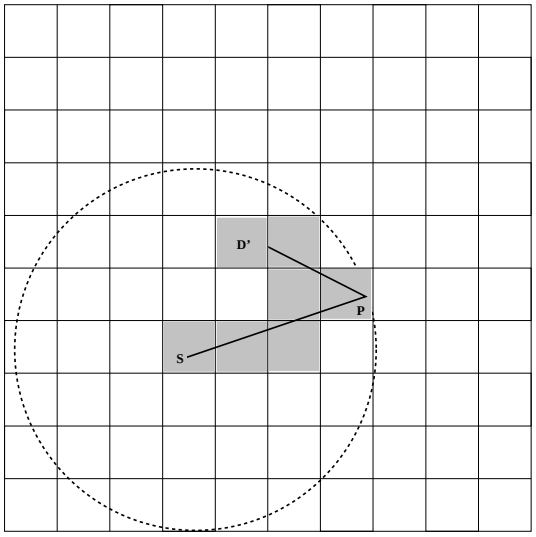

Fig. 3. Illustration of detour routing

be able to guarantee that it can switch from source channel to destination channel. We elaborate further on this issue.

Channel Selection and Transition Strategy: Initially, after each source has chosen a random destination, the flows are sequentially processed and each is assigned an initial source channel, as well as a target destination channel. The destination channel may be chosen from amongst the f channels available at destination D in any manner, e.g., it may be the one with the smallest number of incoming flows assigned to it so far.

Suppose the source S of a flow is assigned channel set $(i,...,i+f-1)$, while the destination D has $(j,...,j+f-1)$. The flow chooses one of the $x \geq \frac{f}{c}$ *preferred* channels available at the source uniformly at random. Let us denote it by l. It also chooses one of the $y \geq \frac{f}{2}$ *preferred* channels available at the destination (let us call it r) as the channel on which the flow reaches the destination. The destination channel choice may be made in any manner, e.g. we may make an i.i.d. choice amongst all channels available at the destination. We assume, without loss of generality, that $l \leq r$. Suppose $r-l = k'\lfloor\frac{f}{2}\rfloor + m(0 \leq m < \lfloor\frac{f}{2}\rfloor)$. Thus $k' = \frac{r-l-m}{\lfloor\frac{f}{2}\rfloor} \leq \frac{c-1}{\frac{f-1}{2}} = \frac{2(c-1)}{f-1} \leq \frac{4c}{f}$. Note that given two preferred channels l and r all channels $l \leq i \leq r$ must also necessarily be preferred. Then, from Lemma 10, it is always possible to transition from l to r in at most $k'+1 \leq$ steps: $l \rightarrow l+\lfloor\frac{f}{2}\rfloor, l+\lfloor\frac{f}{2}\rfloor \rightarrow l+2\lfloor\frac{f}{2}\rfloor, ..., l+k'\lfloor\frac{f}{2}\rfloor \rightarrow l+k'\lfloor\frac{f}{2}\rfloor + m = r$. Thus, the route passes through a sequence of nodes $x_1, x_2, ...x_k$ such that x_1 and x_2 share channel l, x_2 and x_3 share channel $l+\lfloor\frac{f}{2}\rfloor$ and so on. When $l \geq r$, the transitions are of the form $l \rightarrow l-\lfloor\frac{f}{2}\rfloor, ..., r$.

Thus, we stipulate that the straight-line path be followed if either the chosen source and destination channels are the same, or if the straight-line segment SD' comprises $h \geq \frac{4c}{f}$ *intermediate* hops. If S and D' (hence also D) lie close to each other, the hop-length of the straight line cell-to-cell path can be much smaller. In this case, a *detour* path is chosen. Consider a circle of radius $\frac{4c}{f}r(n)$ centered at S. Choose

a point on this circle, say P. In the considered $c = O(\log n)$ regime, P can be any point on the circle [3]. Then the route is obtained by traversing cells along SP and then PD'D. This ensures that the route has at least the minimum required hop-length (provided by segment SP). This situation is illustrated in Fig. 3.

A non-detour-routed flow is initially in a *progress-on-source-channel* mode, and keeps to the source channel till there are only $\frac{4c}{f}$ intermediate hops left to the destination. At this point, it enters *transition* mode, and starts making channel transitions along the remaining hops, till it has transitioned into its chosen destination channel. Thereafter, it remains on that channel. When a flow enters a cell in *progress-on-source-channel* mode, amongst all nodes in that cell capable of switching on that channel, it is assigned to the node which has the least number of flows assigned to it on that channel so far.

A detour-routed flow is always in *transition* mode.

Lemma 13: Given that the high probability event in Lemma 8 holds, suppose a flow is on *preferred* source channel i and needs to finally be on *preferred* destination channel j. Then after having traversed $h \geq \frac{4c}{f} + 1$ cells (recall that $2 \leq f \leq c$) , it is guaranteed to have made the transition.

Proof: By Lemma 10, each cell has at least $12 \log n$ nodes on each pair of preferred channels (i,x), for all $x \leq \lfloor \frac{f}{2} \rfloor$. Thus, given that the chosen source channel is l, the flow packets are transmitted on l on those hops where the flow is in *progress-on-source-channel* mode. When the flow moves into *transition* mode, the first relay node in this phase chooses as first hop a node having channel pair $(l, l + \lfloor \frac{f}{2} \rfloor)$ in the next cell, and transmits flow packets to it using channel l. This node then chooses a next hop having channel pair $(l + \lfloor \frac{f}{2} \rfloor, l + 2\lfloor \frac{f}{2} \rfloor)$, and sends packets to it over channel $l + \lfloor \frac{f}{2} \rfloor$, and the process continues till the flow has found a transition into the chosen destination channel r. This requires at most $\frac{4c}{f}$ intermediate hops, which are obtained by traversing at most $\frac{4c}{f} + 1$ cells. Once the transition to destination channel r is done, flow packets are transmitted on channel r for the remaining hops (if any) to the destination. ■

Lemma 14: The length of any route increases by only $O(\frac{c}{f}) \implies O(\log n)$ hops due to detour routing. The average route length increases by $O(\log n)$ hops.

Proof: The proof proceeds directly from the *detour* routing strategy. ■

Lemma 15: If the number of distinct flows traversing any cell is x in case of pure straight-line routing, it is at most $x + O(n\frac{c^2}{f^2}r^2(n)) \implies x + O(\log^4 n)$ even with detour routing [4].

Proof: Since the detour occurs only upto a circle of radius $\frac{4c}{f}r(n)$ around the source, the extra detour-routed flows that may pass through a cell (compared to straight-line routing) are only those whose sources lie within a distance $\frac{4c}{f}r(n)$ from this cell. Thus all such possible sources fall within a circle of radius $(\frac{4c}{f} + 1)r(n)$, and hence area $a_c(n) = \pi(\frac{4c}{f} + 1)^2 r(n)^2$. Then from Lemma 4, with high probability, any circle of this radius will have $O(na_c(n))$ nodes, and hence at most $O(na_c(n))$ sources. Thus the

[3] We are currently working on extending this work to larger values of c. In that regime it may be beneficial to choose P as per some specific rule, e.g. point of intersection of circle and horizontal line through S

[4] This is a loose upper bound. The actual number of detour-routed flows traversing a cell is much smaller.

number of detour-routed flows that traverse the cell is $O(na_c(n))$, and the total number of flows is $x + O(n\frac{c^2}{f^2}r^2(n)) \implies x + O(\log^4 n)$ w.h.p. ■

Lemma 16: The number of distinct flows traversing any cell is $O(n\sqrt{a(n)})$ even with detour routing.

Proof: Note that $na_c(n) = O(\log^4 n)$. Since the number of flows passing through cell C in the straight-line routing case is $O(n\sqrt{a(n)}) = O(\sqrt{\frac{cn\log n}{f}})$, the number of flows through a cell C is at most $O(\sqrt{\frac{cn\log n}{f}}) + O(\log^4 n) = O(\sqrt{\frac{cn\log n}{f}}) = O(n\sqrt{a(n)})$. ■

Lemma 17: The number of flows traversing any cell in *transition* mode is $O(\log^4 n)$ w.h.p.

Proof: First let us account for the SD' stretch of each flow, without considering the possible additional last hop. We account for it explicitly later in this proof.

By our construction, a non-detour routed flow enters the *transition* mode only when it is $\frac{4c}{f}$ intermediate hops away from its destination. All such flows must have their pseudo-destinations within a circle of radius $\Theta(\frac{c}{f}r(n))$ centered in the cell. The number of destinations that lie within a circle of radius $\Theta(\frac{c}{f})r(n)$ from the cell is $\Theta(n(\frac{c}{f})^2 r^2(n)) \implies O(\frac{c^3}{f^3}\log n)$ w.h.p., (by suitable choice of $\alpha(n) = O(\frac{c^3}{f^3})$ in Lemma 4). Thus the number of non-detour routed flows that may traverse a cell is $O(\frac{c^3}{f^3}\log n)$.

A detour-routed flow is always in *transition* mode. By Lemma 15, there are $O(\log^4 n)$ such flows traversing any cell. Each such flow can only traverse a cell twice along the SD' stretch. This yields $O(\log^4 n)$ detour-routed flows (including repeat traversals).

