COSC 240 Introduction to Algorithms
Problem Set 1 **Total points: 56**
Spring 2019

Assigned on 1/22/2019
Due at start of class on ~~1/29/2019~~ 1/31/2019 (please submit via Canvas)

Question 1. Prove the following statements using the definitions of asymptotic notations learned in class. (Identify constants such as *c*, n_0 so that the statements would hold):

a) (2 points) $2n + 8 = O(n^3)$

b) (2 points) $\frac{n}{100} = \Omega(\pi)$

c) (4 points) $3n^2 + 11n + 6 = \Theta(n^2)$

(Recall: to show f(n) = Θ(g(n)), you must show both that f(n) = O(g(n)) *and* that f (n) = Ω(g (n)).)

Questions 2, 3 and 4 use the recurrence below.

$$T(n) = \begin{cases} 3T\left(\frac{n}{4}\right) + n & if\ n \geq 4 \\ \Theta(1) & if\ n < 4 \end{cases}$$

Question 2. (8 points) Find a big-Θ bound for T(n) using the master method.
Note: Clearly identify ϵ. If case 3 applies, provide a constant c by which the regularity condition holds.

Question 3. (8 points) Find a big-Θ bound for T(n) using the substitution method.

Question 4. (8 points) Assuming that n is a power of 4, draw a recursion tree for the above recurrence, and label it with the following: (a) the number of levels; (b) number of nodes per level; and (c) cost of leaf level. *Please draw at least 1 full level of the recursion tree in addition to the root.*

Question 5. (8 points) Use a loop invariant to prove that the following subroutine returns the sum of the elements of array A, where n is the length of the array.

Sum(A, n)

```
i ← n
sum ← 0
while (i >= 1)
    sum ← sum + A[i]
```

```
        i ← i-1
    return sum
```

Question 6 (8 points)

Find the ~~big-Θ bound on~~ the expected execution time for the following algorithm called KeepRolling(). Use the number of times the While statement is executed as an estimate of the execution time.

$RollSixSidedDie()$ is a Θ(1) function that returns a random integer between 1 and 6. Assume that the function rolls a fair die, i.e., each integer between 1 and 6 (inclusive) has an equal probability to be the outcome of the function.

KeepRolling()

```
flag ← True
While (flag == True)
        p ← RollSixSidedDie()
        If (p == 6)
                flag ← False
```

Question 7 (8 points) Assume that in a particular system the array input A for insertion sort is "almost-sorted" in the following sense:

there exists at most one value of i (1<=i<=n-1) such that A[i]>A[i+1]

where n is the length of the array (the array is assumed to be indexed from 1 to n).

For such inputs, estimate big- Θ bound for worst-case execution time of insertion sort.