COSC 240
Spring 2019
Problem Set 6
Due by class time on April 11, 2019
40 points

1. (20 points) Average consensus

   Consider a set of n nodes (named 1, 2, …, n) connected by an undirected graph G(V,E). Each node i maintains a local variable x_i, which is initialized to the input of node i. The nodes perform an iterative computation described here. The value of x_i after t iterations of the algorithm is denoted as $x_i[t]$. Thus, $x_i[0]$ equals the input of node i.

   In t-th iteration (or round) of the algorithm, each node obtains $x_j[t-1]$ from each neighbor j, that is, from each node j such that edge (i,j) is in E.

   In the following, assume that $a_{ij} = 0$ whenever edge (i,j) is not in E.

   In the t-th iteration, each node i updates its x variable as follows.

   $$x_i[t] = \sum_{i=1}^{n} a_{ji}\, x_j[t-1], \quad \text{for } t>0$$

   Observe that in the above equation $a_{ij} = 0$ whenever edge (i,j) is not in E. Thus, to perform the above computation node i does NOT need x_j if j is not i's neighbor.

   Now let us consider an n-by-n matrix A such that $A[i,j] = a_{ij}$

   The above algorithm ensures that asymptotically the x values at all the nodes converge to the average of the inputs at all the nodes, provided that the following conditions are true:

   - G(V,E) is a connected graph.
   - $a_{ij} >= 0$ for all i,j
   - The elements in each row of A add to 1.
   - The elements in each column of A add to 1.
   - $a_{ij} > 0$ if and only if (i,j) is in E.

   Suppose that V={1,2,3} and E={(1,2), (2,3)}.

   Suppose that the inputs at nodes 1, 2 and 3 are 4, 5, and 6, respectively.

   (a) Will matrix A below result in average consensus? Explain why.

$$A = \begin{bmatrix} 7/8 & 1/8 & 0 \\ 1/8 & 1/4 & 5/8 \\ 0 & 5/8 & 3/8 \end{bmatrix}$$

(b) Determine A^2 and A^4.

(c) Determine the values of variable x at the three nodes after 2 iterations and 4 iterations of the algorithm.
Hint: You should be able to use the answer from part (b).

(d) Should the x values at all the nodes add to 15 after t iterations, for any t? Explain why.

2. (20 points) Consider the memoization algorithm for determining the length of the longest common subsequence, as given on slides 26-27 of https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-introduction-to-algorithms-sma-5503-fall-2005/video-lectures/lecture-15-dynamic-programming-longest-common-subsequence/lec15.pdf

   Using amortized analysis, argue that its execution time is O(mn). You may find it convenient to apply the accounting method to the recursion tree in this case, observing that a constant amount of work is performed (on average) at each node of the recursion tree.

   Note: An informal argument will suffice.