Also, the cell may be re-traversed by some flows on their additional last hop. By an argument similar to Lemma 12, there are $O(na(n))$ pseudo-destinations in the adjacent cells, and thus $O(na(n)) = O(\frac{c\log n}{f}) \implies O(\log^2 n)$ such last hop flow traversals. Thus the number of flows transitioning in any cell is $O(\frac{c^3}{f^3}\log n)) + O(\log^4 n) + O(\log^2 n)$. Also $c = O(\log n)$. Hence all channels have $O(\log^4 n)$ transitioning flows in the cell w.h.p. ■

B. Balancing Load within a Cell

Per-Channel Load: Recall that each cell has $O(na(n))$ nodes w.h.p., and $O(n\sqrt{a(n)})$ flows traversing it w.h.p. Since nodes are contrained in terms of channel switching, the assignment algorithm of [1] does not apply here.

Lemma 18: The number of flows that enter any cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: A flow may be on a channel i in the cell if (1) the flow's source channel is i and it is *progress-on-source-channel* mode, or (2) it is in *transition* mode, and transitioning through i or the flow's destination channel is i, and it has already made a transition.

Note that from our construction, and by our choice of $a(n)$, each flow initially stays on its chosen source channel, and when within c hops of the destination, makes one transition in the right direction (up or down in frequency) at each hop till it transitions into its destination channel. Thus a flow is on its source channel in a given cell if its destination is more than c hops away. Since each source chooses a block of f contiguous channels in an i.i.d. manner, and thereafter chooses one channel uniformly from $x \geq \lceil \frac{f}{2} \rceil \geq \frac{f}{2}$

preferred channels in this channel block, the probability that a non-transitioning flow is on a particular *preferred* channel i is at most $\left(\frac{2}{f}\right)\left(\frac{\min\{f,c-f+1,i,c-i+1\}}{c-f+1}\right) \leq \left(\frac{2}{f}\right)\left(\frac{\min\{f,c-f+1\}}{c-f+1}\right) = \frac{2}{\max\{f,c-f+1\}} \leq \frac{4}{c}$.

Thus, we know that flows in the *progress-on-source channel* mode (i.e. non-transitioning flows) are on a given *preferred* channel with probability at most $\frac{4}{c}$ and at least $\frac{1}{c}$. Also, there are $O(n\sqrt{a(n)}) = O(\sqrt{\frac{cn\log n}{f}})$ flows that traverse the cell. Note that if the number of non-transitioning flows traversing the cell is $m = O(\frac{n\sqrt{a(n)}}{c})$, then trivially in this cell no channel can have more than so many non-transitioning flows. So we shall focus on the case where the number of traversing non-transitioning flows is $m = \omega(\frac{n\sqrt{a(n)}}{c}) = \omega(\sqrt{\frac{n\log n}{cf}})$. Let m be $\omega(\frac{n\sqrt{a(n)}}{c})$ and $O(n\sqrt{a(n)})$. Let X_{ij} be an indicator variable which is 1 if flow j enters the cell on channel i, and is 0 else. We know that $\frac{1}{c} \leq Pr[X_{ij}=1] \leq \frac{4}{c}$. Then $X_i = \sum_j X_{ij}$ denotes the number of non-transitioning flows that enter the cell on channel i, and $\frac{m}{c} \leq E[X_i] \leq \frac{4m}{c}$. The X_{ij}'s are i.i.d. random variables for a given i, as each flow's source channel is chosen in an i.i.d. manner (though they may not be independent for different i, since $X_{ij}=1 \implies X_{ik}=0\ \forall k \neq i$). Hence we may still apply the Chernoff bound from Lemma 3 (with $\beta = 1$), and obtain that:

$$Pr[X_i \geq \frac{8m}{c}] \leq Pr[X_i \geq 2E[X_i]] \leq \exp(-\frac{E[X_i]}{3}) \leq \exp(-\frac{m}{3c}) \tag{21}$$

Taking the union bound over the maximum possible c *preferred* channels, the probability that any channel has more than $\frac{8m}{c}$ flows is at most $c\exp(-\frac{m}{3c}) \leq \exp(-\frac{\omega(\frac{n\sqrt{a(n)}}{c})}{3c} + \log c) = \exp(-\frac{\omega(\sqrt{\frac{n\log n}{cf}})}{3c} + \log c) = \exp(-\omega(\sqrt{\frac{n\log n}{c^3 f}})) < \exp(-\omega(\sqrt{\frac{n}{\log^3 n}}))$. Taking union bound over all $\frac{1}{a(n)} = \frac{fn}{100c\log n}$ cells, this probability is less than $\frac{fn}{100c\log n} exp(-\omega(\frac{\sqrt{n}}{\log^3 n})) = \exp(-\omega(\frac{\sqrt{n}}{\log^3 n}))$. We have thus proved that the number of non-transitioning flows that enter any cell on a given channel is $O(\frac{n\sqrt{a(n)}}{c})$.

We now need to account for the fact that some of these flows may be in the *transition* mode, and may either be transitioning through an intermediate channel or may have transitioned to the destination channel. From Lemma 17 the number of such flows traversing the cell is $O(\log^4 n)$ w.h.p.

Hence the per-channel load in all cells is at most $O(\sqrt{\frac{n\log n}{c}}) + O(\log^4 n) = O(\sqrt{\frac{n\log n}{c}}) = O(\frac{n\sqrt{a(n)}}{c})$ w.h.p. ■

Lemma 19: The number of flows that leave any given cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: The flows that leave a cell fall into two categories (1) those that originate at some node in the cell (2) those that entered the cell but did not terminate there (i.e. were relayed through the cell). The former can be no more than the number of nodes in the cell, i.e. $\Theta(na(n)) = \Theta(\frac{c\log n}{f}) = \Theta(\log^2 n)$. For the latter, note that the flows that leave the cell, must then enter one of the 8 adjacent cells. Thus, the former can be no more than 8 times the maximum number of flows entering a cell on any one channel, which has been established as $O(\frac{n\sqrt{a(n)}}{c}) = O(\sqrt{\frac{n\log n}{cf}})$ in Lemma 18. Hence, the total number of flows leaving any given cell on a given channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p. ■

Per-Node Load:

Lemma 20: The number of flows that are assigned to any one node in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: A node is always assigned the single flow for which it is the source. A node is also assigned flows terminating in that cell and for which it is the destination, and there are at most $D(n) = O(\frac{c\log n}{f}) \implies O(\log^2 n)$ such flows for any node w.h.p.(from Lemma 12). Besides, a node may be assigned flows that are in the *transition* mode, and for which it provides the required adjacent channel pair, or (for flows that have completed the transition) the destination channel. From Lemma 17 there are $O(\log^4 n)$ such flows in a cell w.h.p. Thus a node has $O(\log^4 n)$ assigned flows in *transition* mode.

We now consider the flows that are in *progress-on-source-channel mode* and do not originate in the cell. We have already established in Lemma 18, that the number of flows that are routed on a given channel in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p. Also, non-transitioning flows are necessarily on a *preferred* channel. From Lemma 8, there are at least $12\log n$ nodes on each *preferred* channel in each cell w.h.p. When a flow needs to enter a cell on a certain channel, from amongst all nodes capable of switching on that channel, it is assigned to the node which has the least number of flows assigned on that channel so far. Thus, no node can have more than $O(\frac{n\sqrt{a(n)}}{c\log n})$ non-transitioning flows assigned on any single channel, and no more than $O(\frac{fn\sqrt{a(n)}}{c\log n}) \implies O(\frac{n\sqrt{a(n)}}{c})$ non-transitioning flows assigned overall (recall that $c = O(\log n)$, and $f \leq c$). Thus, the resultant number of assigned flows per node is $1 + D(n) + O(\log^4 n) + O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{n\sqrt{a(n)}}{c})$. ■

C. Transmission Schedule

We noted earlier that each cell can face interference from at most a constant number β of nearby cells. Thus, if we consider the resultant cell-interference graph, it has a chromatic number at most $1+\beta$. We can hence come up with a global schedule having $1+\beta$ unit time slots in each round. In any slot, if a cell is active, then all interfering cells are inactive. The next issue is that of intra-cell scheduling. We need to schedule transmissions so as to ensure that at any time instant, there is at most one transmission on any given channel in the cell. Besides, we also need to ensure that no node is expected to transmit or receive more than one packet at any time instant. We use the following procedure to obtain an intra-cell schedule:

We construct a conflict graph based on the nodes in the active cell, and its adjacent cells (note that the hop-sender of each flow shall lie in the active cell, and the hop-receiver shall lie in one of the adjacent cells), as follows: we create a separate vertex for each flow traversing the cell. Since the flow has an assigned channel on which it operates in that particular hop, each vertex in the graph has an implicit associated channel. Besides, each vertex has an associated pair of nodes corresponding to the hop endpoints. Two vertices are connected by an edge if (1) they have the same associated channel, or (2) at least one of their associated nodes is the same. The scheduling problem thus reduces to obtaining a vertex-coloring of this graph. If we have a vertex coloring, then it ensures that (1) a node is never simultaneously sending/receiving for more than one flow (2) no two flows on the same channel are active simultaneously. Thus, the number of neighbors of a graph vertex is upper bounded by the number of flows entering/leaving the active cell on that channel, and the number of flows assigned to the flow's two hop endpoints (both hop-sender and hop-receiver). Thus, it can be seen from Lemmas 18, 19 and 20 that

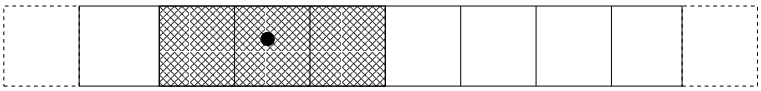

Fig. 4. Untuned Radios: Upper Bound via virtual $(2c+2,3)$ channelization

the degree of the conflict graph is $O(\frac{n\sqrt{a(n)}}{c})+O(\frac{n\sqrt{a(n)}}{c})=O(\frac{n\sqrt{a(n)}}{c})$. Since any graph with maximum degree d is colorable in at most $d+1$ colors, the conlict graph can be colored in $O(\frac{n\sqrt{a(n)}}{c})$ colors.

