COSC 240

Spring 2019

Problem Set 6

Due by class time on April 11, 2019

40 points

1. (20 points) Average consensus

Consider a set of n nodes (named 1, 2, …, n) connected by an undirected graph G(V,E). Each node i maintains a local variable xi, which is initialized to the input of node i. The nodes perform an iterative computation described here. The value of xi after t iterations of the algorithm is denoted as xi[t]. Thus, xi[0] equals the input of node i.  
  
In t-th iteration (or round) of the algorithm, each node obtains xj[t-1] from each neighbor j, that is, from each node j such that edge (i,j) is in E.

In the following, assume that aij = 0 whenever edge (i,j) is not in E.

In the t-th iteration, each node i updates its x variable as follows.

, for t>0

Observe that in the above equation aij = 0 whenever edge (i,j) is not in E. Thus, to perform the above computation node i does NOT need xj if j is not i’s neighbor.

Now let us consider an n-by-n matrix A such that A[i,j] = aij

The above algorithm ensures that asymptotically the x values at all the nodes converge to the average of the inputs at all the nodes, provided that the following conditions are true:

* G(V,E) is a connected graph.
* aij >= 0 for all i,j
* The elements in each row of A add to 1.
* The elements in each column of A add to 1.
* aij > 0 if and only if (i,j) is in E.

Suppose that V={1,2,3} and E={(1,2), (2,3)}.

Suppose that the inputs at nodes 1, 2 and 3 are 4, 5, and 6, respectively.

1. Will matrix A below result in average consensus? Explain why.

A =

1. Determine A2 and A4.
2. Determine the values of variable x at the three nodes after 2 iterations and 4 iterations of the algorithm.  
   Hint: You should be able to use the answer from part (b).
3. Should the x values at all the nodes add to 15 after t iterations, for any t? Explain why.
4. (20 points) Consider the memoization algorithm for determining the length of the longest common subsequence, as given on slides 26-27 of <https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-introduction-to-algorithms-sma-5503-fall-2005/video-lectures/lecture-15-dynamic-programming-longest-common-subsequence/lec15.pdf>

Using amortized analysis, argue that its execution time is O(mn). You may find it convenient to apply the accounting method to the recursion tree in this case, observing that a constant amount of work is performed (on average) at each node of the recursion tree.

Note: An informal argument will suffice.