Thus the cell-slot is divided into $O(\frac{n\sqrt{a(n)}}{c})=O(\frac{\sqrt{\frac{cn\log n}{f}}}{c})$ equal length subslots, and all traversing flows get a slot for transmission. This yields that each flow will get $\Omega(W\sqrt{\frac{f}{cn\log n}})$ throughput.

We thus obtain the following theorem:

Theorem 6: With an adjacent (c,f)-channel assignment, the network capacity is $\Theta(W\sqrt{\frac{f}{cn\log n}})$ per flow.

XI. The Case of Untuned Radios

The untuned channel model is as follows: each node possesses a transceiver with carrier frequency uniformly distributed in the range (F_1,F_2), and admits a spectral bandwidth B. Let $c=\lfloor\frac{F_2-F_1}{B}\rfloor$. Then c is the maximum number of disjoint channels that could be possible. However the channels are untuned and hence partially ovelapping, rather than disjoint. As per the assumption in [3], two nodes can communicate directly if the carrier frequency of one is admitted by the other, i.e., if there is at least 50% overlap between two channels, communication is possible. We consider the issue of capacity of a randomly deployed network of n nodes, where each node has an untuned radio, and each node is the source of one flow, with a randomly chosen destination.

Even though each node only possesses a single radio and stays on a single sub-band, due to the partial overlap between sub-bands, it is still possible to ensure that any pair of nodes will be connected via some path. Contrast this to the case of orthogonal channels, where we argued in Section VII that when $f=1$, and $c>1$, some pairs of nodes are disconnected from each other because they do not share a channel. It is possible to map the partial overlap feature of the untuned channel case to adjacent $(2c+2,3)$ and $(4c+1,2)$ assignment. Note that $f=2$ allows for all nodes to be connected, even with orthogonal channels.

We map the untuned radio scenario to a scenario having $(2c+2,3)$ adjacent channel assignment (Fig. 4).

We perform a virtual channelization of the band (F_1,F_2) into $2c$ orthogonal sub-bands. We add an additional (virtual) sub-band of the same width at each end of the band, to get $2c+2$ orthogonal channels, numbered $1,...,2c+2$. Thus 1 and $2c+2$ are the artificially added channels. If a radio's carrier frequency lies within virtual channel i, it is associated with virtual channel block $(i-1,i,i+1)$, and $i-1$ is called its primary virtual channel. Thus the primary channel can only be one of $1,2,...,2c$ (since the carrier frequency can only fall in $2,..,2c+1$). If a node's primary channel is i, it is capable of communicating with all nodes with primary virtual channel $i-2\leq j\leq i+2$ in the virtual channelization. In the actual situation, the node with the untuned radio would be able to communicate with some subset of those

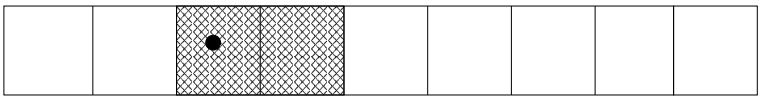

Fig. 5. Untuned Radios: Lower Bound via virtual $(4c+1,2)$ channelization

nodes. Thus, if a pair of nodes cannot communicate directly in the virtual channelization, they cannot do so in the actual situation either, and disconnection events in the former are preserved in the latter. The probability that a node has virtual channel block $(j, j+1, j+2)$ is $\frac{1}{2c}$, i.e., the same as for adjacent $(2c+2,3)$ assignment, and the necessary condition for the (virtual) $(2c+2,3)$ assignment continues to hold for the corresponding untuned radio case. This yields an upper bound on capacity of $O(W\sqrt{\frac{1}{cn\log n}})$.

It can be shown that a schedule constructed for an adjacent $(4c+1,2)$ assignment can be used almost as-is with untuned radios (except that the number of subslots in the cell-slot must increase by a factor of 11 to avoid interference due to overlap).

We perform a virtual channelization of the band (F_1,F_2) into $4c+1$ orthogonal sub-bands. If a radio's carrier frequency lies within virtual channel i, it is associated with virtual channel block $(i,i+1)$, and i is called its primary virtual channel. Note that if a node's primary channel is i, it is always capable of communicating with all nodes with primary virtual channel $i-1 \leq j \leq i+1$, but we will pretend that it can only communicate with those having i or $i+1$. Thus, if a pair of nodes share a channel in the virtual channelization, then they are always capable of direct communication in the actual untuned radio situation. The probability that a radio has virtual channel block $(i,i+1)$ is $\frac{1}{4c}$, same as for adjacent $(4c+1,2)$ assignment. In the adjacent $(4c+1,2)$ assignment, all channel are orthogonal and can operate concurrently. With untuned radios, we assume two nodes can interfere if there is some spectral overlap. Thus, a transmission by a node on carrier frequency F can interfere with transmissions by nodes with carrier frequency in the range $(F-B,F+B)$. Hence, the transmission schedule for untuned radios is made to follow the additional constraint that if a node with primary virtual channel i is active then no node with primary channel $i-5 \leq j \leq i+5$ should be active simultaneously. This would decrease capacity by a factor of 11, but would not affect the order of the asymptotic results. Also, in the actual network involving untuned radios, a transceiver can use upto $B = \frac{F_2-F_1}{c}$ spectral bandwidth, while in the adjacent $(4c+1,2)$ case, it would be $\frac{F_2-F_1}{4c+1}$, leading to the possibility of having a higher data-rate in the former, given the same transmission power, modulation, etc. However this can only affect capacity by a small constant factor, which does not affect the order of the results.

In the adjacent $(4c+1,2)$ case, our construction performs transitions to ensure that a source on channels $(i,i+1)$ and a destination on channels $(i+j,i+j+1)$ can communicate over $j \leq 4c$ hops. In the untuned radio case, transitioning is done through nodes that provide the required virtual channel pair, and the same transition strategy as for $(4c+1,2)$ assignment continues to work. Hence the capacity is $\Omega(W\sqrt{\frac{1}{cn\log n}})$ per flow.

We re-emphasize that even though $f=1$, the untuned nature of the radios allows for a progressive shift in the frequency over which the packet gets transmitted, thereby allowing a step-by-step transition from the source's carrier frequency to a frequency admitted by the destination. The adjacent (c,f) model captures

this progressive frequency-shift characteristic, and is thus able to model the untuned radio situation.

From the upper and lower bounds proved in this section, it follows that the capacity of the untuned radio network, when $c = O(\log n)$, is $\Theta(W\sqrt{\frac{1}{cn\log n}})$ per flow.

XII. Random (c, f) assignment

In this assignment model, a node is assigned a subset of f channels uniformly at random from the set of all possible channel subsets of size f. Thus the probability that a node is capable of switching on a given channel i is $p_s^{rnd}(i) = \frac{f}{c} = p_s^{rnd}, \forall i$, and the probability that two nodes share at least one channel is given by $p_{rnd} = 1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$.

Lemma 21: For $c = O(\log n)$, and $1 < f \leq c$, the following holds:

$$\frac{cp_{rnd}}{f} \leq \min\{\frac{c}{f}, 2f\} \tag{22}$$

Proof: Since $p_{rnd} \leq 1$, we obtain that $\frac{cp_{rnd}}{f} \leq \frac{c}{f}$.

If $f \geq \sqrt{\frac{c}{2}}$, then $\frac{cp_{rnd}}{f} \leq \sqrt{2c} \leq 2f$ follows from the observation that $p_{rnd} \leq 1$. Hence, we focus on the case $f < \sqrt{\frac{c}{2}}$.

$$\begin{aligned} 1 - p_{comm} &= (1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1}) \\ &\geq (1-\frac{f}{c-f+1})^f > (1-\frac{2f}{c})^f \geq 1-\frac{2f^2}{c} \\ &\therefore p_{rnd} \leq \frac{2f^2}{c} \\ &\therefore \frac{cp_{rnd}}{f} \leq 2f \end{aligned} \tag{23}$$

Thus, $\frac{cp_{rnd}}{f} \leq \min\{\frac{c}{f}, 2f\}$. ■

Lemma 22: $\min\{\frac{c}{f}, 2f\} \leq \sqrt{2c}$

Proof: For a given c, we have $2 \leq f \leq c$. Thus, given c, $\frac{c}{f}$ is a monotonically decreasing function of f, while $2f$ is a monotonically increasing function of f. $\frac{c}{f} = 2f = \sqrt{2c}$ at $f = \sqrt{\frac{c}{2}}$. For $f \leq \sqrt{\frac{c}{2}}$, $\min\{\frac{c}{f}, 2f\} = 2f \leq \sqrt{2c}$, and for $f > \sqrt{\frac{c}{2}}$, $\min\{\frac{c}{f}, 2f\} = \frac{c}{f} \leq \sqrt{2c}$. Thus $\min\{\frac{c}{f}, 2f\} \leq \sqrt{2c}$. ■

A. Necessary Condition for Connectivity

Theorem 7: With a random (c, f) channel assignment (when $c = O(\log n)$), if $\pi r^2(n) = \frac{(\log n + b(n))}{pn}$, where $p = p_{rnd} = 1-(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$, and $c = O(\log n)$, and $b = \lim_{n\to\infty} b(n) < +\infty$ then:

$$\liminf_{n\to\infty} Pr[\text{ disconnection }] \geq e^{-b}(1-e^{-b}) > 0$$

where by *disconnection* we imply the event that there is a partition of the network.

Proof: This proof follows arguments similar to the corresponding proof for adjacent (c, f) assignment. We focus on the disconnection event where singleton sets are partitioned from the rest of network. We first make the following observation:

$$p \geq \frac{f}{c} \tag{24}$$

$$\begin{aligned}\pi r^2(n) \leq \frac{2c\log n}{f} \leq \frac{2\alpha\log^2(n)}{n}\\ \because c \leq \alpha\log n \text{ and for some constant } \alpha\\ \text{and } b(n) < \log n \text{ for large } n \because \limsup b(n) < +\infty\end{aligned} \tag{25}$$

The probability that two nodes in range of each other share a channel is $p = p_{rnd}$ where $1 - p_{rnd} = (1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})$. Note that for $f > \frac{c}{2}$, $p = p_{rnd} = 1$, as any two nodes are guaranteed to have at least one common channel. The probability that a node x is isolated, i.e., cannot communicate with any neighbor is give by $p_1 = (1-p\pi r^2(n))^{(n-1)}$. Consider the event that nodes x and y are both isolated. There are three different cases for this, as also considered for the adjacent (c, f) assignment (Fig. 1): (1) x and y lie within distance $r(n)$ of each other, but do not share a common channel (2) x and y do not lie within distance $r(n)$ of each other, but have overlapping neighborhood regions, i.e. lie within distance $2r(n)$ of each other (3) their neighborhood regions are disjoint, i.e., the distance between them is greater than $2r(n)$. Of these, for case (1), consider two sub-cases, viz., (i) y is at distance $d(n) \leq r'(n) = \left(\frac{16\log\log n}{\log n}\right) r(n)$ from x, and (ii) y is at distance $d(n) > r'(n) = \left(\frac{16\log\log n}{\log n}\right) r(n)$ from x. Also, note that the probability a node $z \neq x,y$ within range of both x and y is capable of communicating with either one of x and y, given that they cannot communicate with each other is given by $q = 1-(1-\frac{2f}{c})(1-\frac{2f}{c-1})...(1-\frac{2f}{c-f+1} \geq p$. Then, it can be seen that

$$\begin{aligned}\frac{1-p}{1-q} &= \frac{(1-\frac{f}{c})(1-\frac{f}{c-1})...(1-\frac{f}{c-f+1})}{(1-\frac{2f}{c})(1-\frac{2f}{c-1})...(1-\frac{2f}{c-f+1})}\\ &= \left(1+\frac{\frac{f}{c}}{1-\frac{2f}{c}}\right)\left(1+\frac{\frac{f}{c-1}}{1-\frac{2f}{c-1}}\right)...\left(1+\frac{\frac{f}{c-f+1}}{1-\frac{2f}{c-f+1}}\right)\\ &\geq 1+\frac{\frac{f}{c}}{1-\frac{2f}{c}}+\frac{\frac{f}{c-1}}{1-\frac{2f}{c-1}}+...+\frac{\frac{f}{c-f+1}}{1-\frac{2f}{c-f+1}}\\ &\geq 1+\frac{f}{c}+\frac{f}{c-1}+...+\frac{f}{c-f+1}\\ &\geq 1+\frac{f^2}{c}\end{aligned} \tag{26}$$

Hence:

$$\begin{aligned}q \geq p(1+\frac{\frac{1}{p}-1}{\frac{c}{f^2}+1}) \geq p(1+\frac{\frac{c}{2f^2}-1}{\frac{c}{f^2}+1})(\text{ from Lemma 21})\\ \geq p(1+\frac{\frac{c}{2f^2}(1-\frac{2f^2}{c})}{\frac{c}{f^2}(1+\frac{f^2}{c})})\end{aligned} \tag{27}$$

For sub-case (i) of case (1), the overlap area between the neighborhoods of x and y is at least $(1-\delta)\pi r^2(n)$ for any $\delta > 0$ and large enough n, since the separation $r'(n) \leq \left(\frac{16\log\log n}{\log n}\right) r(n)$. For our purpose, it suffices to take $\delta = \frac{1}{16}$, yielding an overlap area of at least $\frac{15\pi r^2(n)}{16}$. Then the probability that a node can communicate with either x or y or both is at least q times the probability of lying in the overlap area.

Thus, when $\frac{f}{c} \leq \frac{(\log\log n)^3}{\log n}$, then from Eqn. 27:

$$\begin{aligned} q &\geq p(1+\frac{\frac{c}{2f^2}(1-\frac{2f^2}{c})}{\frac{c}{f^2}(1+\frac{f^2}{c})}) \\ &\geq p(1+\frac{1}{3}) = \frac{4p}{3} \text{ for } \frac{f}{c} \leq \frac{(\log\log n)^3}{\log n} \text{ and large } n \end{aligned} \tag{28}$$

Resultantly, the contribution of subcase (i) of case (1) to the probability that both x and y are isolated is upper-bounded as follows:

$$\begin{aligned} p_{21(i)} &\leq \pi r'^2(n)(1-p)(1-q\frac{15\pi r^2(n)}{16})^{n-2} \\ &< \pi r^2(n)(1-\frac{15q\pi r^2}{16}(n))^{n-2} \leq \pi r^2(n)(1-\frac{5}{4}p\pi r^2(n))^{n-2} \\ &\text{for } \frac{f}{c} \leq \frac{(\log\log n)^3}{\log n} \\ &\leq \pi r^2(n) e^{-\frac{5}{4}(n-2)p\pi r^2(n)} \text{ from Lemma 6} \\ &\leq \frac{2\alpha\log^2(n)}{n} e^{-\frac{5}{4}(n-2)p\pi r^2(n)} \\ &\leq e^{-\frac{5}{4}\log n - \frac{5}{4}b - \frac{5(\log n+b)}{2n} - \log n + \log 2\alpha + 2\log\log n} \\ &\leq e^{-\frac{17}{8}\log n - \frac{5}{4}b} \text{ for large } n \end{aligned} \tag{29}$$

For sub-case (i) of case (1), when $\frac{f}{c} > \frac{(\log\log n)^3}{\log n}$, we lower bound the probability of a node being able to communicate with either of x and y by the probability that it is able to communicate with one of them (say x). Thus the probability that both x and y are isolated is at most:

$$\begin{aligned} p_{21(i)} &\leq \pi r'^2(n)(1-p)(1-p\pi r^2(n))^{n-2} \\ &\pi r'^2(n)(1-p\pi r^2(n))^{n-2} \\ &\leq \frac{256(\log\log n)^2}{\log^2 n}\pi r^2(n)(1-p\pi r^2(n))^{n-2} \\ &\leq \frac{256(\log\log n)^2(\log n + b(n))}{pn\log^2 n}(1-p\pi r^2(n))^{n-2} \\ &\leq \frac{256(\log\log n)^2\log n(\log n + b(n))}{n(\log\log n)^3\log^2 n}(1-p\pi r^2(n))^{n-2} \\ &(\text{ as } p \geq \frac{f}{c} > \frac{(\log\log n)^3}{\log n}) \\ &\leq \frac{256(\log\log n)^2(2\log^2 n)}{n(\log\log n)^3\log^2 n}(1-p\pi r^2(n))^{n-2} \\ &\leq \frac{512}{n\log\log n} e^{-(n-2)p\pi r^2(n)} \text{ from Lemma 6} \\ &\leq e^{-\log n - b(n) + \frac{2(\log n+b)}{n} - \log n + \log 512 - \log\log\log n} \\ &\leq e^{-2\log n - b(n) - \frac{1}{2}\log\log\log n} \text{ for large } n \end{aligned} \tag{30}$$

Thus, in sub-case (i), for all f, and large enough n:

$$p_{21(i)} \leq e^{-2\log n - b(n) - \frac{1}{2}\log\log\log n} \tag{31}$$

For sub-case (ii), the situation is depicted in Fig. 2. The probability that some node can talk to either x or y is lower bounded by the probability that it lies in range of x ($\pi r^2(n)$) and shares a channel with it (p), or it lies out of range of x but within range of y (at least $\frac{\sqrt{3}r(n)r'(n)}{2}$ for large enough n [5]), and shares a channel with y (p). The probability that both x and y are isolated is thus at most:

$$\begin{aligned}
p_{21(ii)} &\leq (\pi r^2(n) - \pi r'^2(n))(1-p)(1-p(\pi r^2(n) \\
&\qquad + \frac{\sqrt{3}r(n)r'(n)}{2}))^{n-2} \\
&\leq \pi r^2(n)\left(1 - p\left(\pi r^2(n) + \frac{\sqrt{3}r(n)r'(n)}{2}\right)\right)^{n-2} \\
&\leq \pi r^2(n)\left(1 - p\pi r^2(n)\left(1 + \frac{\sqrt{3}r'(n)}{2\pi r(n)}\right)\right)^{n-2} \\
&\leq \pi r^2(n)\left(1 - p\pi r^2(n)\left(1 + \frac{8\sqrt{3}\log\log n}{\pi\log n}\right)\right)^{n-2} \\
&\leq \pi r^2(n) e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})} \text{ from Lemma 6}(\because \pi < 2\sqrt{3})) \\
&\leq \frac{2\alpha\log^2 n}{n} e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})} \text{ from Eqn. 7} \\
&\leq e^{-(n-2)p\pi r^2(n)(1+\frac{4\log\log n}{\log n})+\log 2\alpha+2\log\log n-\log n} \\
&\leq e^{-\log n - b(n) - 4\log\log n + \frac{2(\log n + b(n))(1+\frac{4\log\log n}{\log n})}{n} + \log 2\alpha + 2\log\log n - \log n} \\
&\leq e^{-2\log n - b(n) - \log\log n} \text{ for large } n
\end{aligned} \tag{32}$$

For case 2, the probability that some node can communicate with either x or y is lower bounded by the probability that it lies in range of x ($\pi r^2(n)$) and shares a channel with it (p), or it lies out of range of x but within range of y (disjunction of the two circles in Fig. 1 (2) is at least $\frac{1}{2}\pi r^2(n)$), and shares a channel with it. Thus the contribution of this case to the probability that both x and y are isolated is upper bounded by:

$$\begin{aligned}
p_{22} &\leq (4\pi r^2(n) - \pi r^2(n))(1 - \frac{3}{2}p\pi r^2(n))^{n-2} \\
&\leq 3\pi r^2(n) e^{-\frac{3}{2}(n-2)p\pi r^2(n)} \text{ from Lemma 6} \\
&\leq e^{-\frac{3}{2}\log n - \frac{3}{2}b + \frac{3(\log n + b)}{n} - \log n + \log 6\alpha + 2\log\log n} \\
&\leq e^{-\frac{9}{4}\log n - \frac{3}{2}b} \text{ for large } n
\end{aligned} \tag{33}$$

[5]The area within range of y but out of range of y is given by $\pi r^2(n) -$ overlap area ; where overlap area = 2 (area of quadrant subtending angle $2\theta -$ area of $\triangle ABC$) $\leq \pi r^2(n) - r^2(n)\sin(2\theta)$. Note that $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$. Thus non-overlap area $\geq r^2(n)\sin(2\theta) = r^2(n)(2\sin\theta\cos theta) = r^2(n)2\sin\theta\frac{d(n)}{2r(n)} \geq 2r^2(n)\sin\frac{\pi}{3}\frac{r'(n)}{2r(n)} \geq \frac{\sqrt{3}r(n)r'(n)}{2}$

For case 3, the probability that both x and y are isolated is upper bounded by:

$$\begin{aligned} p_{23} &\leq (1-4\pi r^2)(1-2p\pi r^2(n))^{n-2} \\ &\leq (1-2p\pi r^2(n))^{n-2} \\ &\leq e^{-2(n-2)p\pi r^2(n)} \text{ from Lemma 6} \leq e^{-2\log n - 2b + \frac{2(\log n+b)}{n}} \end{aligned} \tag{34}$$

Then, the probability p_2 that nodes i and j are both isolated is given by:

$$p_2 = p_{21(i)} + p_{21(ii)} + p_{22} + p_{23} \tag{35}$$

Let us first consider the case where $b(n) = b$ is a constant.

$$\begin{aligned} Pr[\text{ disconnection }] &\geq \sum_x Pr[x \text{ is only isolated node}] \\ &\geq \sum_x Pr[x \text{ isolated }] - \sum_{x,y} Pr[x \text{ and } y \text{ both isolated }] \\ &= np_1 - n(n-1)p_2 \\ &\geq n(1-p\pi r^2(n))^{(n-1)} \\ &\quad -n(n-1)(p_{21(i)} + p_{21(ii)} + p_{22} + p_{23}) \\ &\geq \theta e^{-b} - n(n-1)(e^{-2\log n - b(n) - \frac{1}{2}\log\log\log n} \\ &\quad + e^{-2\log n - b - \frac{\log\log n}{6}\log n} \\ &\quad + e^{-\frac{9}{4}\log n - \frac{3}{2}b} \\ &\quad + e^{-2\log n - 2b + \frac{2(\log n+b)}{n}}) \\ &\geq \theta e^{-b} - (1+\varepsilon)e^{-2b} \\ &\text{for any } \theta < 1, \varepsilon > 0, \text{ and large } n \text{ by Lemmas 6 and 7} \end{aligned} \tag{36}$$

Now, let us consider the case where $b(n)$ is not constant, and $\lim_{n\to\infty} b(n) = b$. Then, for any $\varepsilon > 0$, $b(n) - b \leq \varepsilon$ for large n. Since the probability of disconnection monotonically decreases in $b(n)$, we can take the following bound:

$$\begin{aligned} Pr[disconnection] &\geq \theta e^{-(b+\varepsilon)} - (1+\varepsilon)e^{-2(b+\varepsilon)} \\ &\quad (\text{ for large enough } n) \end{aligned} \tag{37}$$

Thus, if $\limsup_{n\to\infty} b(n) < +\infty$, the network is asymptotically disconnected with some positive probability. ■

Corollary 2: With a random (c, f) assignment, the necessary condition for connectivity is that $r(n) = \Omega(\sqrt{\frac{\log n}{p_{rnd} n}})$, else the network is disconnected with some positive probability.

B. Sufficient Condition for Connectivity

Theorem 8: With random (c, f) assignment ($c = O(\log n)$), if $\pi r^2(n) = \frac{800\pi\log n}{p_{rnd} n}$, then:

$$Pr[\text{ network is connected }] \to 1$$

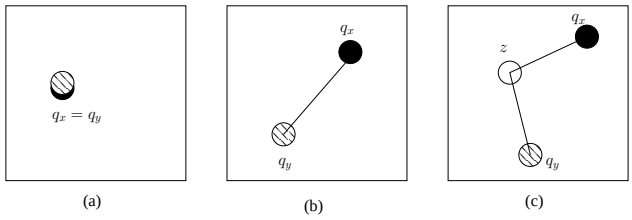

Fig. 6. Three of the ways in which $\mathcal{B}(x)$ and $\mathcal{B}(y)$ can be connected

Proof: We present a construction based on a notion of per-node backbones. Consider a subdivision of the toroidal unit area into square cells of area $a(n) = \frac{100 \log n}{p_{rnd} n}$. Then by setting $\alpha(n) = \frac{1}{p_{rnd}}$ in Lemma 3 there are at least $\frac{50 \log n}{p_{rnd}}$ nodes in each cell with high probability. Set $r(n) = \sqrt{8a(n)}$. Then a node in any given cell has all nodes in adjacent cells within its range. Within each cell, choose $\frac{2 \log n}{p_{rnd}}$ nodes uniformly at random, and set them apart as *transition facilitators* (the meaning of this term shall become clear later). This leaves at least $\frac{48 \log n}{p_{rnd}}$ nodes in each cell that can act as *backbone candidates*.

Consider any node in any given cell. The probability that it can communicate to any other random node in its range is p_{rnd}. Then the probability that in an adjacent cell, there is no backbone candidate node with which it can communicate is less than $(1 - p_{rnd})^{\frac{48 \log n}{p_{rnd}}} \leq \frac{1}{e^{48 \log n}} = \frac{1}{n^{48}}$. The probability that a given node cannot communicate with any node in some adjacent cell is thus at most $\frac{8}{n^{48}}$ (as there are upto 8 adjacent cells per node). By applying the union bound over all n nodes, the probability that at least one node is unable to communicate with any backbone candidate node in at least one of its adjacent cells is at most $\frac{8}{n^{47}}$.

We associate with each node x a set of nodes $\mathcal{B}(x)$ called the primary backbone for x. $\mathcal{B}(x)$ is constituted as follows. Throughout the procedure, cells that are already covered by the under-construction backbone are referred to as *filled* cells. x is by default a member of $\mathcal{B}(x)$, and its cell is the first *filled* cell. From each adjacent cell, amongst all backbone candidate nodes sharing at least one common channel with x, one is chosen uniformly at random is added to $\mathcal{B}(x)$. Thereafter, from each cell bordering a filled cell, of all nodes sharing at least one common channel with some node already in $\mathcal{B}(x)$, one is chosen uniformly at random, and is added to $\mathcal{B}(x)$; the cell gets added to the set of filled cells. This process continues iteratively, till there is one node from every cell in $\mathcal{B}(x)$. From our earlier observations, $\mathcal{B}(x)$ eventually covers all cells with probability at least $1 - \frac{8}{n^{47}}$.

Now consider any pair of nodes x and y. If $\mathcal{B}(x) \cap \mathcal{B}(y) \neq \phi$, i.e., the two backbones have a common node (Fig. 6 (a)), then x and y are obviously connected, as one can proceed from x on $\mathcal{B}(x)$ towards one of the intersection nodes, and thence to y on $\mathcal{B}(y)$, and vice-versa.

Suppose, the two backbones are disjoint. Then x and y are still connected if there is some cell such that the member of $\mathcal{B}(x)$ in that cell (let us call it q_x) can communicate with the member of $\mathcal{B}(y)$ in that cell (let us call it q_y), either directly, or through a third node. q_x and q_y can communicate directly with probability 1 if they share a common channel (Fig. 6 (b)). Thus the case of interest is one in which no cell has q_x and q_y sharing a channel. If they do not share a common channel, we consider the event that there exists a third node z amongst the *transition facilitators* in the cell through whom they can

communicate (Fig. 6 (c)).

Note that, for two given backbones $\mathcal{B}(x)$ and $\mathcal{B}(y)$, the probability that in a network cell, given q_x and q_y that do not share a channel, they can both communicate with a third node z that did not participate in backbone formation and is known to lie in the same cell, is independent across cells. Therefore, the overall probability can be lower-bounded by obtaining for one cell the probability of q_x and q_y communicating via a third node z, given they have no common channel, considering that each cell has at least $\frac{2\log n}{p_{rnd}}$ possibilities for z, and treating it as independent across cells. We elaborate this further.

Let q_x have the set of channels $C(q_x) = \{c_{x_1}, ..., c_{x_f}\}$, and q_y have the set of channels $C(q_y) = \{c_{y_1}, ..., c_{y_f}\}$, such that $C(q_x) \cap C(q_y) = \phi$. Consider a third node z amongst the transition facilitators in the same cell as q_x and q_y. We desire z to have at least one channel common with both $C(q_x)$ and $C(q_y)$. Then let us merely consider the possibility that z enumerates its f channels in some order, and then inspects the first two channels, checking the first one for membership in $C(q_x)$, and checking the second one for membership in $C(q_y)$. This probability is $\left(\frac{f}{c}\right)\left(\frac{f}{c-1}\right) > \frac{f^2}{c^2}$. Thus q_x and q_y can communicate through z with probability $p_z > \frac{f^2}{c^2} = \Omega(\frac{1}{\log^2 n})$. There are $\frac{2\log n}{p_{rnd}}$ possibilities for z within that cell, and all the possible z nodes have i.i.d channel assignments. Thus, the probability that q_x and q_y cannot communicate through any z in the cell is at most $(1-p_z)^{\frac{2\log n}{p_{rnd}}}$, and the probability they can indeed do so is $p_{xy} > 1-(1-p_z)^{\frac{2\log n}{p_{rnd}}}$.

Thus, the probability that this happens in none of the $\frac{1}{a(n)} = \frac{p_{rnd}n}{100\log n}$ cells is at most $(1-p_{xy})^{\frac{p_{rnd}n}{100\log n}} < (1-p_z)^{\frac{2\log n}{p_{rnd}}\frac{p_{rnd}n}{100\log n}} < (1-\frac{1}{c^2})^{\frac{2\log n}{p_{rnd}}\frac{p_{rnd}n}{100\log n}} \to e^{-\Omega(\frac{n}{\log^2 n})}$ (recall that $c = O(\log n)$). Applying union bound over all $\binom{n}{2} < \frac{n^2}{2}$ node pairs, the probability that some pair of nodes are not connected is at most $\frac{n^2 e^{-\Omega(\frac{n}{\log^2 n})}}{2} < \frac{1}{2}e^{-\Omega(\frac{n}{\log^2 n})+2\log n} \to 0$. Thus the probability of a connected network converges to 1.

■

Remark: The constant in the sufficient condition for connectivity can be made much smaller than the value 800 that we have used in Theorem 8.

XIII. Random (c, f) Assignment: Capacity Upper Bound

Since the necessary condition for connectivity requires that $r(n) = \Omega(\frac{\log n}{p_{rnd}n})$, the per flow capacity is $O(W\sqrt{\frac{p_{rnd}}{n\log n}})$ from the discussion on the Connectivity Upper Bound in Section VII.

XIV. Random (c, f) Assignment: Capacity Lower Bound

We present a constructive proof that achieves $\Omega(W\sqrt{\frac{f}{cn\log n}})$. This construction is quite similar to that for adjacent (c, f) assignment. The surface of the unit torus is divided into square cells of area $a(n)$ each. The transmission range is set to $\sqrt{8a(n)}$, thereby ensuring that any node in a given cell is within range of any other node in any adjoining cell. As discussed for the adjacent assignment case, the number of cells that interfere with a given cell is only some constant (say β). We choose $a(n) = \frac{100c\log n}{fn}$ (resultantly $r(n) = \sqrt{\frac{800c\log n}{fn}}$).

Lemma 23: If there are $\frac{50c\log n}{f}$ nodes in every cell, then with probability at least $1-O(\frac{1}{n^4})$, there are at least $25\log n$ nodes in each cell on each of the c channels.

Proof: Let us consider one particular cell D. Let $X_{ij}=1$ if node j is on channel i, and 0 else. Thus $Pr[X_{ij}=1]=\frac{f}{c}$. Let $X_i=\sum_{j\in D} X_{ij}$.Then $E[X_i]\geq 50\log n$. By application of the Chernoff bound in Theorem 4 (with $\beta=\frac{1}{2}$) , we obtain:

$$Pr[X_j\leq 25\log n]\leq \exp(-\frac{50\log n}{8})<\frac{1}{n^6} \tag{38}$$

Since there are $c=O(\log n)$ channels, the union bound yields that $Pr[X_i\leq 25\log n$ for any $i\in 1,2,...,c]\leq \frac{c}{n^6}=O(\frac{\log n}{n^6}) \implies O(\frac{1}{n^5})$. Also, there are $\frac{1}{a(n)}=\frac{fn}{100c\log n}<n$ cells, and thus another application of the union bound yields that:

$$Pr[\text{ less than } 25\log n \text{ nodes per channel in any cell}]\leq O(\frac{1}{n^4}) \tag{39}$$

■

A. Routing

Observe that Lemmas 11 and 12 stated in Section X for SD'D routing are applicable here too. In case of (c,f) random assignment, as with adjacent assignment, we cannot stipulate that *all* flows be routed along the (almost) straight-line path SD'D. The flow may be required to traverse a minimum number of hops to be able to ensure that it will find an opportunity to make the switch from source channel to destination channel.

Channel Selection and Transition Strategy: Initially, after each source has chosen a random destination, the flows are sequentially processed and each is assigned an initial source channel, as well as a target destination channel. The source channel for a flow originating at node S is chosen according to the uniform distribution from the f channels available at S. The destination channel may be chosen from amongst the f channels available at destination D in any manner, e.g., it may be the one with the smallest number of incoming flows assigned to it so far.

We stipulate that a non-detour-routed flow is initially in a *progress-on-source-channel* mode, and keeps to the source channel till there are only $\lceil\frac{4c}{25f}\rceil$ intermediate hops left to the destination. At this point, it enters a *ready-for-transition* mode, and actively seeks opportunities to make a channel transition along the remaining hops. It makes use of the first opportunity that presents itself, i.e., if a node in a on-route cell provides the source-destination channel pair, the flow is assigned to that node for relaying (it enters the node on the source channel, and leaves it on the destination channel). Once it has made the transition into the destination channel, it remains on that channel. During the *progress-on-source-channel* phase, the next hop node is chosen to be the node in the next cell which has the smallest number of flows assigned so far on that channel, amongst all nodes that can switch on the source channel. In the *ready-for-transition* phase, it may be assigned to *any eligible node* that provides either the transition opportunity,

or the source channel (for flows yet to find a transition), or the destination channel (for flows that have already transitioned into their destination channel).

A detour-routed flow is always in *ready-for-transition* mode.

Lemma 24: Suppose a flow is on source channel i and needs to finally be on destination channel j. Then after having traversed $h \geq \lceil \frac{2(c-1)}{(f-1)} \rceil$ distinct cells (recall that $2 \leq f \leq c$, an hence $h = O(\log n)$), it will have found an opportunity to make the transition w.h.p.

Proof: Let us consider a given flow. From Lemma 3, each cell has at least $\frac{50c\log n}{f}$ nodes w.h.p. The probability that there is no node with both channels i and j in a given cell along the flow's route is at most $(1-\frac{f(f-1)}{c(c-1)})^{\frac{50c\log n}{f}}$ (since nodes are assigned channels in an i.i.d. manner). Thus the probability of not finding such a node after h hops is at most $(1-\frac{f(f-1)}{c(c-1)})^{(\frac{50hc\log n}{f})}$. If $h \geq \lceil \frac{2(c-1)}{25(f-1)} \rceil$, then after traversing h distinct cells, the probability of not finding such a node is at most $(1-\frac{f(f-1)}{c(c-1)})^{\frac{4c(c-1)\log n}{(f(f-1)}} \leq \exp(-4\log n) \leq \frac{1}{n^4}$. Then, by applying the union bound over all n flow, the probability that this should happen for even one flow is less than $\frac{1}{n^3}$. Hence all flows will have be able to make the required transition w.h.p. after traversing $h \geq \lceil \frac{2(c-1)}{(f-1)} \rceil$ distinct hops. Note that $h = O(\log n)$, and h hops suffice for transition w.h.p. This yields the proof. ■

Note that $\frac{2(c-1)}{25(f-1)} \leq \frac{4c}{25f}$. Thus, the (almost) straight-line SD'D path is followed if either source and destination channels are the same, or if the straight-line segment SD' provides $h \geq \lceil \frac{4c}{25f} \rceil$ *intermediate* hops. If S and D' (hence also D) lie close to each other, the hop-length of the straight line cell-to-cell path can be much smaller. In this case, a *detour* path is chosen. Consider a circle of radius $\lceil \frac{4c}{25f} \rceil r(n)$ centered at S. Choose any point on this circle, say P, so long as P does not lie in the same cell as D (this guarantees at least one intermediate hop even if $\frac{4c}{25f} \leq 1$). Then the route is obtained by traversing cells along SP and then PD. This ensures that the route has at least the minimum required hop-length (since the segment SP always provides at heast $\lceil \frac{4c}{25f} \rceil$ distinct hops(cells). This situation is illustrated in Fig. 3.

The need to perform *detour* routing for some source-destination pairs does not have any substantial effect on the average hop-length of routes or the relaying load on a cell.

Lemma 25: The length of any route increases by at most $O(\frac{c}{f})$ hops. The average route length increases by at most $O(\frac{c}{f})$ hops.

Proof: The proof proceeds directly from the *detour* routing strategy. Recall that the area of a cell is $\frac{100c\log n}{fn}$, i.e. the side of each cell is $10\sqrt{\frac{c\log n}{f}} = \frac{r(n)}{\sqrt{8}}$. The distance *SP* is at most $\lceil \frac{4c}{25f} \rceil r(n) \leq (1+\frac{4c}{25f})r(n)$ (radius of circle), yielding $O(\frac{c}{f})$ additional hops while *PD* is *at most* $2(\lceil \frac{4c}{25f} \rceil)r(n)$ (diameter of circle) yielding another $O(\frac{c}{f})$ additional hops. This increases route length by at most $O(\frac{c}{f})$ hops. ■

Lemma 26: If the number of distinct flows traversing any cell is x in case of only straight-line routing, it is at most $x + O(n\left(\frac{c}{f}\right)^2 r(n)^2) \implies x + O(\log^4 n)$ even with detour routing[6].

[6]This is a loose upper bound. The actual number of detour-routed flows traversing a cell is much smaller.

Proof: The detour occurs only when the straight-line route has less than $\lceil \frac{4c}{25f} \rceil$ intermediate hops, and the new route lies entirely within a circle of radius $\lceil \frac{4c}{25f} \rceil r(n)$ around the source. Thus the extra flows that may pass through a cell (compared to straight-line routing) are only those whose sources lie within a distance $\lceil \frac{4c}{25f} \rceil r(n)$ from some point in this cell. Thus all such possible sources fall within a circle of radius $(1+\lceil \frac{4c}{25f} \rceil)r(n)$, and hence area $a_c(n) = \pi \left(1+\lceil \frac{4c}{25f} \rceil\right)^2 r^2(n)$. By Lemma 4, any circle of this area has at most $O(na_c(n))$ nodes, and hence at most $O(na_c(n))$ sources. Thus the number of extra flows that traverse the cell due to detour routing is $O(na_c(n))$, and the total number of flows is $x + O(n\left(\frac{c}{f}\right)^2 r(n)^2) \implies x + O(\log^4 n)$ w.h.p. ■

Lemma 27: The number of distinct flows traversing any cell is $O(n\sqrt{a(n)})$ even with detour routing.

Proof: Note that $na_c(n) = O(\log^4 n)$. Since the number of flows passing through cell C in the straight-line routing case is $O(n\sqrt{a(n)}) = O(\sqrt{\frac{cn\log n}{f}})$, the number of flows through a cell C is at most $O(\sqrt{\frac{cn\log n}{f}}) + O(\log^4 n) = O(\sqrt{\frac{cn\log n}{f}})$ in this case too. ■

Lemma 28: The number of flows traversing any cell in *ready-for-transition* mode is $O(\log^4 n)$ w.h.p.

Proof: We first account for the flows traversing the cell along SD', and shall explicitly account later for the possible additional D'D hop.

By our construction, a non-detour-routed flow enters the *ready-for-transition* mode only when it is $\Theta(\frac{c}{f})$ hops away from its destination. All such flows must have their pseudo-destinations within a circle of radius $\Theta(\frac{c}{f}r(n))$ centered in the cell. The number of pseudo-destinations that lie within a circle of radius $\Theta(\frac{c}{f}r(n))$ from the cell is $\Theta(n\frac{c^2}{f^2}r^2(n)) = O(\frac{c^3}{f^3}\log n) \implies O(\log^4 n)$ w.h.p. (by suitable choice of $\alpha(n) = O(\frac{c^3}{f^3}) \geq 1$ in Lemma 4, and by observing that $c = O(\log n)$).

A detour-routed flow is always in *ready-for-transition* mode. From Lemma 26, there are at most $O(\log^4 n)$ such flows, and they traverse a cell at most twice, yielding $O(\log^4 n)$ traversals.

We now account for the fact that all the above routed flows could have an additional last hop that may re-traverse a cell, and needs to be counted separately. This yields at most $O(na(n)) = O\frac{c\log n}{f} \implies O(\log^2 n)$ additional traversals. Hence all channels have $O(\log^4 n)$ transitioning flows in the cell w.h.p. ■

B. Balancing Load within a Cell

Per-Channel Load: Recall that each cell has $O(na(n))$ nodes w.h.p., and $O(n\sqrt{a(n)})$ flows traversing it w.h.p. Due to constrained channel switching, the assignment algorithm of [1] does not apply here.

Lemma 29: The number of flows that enter any cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: A flow may be on a channel i in the cell if (1) the flow's source channel is i and it is *progress-on-source-channel* mode, or it is in *ready-for-transition* mode, but is yet to find a transition into the destination channel (2) the flow's destination channel is i, and it has already made a transition. A flow's source channel can be any of $1, 2, ..., c$ with equal probability, and the source channels for different flows are independent. However, the destination channels of flows are not necessarily independent, since two flows with the same destination are more likely to have the same destination channel.

Thus in a given cell, if a flow is in *progress-on-source-channel* mode, it is equally likely to be on any channel, i.e., $Pr[$ flow is on channel $i] = \frac{1}{c}, \forall 1 \leq i \leq c$. Note that if the number of such (non-transitioning) flows traversing the cell is $m = O(\frac{n\sqrt{a(n)}}{c})$, then trivially in this cell no channel can have more than so many flows. So we shall focus on the case where the number of traversing non-transitioning flows is $m = \omega(\frac{n\sqrt{a(n)}}{c}) = \omega(\sqrt{\frac{n\log n}{cf}})$.

Let m be $\omega(\frac{n\sqrt{a(n)}}{c})$ and $O(n\sqrt{a(n)})$. Viewing this situation as that of m balls being thrown into c bins, we know from Lemma 2 that the probability of having more than $\frac{2m}{c}$ flows on a channel is at most $c \cdot \exp(-\frac{m}{3c}) \leq \exp(-\frac{\omega(\sqrt{\frac{n\log n}{cf}})}{3c} + \log c) = \exp(-\omega(\sqrt{\frac{n\log n}{c^2 f}})) < \exp(-\omega(\sqrt{\frac{n}{\log^2 n}}))$. Taking union bound over all $\frac{1}{a(n)} = \frac{fn}{100c\log n}$ cells, the probability that this happens in any cell is less than $\frac{fn}{100c\log n}\exp(-\omega(\frac{\sqrt{n}}{\log^2 n})) = \exp(-\omega(\frac{\sqrt{n}}{\log^2 n}))$. Since $m = O(n\sqrt{a(n)})$, this proves the load-balance of non-transitioning flows in all cells.

We now need to account for the fact that some of these flows may be in the *ready-for-transition* mode. From Lemma 28 there are $O(\log^4 n)$ flows traversing the cell in this mode w.h.p. Thus, the additional transitioning overhead per channel is $O(\log^4 n)$ w.h.p.

Hence, the per-channel load in all cells is at most $O(\frac{n\sqrt{a(n)}}{c}) + O(\log^4 n) = O(\frac{n\sqrt{a(n)}}{c})$ w.h.p. ■

Lemma 30: The number of flows that leave any given cell on any single channel is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: The proof follows the same argument as that for Lemma 19. ■

Per-Node Load:

Lemma 31: The number of flows that are assigned to any one node in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Proof: A node is always assigned the single flow for which it is the source. A node is also assigned flows terminating in that cell and for which it is the destination, and there are at most $D(n) = O(\frac{c\log n}{f}) \Longrightarrow O(\log^2 n)$ such flows for any node w.h.p. (from Lemma 12). Besides, a node may be assigned to relay flows that are in the *ready-for-transition* mode: for some it may provide the required channel pair to facilitate a transition, and for some it may provide the source channel (flows yet to find a transition) or destination channel (flows that have already transitioned). From Lemma 28, there are $O(\log^4 n)$ such flows w.h.p. Thus a node or channel can only have $O(\log^4 n)$ transitioning flows assigned for relaying.

We now consider the flows that are in *progress-on-source-channel* mode, and do not originate in the cell. We have already established in Lemma 29, that the number of flows that enter on a given channel in any cell is $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p. By construction, we have chosen cell sizes such that there are at least $25\log n$ nodes on each channel in each cell w.h.p. Also $c = O(\log n)$. A flow is always assigned to the node with least load on that channel so far (from amongst all nodes capable of switching on that channel). Then from Lemma 29, and the fact that each node can switch on only f channels, the number of such flows that are assigned to any one node is $O(\frac{fn\sqrt{a(n)}}{c\log n})$ and hence $O(\frac{n\sqrt{a(n)}}{c})$ w.h.p.

Thus, the resultant number of assigned flows per node is $1+D(n)+O(\log^4 n)+O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{n\sqrt{a(n)}}{c})$. ■

C. Transmission Schedule

The transmission schedule is obtained in the same manner as for adjacent (c, f) assignment, by first obtaining a global inter-cell schedule (recall that the cell-interference graph has chromatic number at most $1+\beta$), and then constructing a conflict graph for intra-cell scheduling. Thus, it can be seen from Lemmas 29, 30 and 31 that the degree of the conflict graph is $O(\frac{n\sqrt{a(n)}}{c})+O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{n\sqrt{a(n)}}{c})$. Thus the graph can be colored in $O(\frac{n\sqrt{a(n)}}{c})$ colors.

Thus the cell-slot is divided into $O(\frac{n\sqrt{a(n)}}{c}) = O(\frac{\sqrt{\frac{cn\log n}{f}}}{c})$ equal length subslots, and all traversing flows get a slot for transmission. This yields that each flow will get $\Omega(\sqrt{\frac{f}{cn\log n}}W)$ throughput. We thus obtain the following theorem:

Theorem 9: With a random (c, f) channel assignment, the described simple protocol achieves throughput of $\Omega(W\sqrt{\frac{f}{cn\log n}})$ per flow.

XV. A Remark on the Proof Technique

In the proofs for both the adjacent and random assignment models, it is to be noted that many of our intermediate lemmas are conditioned on certain desirable events proved to occur w.h.p. in some of the lemmas proved before them, e.g., most intermediate lemmas are conditioned on the event in Lemma 3. Let a generic undesirable event be denoted by $\mathcal{E}_i$ (i.e., $\neg\mathcal{E}_i$ is the desirable event). Note that the following is always true:

$$Pr[\mathcal{E}_1 \cup \mathcal{E}_2] = Pr[\mathcal{E}_1] + Pr[\neg\mathcal{E}_1]Pr[\mathcal{E}_2|\neg\mathcal{E}_1] \leq Pr[\mathcal{E}_1] + Pr[\mathcal{E}_2|\neg\mathcal{E}_1] \tag{40}$$

In light of this, it is not hard to see that the probability that even one of the undesirable events from any of these lemmas occurs, can be upper-bounded via by summing up the individual (in some cases, conditional) probability of occurrence of each undesirable event, as bounded by each lemma (i.e., by essentially applying a union bound on the possibly conditiona probabilities). Since we have in all only as small constant number of lemmas, and each lemma shows that the (possibly conditional on previous lemmas) probability of occurrence of some undesirable event goes to 0 (or equivalently shows that the probability of occurrence of the complementary desirable event goes to 1), the sum will also go to zero. Hence, the probability that even one of the undesirable events happens goes to 0.

XVI. Discussion

The lower bound constructions for the two assignment models yield some interesting insights. As is intuitive, when all nodes cannot switch on all channels, the transmission range needs to be larger (or alternatively, the network density must be higher) to preserve network connectivity. This leads to a loss of capacity compared to the case of unconstrained switching. Also, it may no longer be possible to use the shortest route towards the destination, and a flow may need to take a circuitous path (*detour routing*) in order to ensure that the destination is reached. However, when the number of channels is much smaller

than the number of nodes, the increase in the length of the routes is not asymptotically significant, and only affects the capacity by a constant factor. Taking all factors into account, when $c = O(\log n)$, given a sufficiently dense network, it is beneficial to attempt to use all channels by assigning different channel subsets to different nodes, rather than follow the naive approach of using the same f channels at all nodes. In the latter case, the per-flow capacity would be reduced to $\Theta(W\frac{f}{c\sqrt{n\log n}})$. Thus the *use-all-channels* approach outperforms the *f-common-channels* approach by a factor of $\sqrt{\frac{c}{f}}$. As an example, even when $f = 2$, utilizing all channels yields a capacity of the order of $\sqrt{c}$ channels.

It is also to be noted that when $f = c$, our models reduce to the unconstrained switching model in [1] with a single interface per node. For this case, our per-flow capacity results yield $\Theta(\sqrt{\frac{W}{n\log n}})$, as also obtained in [1] for $\frac{c}{m} = O(\log n)$. However, we are able to achieve the optimal capacity by using a much simpler flow-channel mapping. We also note that the notions of random flow-channel assignment and detour routing devised for the models in this paper, can be applied to other situations, e.g., the deterministic fixed assignment considered in [18].

Another interesting insight is yielded by the results for random (c, f) assignment. Note that a transmission range of $\Theta(\sqrt{\frac{\log n}{p_{rnd}n}})$ is both necessary and sufficient for connectivity. However, at this transmission range, it is possible that some cells may have some channels missing. Thus, the subgraph induced by a certain channel (obtained by retaining only nodes capable of switching on that channel, and assuming this is the only channel they can use) may not necessarily be connected, but the overall network graph is always connected at this transmission range. This may perhaps sometimes make it necessary (due to connectivity concerns) to schedule different links of a flow on different channels, even if the source and destination share a channel. Note that if we set $r(n) = \Theta(\sqrt{\frac{c\log n}{fn}})$, then a source-destination pair that share a channel always have a route with all links using that channel (though it may not necessarily be capacity-optimal to use it), since each channel is available on some nodes in each cell.

XVII. Conclusion

In this paper we have presented a case for the study of multi-channel networks with channel switching constraints. We introduced some models for channel switching constraints, and presented connectivity and capacity results for two such models, viz. adjacent (c, f) assignment, and random (c, f)-assignment, when $c = O(\log n)$. While originally derived for channelization in the frequency domain, our results can also be interpreted in the time domain, and provide insights about energy-capacity trade-offs in networks with low-duty-cycle nodes. We believe there is much potential for extension of the current models, as well as study of a wider range of switching constraints.

Appendix

Proof of Lemma 7

This is basically the proof of Lemma 2.2 from [17], as presented in [17], with the minor change that $\pi r^2(n)$ is replaced with $p\pi r^2(n)$. Taking the log of the L.H.S. and using the Taylor Series expansion, we

have:

$$\begin{aligned}
\log L.H.S. &= \log n + (n-1)\log(1-p\pi r^2(n)) \\
&= \log n - (n-1)\sum_{i=1}^{\infty}\frac{(p\pi r^2(n))^i}{i} \\
&= \log n - (n-1)\left(\sum_{i=1}^{2}\frac{(\log n+b)^i}{in^i} + \varepsilon(n)\right) \\
\text{where } \varepsilon(n) &= \sum_{i=3}^{\infty}\frac{p\pi r^2(n))^i}{i} = \sum_{i=3}^{\infty}\frac{(\log n+b)^i}{in^i} \\
&\leq \frac{1}{3}\int_{i=2}^{\infty}\left(\frac{\log n+b}{n}\right)^x dx \\
&\leq \frac{1}{3}\left(\frac{\log n+b}{n}\right)^2 \\
&\qquad \text{for large } n
\end{aligned}$$

Then, substituting, we get:

$$\begin{aligned}
\log L.H.S. &\geq \log n - (n-1)\left(\frac{\log n+b}{n} + \frac{5(\log n+b)^2}{6n^2}\right) \\
&\geq -b - \frac{(\log n+b)^2 - (\log n+b)}{n} \geq -b-\delta
\end{aligned}$$

Setting $\delta = \ln\frac{1}{\theta}$, and taking exponents on both sides will yield that L.H.S. $\geq \theta e^{-b}$ for large n.

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